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EXPERIMENTAL STUDIES OF MASS TRANSPORT IN POROUS MEDIA WITH LOCAL HETEROGENEITIES

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ABSTRACT

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The effect of local heterogeneities in a porous medium, which are randomly distributed, upon longitudinal spreading in groundwater transport problems is described and related to the permeability ratio between the heterogeneities and the surrounding porous medium. Numerous experimental runs using simplified model aquifers provide a large data base. The measured breakthrough curves indicate the transport behavior of the system. They are compared with the solution of the classical Fickian advection-dispersion equation for the given boundary conditions as well as with a numerical approach of a modified transport formulation following the dual porosity concept of Coats and Smith. A significant difference in the transport behavior depending on the permeability of the heterogeneities is observed: while mass transport in the model aquifers with local heterogeneities of higher relative permeability remained Fickian, local heterogeneities of lower relative permeability caused a marked tailing and a significant spreading of the tracer. In the dual porosity formulation of transport the transfer coefficient had to be fitted, while the remaining transport parameter could be determined directly.

INTRODUCTION

Mass transport in groundwater is strongly influenced by the geological structure of the porous medium. Aquifer heterogeneities are often distinguished from each other by their individual size (Bear, 1972). This is shown schematically in Figure 1.

Numerous investigations have been carried out in order to describe mass transport in ideal homogeneous media (heterogeneities of first order) (Pfannkuch, 1963; for overview see Bear, 1972; Spitz, 1985). These data confirm a functional relationship between mean grain size and dispersivity of the solid matrix. The dispersive flux can be expressed by Fick's law and, if the range of

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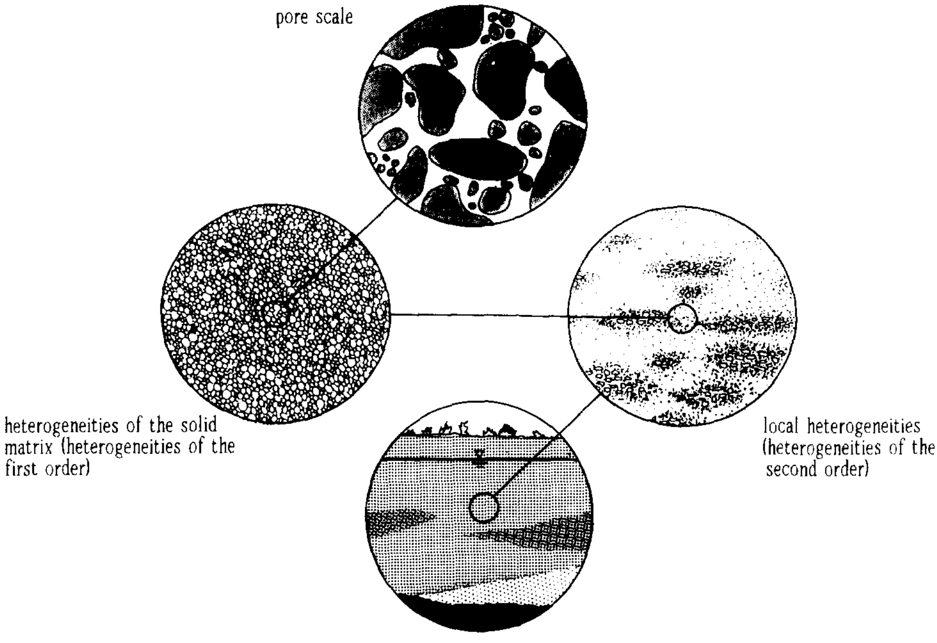

Fig. 1. Magnitude of heterogeneities in a natural aquifer (Spitz, 1985).

extremely small velocities is excluded, the dispersion coefficients can be described approximately by the product of the mean velocity and the corresponding dispersivity.

Considering a large-scale aquifer, each formulation of the transport behavior has to be taken with respect to the geological structure. The study of mass transport in heterogeneous materials is subject of intensive research (e.g. Villermaux and Swaaij, 1969; Renault et al., 1975; Dagan, 1982; Gueven et al., 1984; Sudicky, 1986; Gelhar, 1986). In light of the dispersion concept described by Fickian law, the velocity variation due to the heterogeneities results in an increase of the dispersion coefficient with travel length or with time (e.g., Mercado, 1967; Dagan, 1982; Gelhar and Axness, 1983). If no larger-scale heterogeneities are present, it is found that an asymptotical Fickian transport regime with constant dispersion coefficients may occur after a travel distance of approximatively 50 times of the horizontal extent of a typical heterogeneity (e.g., Frind et al., 1987).

In the present paper extensive systematical experiments on dispersion in heterogeneous porous media are presented. The purpose of the investigations is to improve the physical understanding of the effect of local aquifer heterogeneities (heterogeneities of second order, see Fig. 1) upon longitudinal spreading and to give qualitative results. The considerations assume that the heterogeneities are in the form of local lenses with permeabilities that differ from the surrounding aquifer.

Transport behavior was studied in different heterogeneous model aquifers with the main purpose being to determine the influence of permeability differences upon longitudinal dispersion. To isolate this effect, each individual model aquifer was built up with only two different kinds of porous media for each run. Porous ceramic cubes represented the heterogeneities while the aquifer was built up with homogeneous sand. Initially experiments following this method were performed by Herr (1985). Besides the investigations on the effect of change in permeability ratios, more studies were carried out focusing on the influence of geometry and flow velocity.

PHYSICAL MODEL AND BREAKTHROUGH CURVES

A systematic experimental investigation on an elementary flow condition, as shown in Figure 2, has been conducted. The chosen flow configuration resembles the classical configuration for experiments on longitudinal dispersion (Harleman and Rumer, 1963). The experiments were performed using salt ($c_0 < 1\,\mathrm{g\,L^{-1}}$) as tracer. A step function was chosen as the concentration boundary at the model inlet. The measurement of the breakthrough curves was performed directly at the model outlet. The cylindrical laboratory tank having a length of 1 m and a diameter of 0.1 m was filled with a mixture of a uniform sand and porous ceramic cubes. The permeability and the dispersivity of the model sands as well as the permeability of the ceramic materials were tested in seperate test series. The mixture of both media represents in an abstract manner a heterogeneous aquifer where the heterogeneities are randomly distributed. Such random distributions of local heterogeneities may be found in

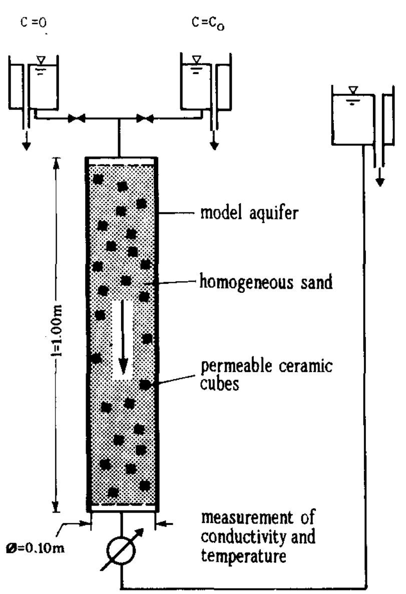

Fig. 2. Flow configuration.

heterogeneous sands or, on a larger scale, in an aquifer with local fine sand or clay lenses.

Table 1 presents the characteristics of the sands and ceramic cubes used. It contains also an overview of all test aquifers described by the relationship of k_h/k_s and θ_h/θ_s, where θ_s is the pore volume of the sand per total volume of the laboratory tank while θ_h is the total pore volume of the ceramic heterogeneities in comparison to the total tank volume. k_s is the hydraulic conductivity of the sand, k_h is that of the heterogeneities (in metres per day). The porosity of the ceramic cubes were constant (circa 0.4). Impermeable heterogeneities ($k_h/k_s = 0$) was represented by PVC-cubes (see Table 1). To meet the objectives of the investigations, the ratio of the permeabilities k_h/k_s was changed in the experimental runs. In additional experiments, ceramic plates (80 × 50 × 10 mm) replaced the cubes (Schäfer, 1987). The cube sizes were equal for each individual model aquifer and varied from 15 mm to 18 mm (run 1 and runs 3–9, Table 1) and from 21 mm to 23 mm for runs 10–14. Each model aquifer contained approximately 200 cubes. For the experiments (runs 10–14) the model sand used as filling differed from the one for runs 1–9 (see Table 1).

Figure 3 shows the measured breakthrough curves for the experimental series, where the permeability of the local heterogeneities is higher than the permeability of the surrounding sand. The experimental results in Figure 4 are obtained for the reverse relationship. Both figures are dimensionless. Measured concentrations are normalized to the initial concentration c_0, while time refers to the average travel time calculated by dividing the model length l with the average pore velocity.

The sand with highly permeable heterogeneities shows a marked increase of the mixing zone at the tracer front by increasing the relationship k_h/k_s. A

TABLE 1

Characteristics of the model aquifer

characteristics of the uniform sand	mean grain diameter $d_m = 5.7\ 10^{-4}$ [m]; k-value $k = 1.5\ 10^{-3}$ [m/s]; longitudinal dispersivity $A_L = 0.0006$ [m]									$d_m = 2.8\ 10^{-4}$ [m]; $k = 2.2\ 10^{-4}$ [m/s]; $A_L = 0.0003$ [m]				
	porous media containing less permeable heterogeneities									porous media containing more permeable heterogeneities				
run	1	2	3	4	5	6	7	8	9	10	11	12	13	14
geometry of ceramic heterogeneities	cubes*	plates	cubes	cubes	cubes	cubes	cubes	cubes	cubes	cubes	cubes	plates	cubes	cubes
ratio of permeabilities k_h/k_s	0.0	0.013	0.013	0.053	0.053	0.2	0.33	0.33	0.66	1.33	2.5	2.5	3.0	4.0
ratio of volumes θ_h	0.0	0.049	0.074	0.094	0.073	0.059	0.112	0.073	0.072	0.059	0.059	0.051	0.059	0.059
θ_s	0.332	0.331	0.315	0.304	0.324	0.320	0.273	0.311	0.327	0.334	0.334	0.342	0.334	0.334

* PVC-cubes

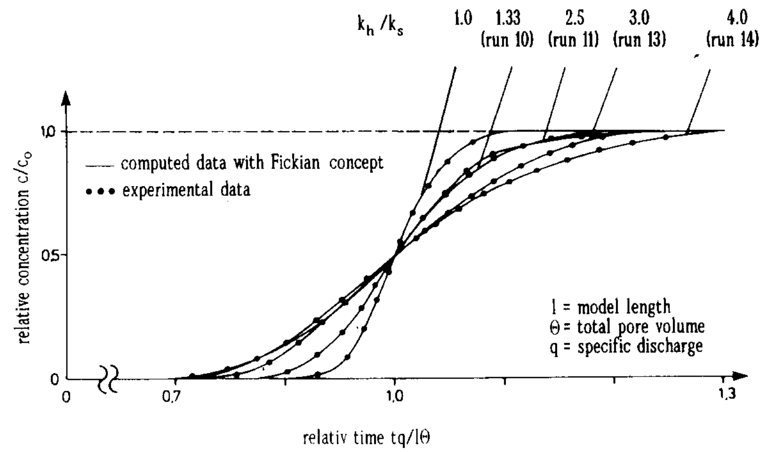

Fig. 3. Breakthrough curves in the model aquifers where the permeability of the local heterogeneities is higher than that of the surrounding aquifer.

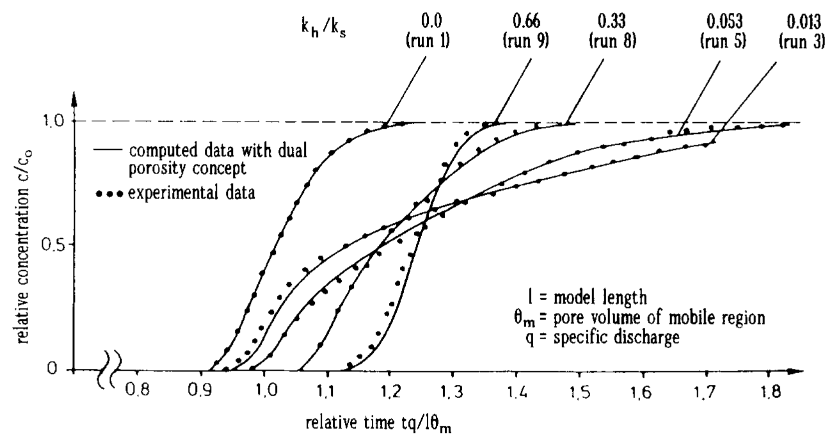

Fig. 4. Breakthrough curves in the model aquifers where the permeability of the local heterogeneities is lower than that of the surrounding aquifer.

change in the velocities did not influence the dimensionless breakthrough curves in the experiments. The measured concentration curves for these model configurations have a strong resemblance to breakthrough curves which can be described with the classical dispersion concept.

The curves for model aquifers, where the heterogeneities have a lower permeability than the model sand, illustrate a completely different characteristic (Fig. 4). The heterogeneities cause a concentration tailing which cannot be described with the classical dispersion concept. The tailing seems to be more marked with decreasing values of k_h/k_s. Such differences in transport behavior in heterogeneous aquifers where the permeability of the heterogeneities are either higher or lower than the surrounding aquifer are also described theoretically by Spitz (1985).

SELECTION OF THE MATHEMATICAL MODEL

The transport of a single solute through a porous medium, considering only a non-interactive solute, is usually analyzed by means of the classical advection-dispersion equation. A general form of this equation is given for example by Bachmat and Bear (1964). Regarding only the one dimensional transport case in a steady-flow regime, the equation can be written as:

$$\frac{\partial c}{\partial t} = D_{\mathrm{L}} \frac{\partial^2 c}{\partial x^2} - \frac{q}{\theta} \frac{\partial c}{\partial x} \tag{1}$$

where $c(x, t)$ is the concentration of the solute, t is time, x is the space coordinate, q is the flux, D_{L} is the longitudinal dispersion coefficient and θ is the water content. In this equation D_{L} represents the spreading effect caused by the porous medium.

The description of mass transport in a heterogeneous aquifer using the Fickian dispersion concept is inaccurate if transport occurs at a small scale or if the permeability distribution tends to be bimodal. A number of mathematical concepts are developed to improve the simulation of dispersive transport in these configurations. The principal idea of most of these concepts is to split the porous medium into two regions with different hydraulic characteristics (Deans, 1963; Coats and Smith, 1964; Skopp and Warrick, 1974; Tang et al., 1981). Each region represents a characteristic part of the aquifer. The transport in a region is described by an individual transport equation which is coupled with a transfer condition along the interface between adjacent regions.

The simplest approach is to assume that there is a mobile and an immobile zone (Coats and Smith, 1964) (Fig. 5). It is further assumed that the flux between the two zones is proportional to the difference in concentration. The concentration in the mobile zone is in instantaneous equilibrium so that the one-dimensional transport equation can be expressed now as:

$$\frac{\partial c_{\mathrm{m}}}{\partial t} + \frac{\theta_{\mathrm{im}}}{\theta_{\mathrm{m}}} \frac{\partial c_{\mathrm{im}}}{\partial t} = D_{\mathrm{L}} \frac{\partial^2 c_{\mathrm{m}}}{\partial x^2} - \frac{q}{\theta_{\mathrm{m}}} \frac{\partial c_{\mathrm{m}}}{\partial x} \tag{2}$$

$$\frac{\partial c_{\mathrm{im}}}{\partial t} = \frac{\alpha}{\theta_{\mathrm{im}}} (c_{\mathrm{m}} - c_{\mathrm{im}}) \tag{3}$$

where c_{m} and c_{im} are the concentrations of the solute in the mobile and immobile zone, θ_{m} and θ_{im} are the water contents in these zones and α is the mass transfer coefficient (in day^{-1}).

Equation (1) as well as eqns. (2) and (3) are used in the present study to calculate the experimental breakthrough curves. For the discussion of the experimental results, eqn. (1) can be solved analytically using the initial and

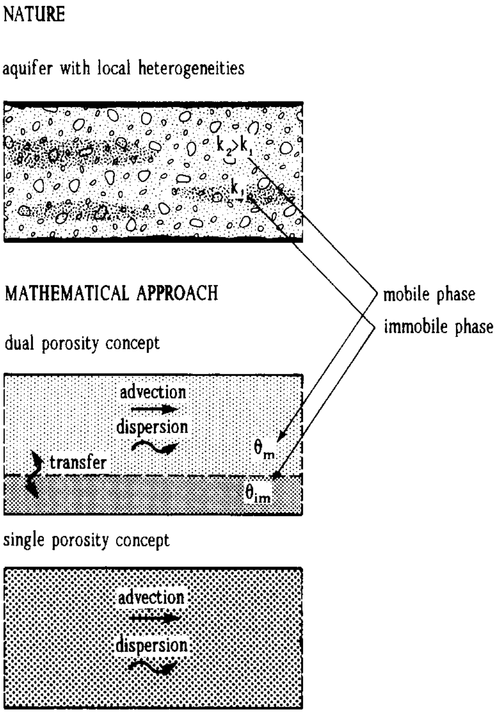

Fig. 5. Mathematical approaches of the heterogeneous porous media.

boundary conditions given by the experimental setup. Since the range of the Peclet number in the experiments was in the order of 2000, the analytical solution of Ogata and Banks (1961) approximately satisfies the physical boundary conditions (van Genuchten and Parker, 1984):

$$\frac{c}{c_0} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x - (q/\theta)t}{\sqrt{2D_\mathrm{L}t}}\right) + \frac{1}{2}\exp\frac{(q/\theta)l}{D_\mathrm{L}}\,\mathrm{erfc}\left(\frac{x + (q/\theta)t}{\sqrt{2D_\mathrm{L}t}}\right) \tag{4}$$

Due to the high column Peclet numbers in the experiments, the second part of eqn. (4) is negligible (Bear, 1972).

Van Genuchten and Wierenga (1976) presented an analytical solution for eqns. (2) and (3) for the given boundary conditions. In our study however, we solved the equations numerically with a finite difference scheme. The reason for this preference was that the analytical solution becomes unhandy for high Peclet numbers. For the one-dimensional case, the equations can be very easily and accurately solved numerically.

EFFECTS OF LOCAL HETEROGENEITIES

The experiments were carried out in a porous medium where the total volume of the heterogeneities is small in comparison to the total aquifer volume. Therefore, the experimental studies allow the following assumptions related to the transport coefficients:

In runs with a permeability of the heterogeneities greater than the corresponding value of the surrounding sand ($k_h/k_s > 1$), the sand part of the aquifer having the lower permeability, cannot be considered as an immobile zone. This is also true for the local, more permeable heterogeneities. In the view of a dual porosity concept, no immobile region exists in these model aquifers. Setting θ_{im} equal to zero, the transport equations (2) and (3) are simplified to the classic advection dispersion equation (1). In eqn. (1) only the dispersion coefficient expresses the dispersivity of the heterogeneous model aquifers. D_L must be fitted for each measured curve which is shown in Figure 3. The analytical curves using the Fickian dispersion concept can be seen to describe the experimental values extremely well. The investigations seem to show a marked increase of D_L with increasing values of k_h/k_s. The dispersivity of the model aquifer grows to 5-times that of the dispersivity of the uniform sand used in the experiments, when the ceramic cubes become 4 times more permeable than sand.

Considering the experimental values for all runs which exhibit a relationship k_h/k_s less than 1 (heterogeneities with low permeability), no realistic curves could be fitted with eqn. (4). Here the observed tailing can hardly be described with the Fickian dispersion concept. This is shown in Figure 4. However, as Figure 4 illustrates, the experimental curves can be approximated in a qualified manner with the present dual porosity concept (equations 2 and 3) even though we find in reality a bimodal velocity distribution within the porous medium. It is evident, that when we use the dual porosity concept, θ_{im}/θ_m is equal to the ratio of the volumes of each porous medium in the model aquifer (θ_h/θ_s). The dispersion coefficient D_L is fixed by the dispersivity of the sand which is also well known. Therefore, in these studies the transfer coefficient α is the only parameter which must be fitted. For some characteristic runs the transfer coefficient is given in Table 2. Note that while the Darcy velocity for each experiment remains approximately constant, the transfer coefficient α decreases rapidly with decreasing values of the permeability of the heterogeneities.

TABLE 2

Fitted transfer coefficient (α) in experiments with low-permeability heterogeneities ($k_h/k_s < 1$)

run	3	5	6	8	9
ratio of permeabilities k_h/k_s	0.013	0.053	0.2	0.33	0.66
Darcy velocity in m/min	0.0138	0.0143	0.0125	0.0144	0.0146
transfer coefficient α in min^{-1}	0.017	0.032	0.15	0.252	2.0

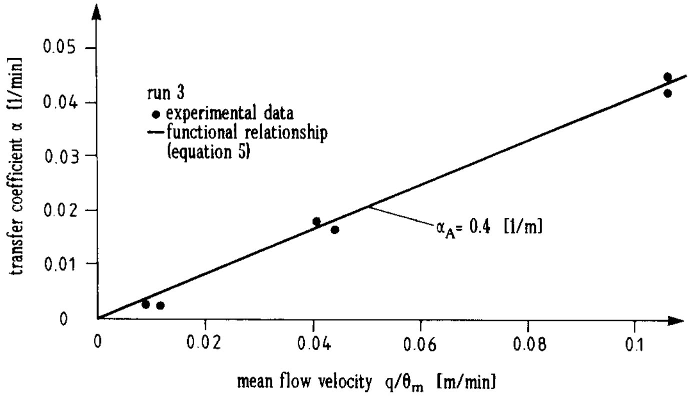

Fig. 6. Dependence of the transfer coefficient (α) on the mean flow velocity (Herr, 1985).

The experimental studies with different velocities indicate a simple functional dependence of α on the mean flow velocity as shown in Figure 6 for run 3:

$$\alpha = \alpha_A q/\theta_m \tag{5}$$

where α_A is a parameter that depends only on the aquifer properties. q expresses the influence of velocity. Such a linear relationship is also described by Brissaud, et al. (1978). Note that the experiments were carried out within a range of velocities where the effect of diffusion could be excluded. In flow regimes with small velocities the influence of diffusion must be considered.

CONCLUSIONS AND SUMMARY

The present investigations yield a number of conclusions with regard to the effect of local heterogeneities in an aquifer upon longitudinal mixing:

(a) Longitudinal dispersive transport can be described well with the Fickian concept when local heterogeneities with high permeability occur. An increase in the k-values of the heterogeneities causes an increase in the dispersion coefficient.

(b) Laboratory experiments with model aquifers where the permeability of the heterogeneities is less than the value of the surrounding aquifer show a strong tailing in the concentrations. Here the Fickian dispersion concept failed to describe the experimental values.

(c) The effect of the less permeable heterogeneities on the longitudinal mixing seems to be more pronounced and increases with decreasing values of k_h/k_s.

(d) The dual porosity concept introduced by Coats and Smith (1964) describes the observed breakthrough curves for $k_h/k_s < 1$ extremely well. In these experiments only the transfer coefficient α had to be fitted while all other transport parameters were well known.

(e) In the experiments with various velocities, the transfer coefficient α must be written as a linear product of the flow velocity and a constant aquifer parameter in order to fit the curves for each velocity.

(f) The investigations indicate that transport in a porous medium with a bimodal distribution of hydraulic conductivities can be described with the dual porosity concept in a qualitative manner.

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