# A Flow Bench for Measuring Fan Curves and System Curves for Air-Cooled Electronic Equipment

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#### Abstract

The design and operation of a flow bench for measuring system curves and fan curves for air-cooled electronic equipment is described. The flow bench is capable of controlling and measuring the flow rate of air up to  $2.8 \text{ m}^3/\text{s}$  (100 CFM). Step-by-step procedures for measuring fan curves and system curves are documented. Detailed calculations for data reduction are presented.

# 1 Introduction

A flow bench is a piece of laboratory equipment used to determine the *fan curve* of a fan or the *system curve* of an enclosure that has air moving through it. A flow bench might also be used to provide a controllable and measurable flow of air through a piece of electronic equipment undergoing thermal testing.

The flow bench described in this article was designed and constructed as part of a senior design project during the Winter and Spring of 1999 [4]. It was constructed in accordance with the joint ANSI/AMCA/ASME<sup>1</sup> Standard 210-99 [1]. In the remainder of this document the joint 210-99 standard will simply be referred to as *standard 210-99*.

The flow bench is capable of measuring flow rates between 0 and  $2.83 \text{ m}^3/\text{min}$  (0 and 100 CFM) and pressure drops up to 7.6 cm H<sub>2</sub>O(3 in H<sub>2</sub>O). This operating range is suitable for small electronic enclosures such as desktop computers, projectors, and small laboratory test equipment. The design could easily be scaled up to test equipment with larger ranges of flow rate and pressure drop.

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<sup>&</sup>lt;sup>1</sup>ANSI is the American National Standards Institute. AMCA is the Air Movement and Control Association. ASME is the American Society of Mechanical Engineers. AMCA Standard 210-99can be purchased from www.amca.org.



Figure 1: Schematic of flow bench configured for measurement of the system curve of a an electronic enclosure, the Device Under Test (DUT).

# 2 Apparatus

Figure 1 is a schematic of the flow bench with an Device Under Test (DUT) attached to the inlet. The primary components of the bench are a blower, a nozzle wall, plenums, and a blast gate (damper) for adjusting the flow rate.

### 2.1 Principle of Operation

The blower draws air through the flow bench, and in so doing, overcomes the flow resistance of the DUT, the flow bench ducting, and the metering nozzle(s). The nozzles are used to measure the volumetric flow rate of the air. The blower operates at constant speed and the blast gate is opened and closed to control the flow through the system.

If the DUT is an enclosure, i.e. if the purpose of the experiment is to measure the system curve of the enclosure, the blower provides the flow through the flow bench and the DUT. The system curve is obtained by performing a range of measurements at different flow rates to obtain

$$\Delta p_{\rm sys} = f(Q) \tag{1}$$

where  $\Delta p_{\rm sys} = p_{\rm atm} - p_1$  is the pressure drop necessary to overcome the impedance of the DUT,  $p_{\rm atm}$  is the local ambient pressure,  $p_1$  is the pressure in the upstream plenum, and Q is the volumetric flow rate. For a given flow rate through the DUT (and therefore the flow bench),  $\Delta p_{\rm sys}$  is a characteristic of the system impedance.

If the DUT is a fan, i.e. if the purpose of the experiment is to measure the fan curve, the fan pushes air into the flow bench thereby creating an elevated pressure in the first plenum. The purpose of the flow bench blower in a fan curve experiment is to maintain the pressure in the first plenum to some desired level

by drawing air through the flow bench. The fan curve is obtained by performing a range of measurements at different flow rates to obtain

$$\Delta p_{\rm fan} = f(Q) \tag{2}$$

where  $\Delta p_{\text{fan}} = p_1 - p_{\text{atm}}$  is the pressure *rise* provided by the fan. Note that when the DUT is a fan,  $p_1 > p_{\text{atm}}$ , and when the DUT is an electronic enclosure,  $p_{\text{atm}} > p_1$ .

For both fan curve and system curve experiments, the flow rate, Q, is determined by measuring the pressure drop across the nozzle wall. The nozzle wall has three, ASME standard, long radius nozzles with nominal diameters of 0.75, 1.0, and 1.6 in (1.91, 2.54, 4.06 cm). Usually only one nozzle is used at a time, although simultaneous use of two or three nozzles in parallel is allowed by standard 210-99. The inactive nozzles are sealed with plugs that are inserted manually via an access door on the downstream plenum.

To compute the flow rate through the flow bench one needs to measure  $\Delta p_n$ , the pressure drop across one (or more) of the flow nozzles,  $p_1$ , the pressure in the upstream plenum, and  $T_1$ , the temperature of the air in the upstream plenum. Note that  $p_1$  needs to be an absolute pressure. In this flow bench a differential pressure gage is used to measure  $p_1 - p_{\text{atm}}$  (or  $p_{\text{atm}} - p_1$ ) and the pressure in the laboratory is measured with a barometer. In addition to the fluid property values one must know the diameter of the nozzle throat and a correlation for the flow coefficient of the nozzle. The data reduction procedure is described in §4, below.

### 2.2 Instrumentation

#### Upstream Plenum Pressure

The pressure in the upstream plenum,  $p_1$ , is needed for both the fan curve measurement and system curve measurement. For system curve measurement,  $p_1 < p_{\text{atm}}$ . For fan curve measurement  $p_1 > p_{\text{atm}}$ . The value of  $p_1$  is also needed to compute the density of the air flowing through the flow nozzles.

A ring of four pressure taps, one on the center of each of the four plenum walls, is used to sense  $p_1$ . All four taps in the ring are at the same axial position (same location along the direction of flow) of the flow bench. The four taps are connected by a common tube which is connected to the pressure sensors. This configuration results in a measurement of the average of the pressures sensed at the four tap locations. The value of  $p_1$  is measured with an inclined manometer (Dwyer, Model 202.5) and with a capacitance type electronic sensor (Omega, Model PX653-2.5BD5V). These redundant pressure sensors allow for manual reading and automated data acquisition of the pressure values.

#### Nozzle Pressure Drop

Two pairs of pressure tap rings (four taps per ring) are used to sense  $\Delta p_n$ , the pressure drop across the nozzle wall. One set of four taps is located just



Figure 2: End view of plenum cross section showing location of pressure taps.

upstream of the nozzle wall in the upstream plenum<sup>2</sup>. Another set of four taps is located just downstream of the nozzle wall in the downstream plenum. The value of  $\Delta p_n$  is measured with an diaphragm type dial gage (Dwyer Magnehelic, Model 2003C) and a capacitance type electronic sensor (Omega, Model PX653-10D5V).

#### Plenum Air Temperature

The temperature in the upstream plenum is measured with 24 gage type T thermocouples. One thermocouple is suspended upstream of each flow nozzle in the space downstream from the settling screens. (See Figure 1.) The user must choose the appropriate thermocouple for the nozzle used during any one flow rate measurement.

## 2.3 Nozzle Selection

Long radius flow nozzles manufactured by Helander Metal Spinning<sup>3</sup> are used as primary flow elements. The nozzles are manufactured to the specifications in standard 210-99, and are not independently calibrated. Three nozzles are installed in the nozzle wall of the flow bench (cf. Figure 1).

Each nozzle is suitable for measurements over a limited range of flow rates. Table 1 gives the ranges of flow rates for each nozzle, and for combinations of nozzles. Below the minimum flow rate (for a particular nozzle) the pressure drop across the nozzle is too small to be measured accurately by the pressure sensors. Above the maximum flow rate (for a particular nozzle) the pressure

 $<sup>^{2}</sup>$ The distance from the nozzle wall to the upstream and downstream pressure taps is 3.81 cm(1.5 in) in accordance with standard 210-99.

<sup>&</sup>lt;sup>3</sup>Addresses and phone numbers of component manufacturers are supplied in the Appendix.

Single Nozzle Operation:

flow rate CFM	nozzle diameter (inch)
9-19	0.75
15 - 35	1
39-96	1.6

#### Multiple Nozzle Operation:

flow rate CFM	nozzle diameters (inch)
24-40	$0.75~\mathrm{and}~1$
48-71	0.75  and  1.6
63 - 150	0.75, 1, and 1.6

Table 1: Recommended nozzle selection as a function of flow rate.

drop is greater than the maximum allowable pressure for the pressure sensor<sup>4</sup>. In addition, as the flow rate through a given nozzle increases, the velocity in the nozzle throat may become large enough for compressibility effects to become significant.

# 3 Experimental Procedure

# 3.1 Attachment of DUT

The inlet of the upstream plenum is a 12 in (30.48 cm) square opening. An adapter plate is used to attach a fan or an electronic enclosure to this opening. The adapter plate can be any convenient sheet material (sheet metal, plywood, press board, cardboard) with sufficient structural integrity to maintain a seal during the test. The adapter plate must be large enough to cover the opening in the upstream plenum. In the center of the adapter plate should be a hole of the same size and shape as the exhaust hole of the DUT.

It is very important to provide leak-proof joints between the DUT and the adapter plate, and between the adapter plate and the upstream plenum. The easiest way to seal the joints is with duct tape. For a fan test the adapter plate can be held onto the upstream plenum with duct tape. For a system curve measurement, the DUT should be supported on a stand or blocks, before the joints are sealed with duct tape.

 $<sup>^4 \</sup>rm For \ each \ nozzle, the limiting pressure sensor is the Magnehelic gage, which has a maximum pressure difference of 3 inches of water.$ 

## **3.2** Data Collection

The following steps describe the procedure for both fan curve and system curve measurements. The instructions assume that the DUT is already attached to the upstream plenum.

- 1. Fill the ice bath with a mixture of (mostly) ice, and water. Make sure the spaces between the ice are filled with water. Insert the thermocouple reference junction in the ice bath.
- 2. Record ambient pressure and temperature in the laboratory.
- 3. Attach the tubing from the pressure taps on the upstream plenum to the appropriate side of the inclined manometer
  - System Curve Measurement: Connect the  $p_1$  pressure tap to the *low* pressure side of the inclined manometer.
  - Fan Curve measurement: Connect the  $p_1$  pressure tap to the *high* pressure side of the inclined manometer.
- 4. Using Table 1, select a nozzle appropriate for the range of flow rates to be measured. Open the downstream hatch. Remove the plug from the desired nozzle, and check that all other nozzles are plugged.
- 5. Adjust the blast gate to a reasonable position (half open is good), and turn on the blower. Turn on the power supply to the pressure transducers
- 6. Wait for the pressure and temperature values to settle, indicating that the flow bench is operating in steady state
- 7. Record thermocouple voltage, and the two pressure readings. Be sure to read the voltage of the thermocouple upstream of the *active* nozzle(s) as well as the voltage of the reference junction.
- 8. Prepare for the next data point by moving the blast gate to a new position. If the flow rate to be measured is out of range for the current flow nozzle, turn off the blower and return to step 4. If the current nozzle is still appropriate for the desired flow rate, return to step 6.
- 9. At the end of the experiment, record ambient pressure and temperature in the laboratory.

# 4 Data Reduction

The final result of a flow bench experiment is either a system curve or a fan curve for the DUT. To obtain these curves the pressure and temperature measurements for the nozzles must be converted to flow rates. The pressure transducer and thermocouple conversion involve standard practices and will not be discussed here.

### 4.1 Flow Rate

The flow rate through the flow bench is measured by long radius flow nozzles built to the specifications in standard 210-99. The volumetric flow rate through the nozzle is (see, e.g. [6])

$$Q = C_d Y A_n \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \tag{3}$$

where  $C_d$  is the nozzle discharge coefficient, Y is the expansion factor (see below),  $A_n = (\pi/4)d_t^2$  is the area of the nozzle throat,  $d_t$  is the throat diameter,  $\Delta p$  is the measured pressure drop across the nozzle,  $\rho$  is the fluid density upstream of the nozzle,  $\beta = d_t/D$  is the contraction ratio, and D is the diameter of the upstream duct.

The expansion factor accounts for the compressibility of the air as it flow through the nozzle. For typical ranges of operation of the flow bench,  $Y \sim 0.99$ or greater. The formula for the expansion factor is [1]

$$Y = \left[\frac{\gamma}{\gamma - 1} \alpha^{2/\gamma} \frac{1 - \alpha^{(\gamma - 1)/\gamma}}{1 - \alpha}\right]^{1/2} \left[\frac{1 - \beta^4}{1 - \beta^4 \alpha^{2/\gamma}}\right]^{1/2} \tag{4}$$

where  $\gamma$  is the ratio of specific heats (taken as 1.4 for air), and  $\alpha$  is the ratio of the nozzle exit pressure to the pressure upstream of the nozzle.

$$\alpha = \frac{p - \Delta p}{p} \tag{5}$$

Note that the  $1 - \beta^4$  term can safely be neglected in the current application. Take *D* to be the hydraulic diameter of the upstream plenum, which has a square cross section 24 in (60.96 cm) on a side. For the largest nozzle in the flow bench,  $d_t = 1.6$  in giving  $\beta = 0.0667$  and  $1 - \beta^4 = 0.999980$ .

Ideally, each of the nozzles would be calibrated. The result would be a unique relationship between  $C_d$  and nozzle Reynolds number

$$\operatorname{Re}_{t} = \frac{V_{t}d_{t}}{\nu} = \frac{4Q}{\pi d_{t}\nu} \tag{6}$$

where  $V_t = Q/A_n$  is the velocity in the throat,  $\rho$  is the fluid density, and  $\mu$  is the fluid viscosity evaluated at the pressure and temperature upstream of the



Figure 3: Discharge coefficient for a standard, long radius nozzle.

nozzle. The nozzles in the flow bench are not calibrated, but they have the geometry of flow nozzles specified by standard 210-99. The equation relating  $C_d$  to  $\operatorname{Re}_t$  for standard nozzles is [1]

$$C_d = 0.9986 - \frac{7.006}{\sqrt{\text{Re}_t}} + \frac{134.6}{\text{Re}_t} \tag{7}$$

Figure 3 is a plot of equation (7).

Calculation of Q requires a value of  $C_d$ , but calculation of  $C_d$  requires a value of Q. Use of equation (3) through equation (7) involves the following iterative procedure.

For each measured  $\Delta p$ :

- 1. Compute and store  $K_Q = A_n \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}}$
- 2. Guess a value of  $C_d$  (say  $C_d = 0.9$ )
- 3. Compute  $Q = C_d K_Q$
- 4. Compute  $Re_t$  from equation (6)
- 5. Compute  $C_d$  from equation (7)
- 6. If the new  $C_d$  is "close enough" to the old  $C_d$ , stop. Otherwise, return to step (3)

The preceding algorithm is implemented in the MATLAB function called nozzleFlow, which is listed in Listing 1 in Appendix B. The expansion factor Y and the viscosity of air are computed with utility functions also listed in Appendix B.

### 4.2 Curve Fit for System Curve

Figure 4 depicts a DUT attached to the front of the flow bench as appropriate for measurement of a system curve. The incompressible, steady flow energy equation for a fluid system is (see, e.g. [6])

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_1 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_2 + h_{f,\text{tot}} - h_p \tag{8}$$

where p is the static pressure,  $\rho$  is the fluid density, g is the acceleration of gravity, V is the average velocity at a cross-section, z is the elevation above a common datum,  $h_{f,\text{tot}}$  is the total head loss,  $h_p$  is the head supplied by a fan (or pump), and stations 1 and 2 are at upstream and downstream positions in the flow.

Applying equation (8) to stations 1 and 2 in Figure 4 gives

$$h_{f, ext{tot}} = rac{p_{ ext{atm}} - p_{ ext{plenum}}}{
ho g} - rac{V_2^2}{2g}$$

During the experiment to measure the system curve the fan is either removed or unpowered, so  $h_p = 0$ . Since the cross sectional area of the plenum is large compared to the fan opening in the DUT,  $V_2 \approx 0$  so that the preceding equation reduces to<sup>5</sup>

$$h_{f,\text{tot}} = \frac{p_{\text{atm}} - p_{\text{plenum}}}{\rho g}$$

Thus, the pressure difference between the ambient and the first plenum is a measure of the head loss for the DUT.

The head loss in the DUT is related to the average velocity through the DUT by the standard equation for a minor loss element.

$$h_{f,\text{tot}} = K_1 \frac{V^2}{2g}$$

where V is the average velocity in the DUT and  $K_1$  is the minor loss coefficient for the DUT. In general, the geometry of the flow passages inside the DUT are very complicated. This makes definition of an average velocity problematic. Instead of the preceding equation, we use

$$h_{f,\text{tot}} = K_2 Q^2 \tag{9}$$

where  $K_2$  is another constant,  $K_2 = K_1/(2gA_{\text{effective}}^2)$ , and Q is the volumetric flow rate through the DUT. The flow rate is more useful than V because Q is a

<sup>&</sup>lt;sup>5</sup>When the DUT is operating normally, i.e. when the flow through it is induced by a fan inside the DUT, there will be an additional loss  $V_e^2/2g$  due to the loss of kinetic energy leaving the fan.  $V_e = Q/A_{\text{exhaust}}$  is the velocity of fluid leaving the DUT when the fan is operating.



Figure 4: Configuration of a DUT for measurement of a system curve. Stations 1 and 2 should not be confused with  $p_1$  and  $p_2$  in Figure 1. Here stations 1 and 2 are used to temporarily designate the upstream and downstream conditions for flow through the DUT.

performance parameter of fans, and Q is measured by the flow bench. Since we directly measure pressure difference instead of head loss, it is more convenient to multiply equation (9) by  $\rho g$  to get

$$\Delta p_{\rm sys} = cQ^2 \tag{10}$$

where  $\Delta p_{\text{sys}} = p_{\text{atm}} - p_{\text{plenum}}$  is the pressure difference across the system depicted in Figure 4.

Equation (10) is the system curve for a DUT. The flow bench is designed to measure  $\Delta p_{\rm sys}$  and Q. The value of c that characterizes a particular DUT can be obtained from a least squares curve fit of  $\Delta p_{\rm sys}$  versus Q. Many different types of analysis software (e.g. spreadsheets) have built-in routines for obtaining polynomial curve fits of a set of data. Thus, it is easy to obtain a curve fit of the form

$$\Delta p = c_1 + c_2 Q + c_3 Q^3 \tag{11}$$

Unfortunately this *easy* curve fit usually gives nonzero values of  $c_1$  and  $c_2$ . A nonzero  $c_1$  implies that there is a pressure drop across the DUT when the flow is zero — a physical impossibility. A nonzero  $c_2$  is not so problematic, but it does not fit the simple form of the system curve in equation (10). As an example, Figure 5 shows data for two system curves fit to equation (10) and equation (11). The curve fits to equation (11) have a nonzero intercept at Q = 0. In addition, as shown in Figure 5, attempting a curve fit with a degree two polynomial may result in a fit equation that predicts a negative  $\Delta p_{\rm sys}$  for a positive flow rate.

Rather than use curve-fitting software that adds a constant term and linear term to equation (10), it is desirable to fit equation (10) directly. It is not hard to show that the value of c that obtains the least squares fit to equation (10) is

$$c = \frac{\sum_{i=1}^{n} Q_i^2 \Delta p_{\text{sys},i}}{\sum_{i=1}^{n} Q_i^4}$$
(12)

where  $(\Delta p_{\text{sys},i}, Q_i)$  is a single data point from the system curve measurements, and there are *n* data points in total.

# References

- Air Movement and Control Association. Laboratory Methods of Testing Fans for Aerodynamic Performance Rating, ANSI/AMCA Standard 210-99. Air Movement and Control Association International, Inc, Arlington Heights, IL, 2000.
- [2] American Society for Testing and Materials. A Manual on the Use of Thermocouples in Temperature Measurement. ASTM, Philadelphia, New York, fourth edition, 1993.
- [3] G. Burns, M. Scroger, and G. Strouse. Temperature-electromotive force reference functions and tables for the letter-designated thermocouple types based on the its-90. NIST Monograph 175, National Institute for Standards and Technology, Washington, DC, 1993.
- [4] T. Landolt, M. Ma, K. Nguyen, and J. E. Wright. Flow bench design project. Technical report, Portland State University, Department of Mechanical Engineering, Portland, Oregon, Jun 1999.
- [5] R. J. Moffat. Uncertainty analysis. In K. Azar, editor, *Thermal Measure*ments in Electronic Cooling, pages 45–80. CRC Press, Boca Raton, FL, 1997.
- [6] F. M. White. *Fluid Mechanics*. McGraw-Hill, New York, fourth edition, 1999.



Figure 5: Curve fits of system curve data to quadratic polynomial and to equation (10).

# A Appendix

# A.1 Suppliers of Primary Components

### Flow Nozzles

Helander Metal Spinning Company 630-268-9292 (voice) 708-268-9393 (FAX) www.helandermetal.com

### Sheet Metal Fabrication

Arjae Sheet Metal Company, Inc. Portland, OR 503-231-7717

#### **Pressure Measurement**

Dwyer Instruments 101 Highway 212 P.O. Box 373 Michigan City, IN 46361 (219) 879-8000 www.dwyer-inst.com

Omega Engineering, Inc. One Omega Drive P.O. Box 4047 Stamford, CT 06907-0047 (800) 826-6342 www.omega.com

### **Temperature Measurement**

Omega Engineering, Inc. (see Pressure Measurement, above)

# **B** MATLAB **Programs**

```
function Q = nozzleFlow(d,D,dp,p,T)
% nozzleFlow Volumetric flow rate of air through a long radius nozzle.
%
% Synopsis: Q = nozzleFlow(d,D,dp)
              Q = nozzleFlow(d,D,dp,p)
%
%
             Q = nozzleFlow(d,D,dp,p,T)
%
% Input: d = throat diameter, (m)
%
         D = upstream pipe (or plenum) diameter (m)
%
         dp = pressure drop across nozzle, (Pa)
         p = (optional) pressure upstream of nozzle; Default: p = 101325 (Pa)
T = (optional) temperature upstream of nozzle; Default: T = 20 (C)
%
%
%
% Output: Q = volumetric flow rate, (m^3/s)
if nargin<4, p = 101325; end \% standard atmosphere, (Pa) if nargin<5, T = 20; end \% degrees C
if dp<eps || d<eps, Q=0; return; end % allows for no-flow case on fan curves
rho = p/(287*(T+273.15)); % kinematic viscosity
bbta = d/h.
% --- Evaluate fluid properties and other constants
                               % air density from ideal gas law
bbeta = d/D;
y = expansionFactor(p,dp,bbeta,1.4);
area = 0.25*pi*d^2;
qcon = area*y*sqrt(2*dp/(rho*(1-bbeta<sup>4</sup>))); rcon = rho*d/(area*mu);
% --- Initialize and loop until cd converges
tol = 5e-6; it = 0; maxit = 25; cdold = 0; cd = 0.9;
while abs(cdold-cd)>tol && it<maxit
 cdold = cd;
  Q = cd*qcon;
 Re = rcon*Q;
 cd = 0.9986 - 7.006/sqrt(Re) + 134.6/Re;
 it = it + 1;
end
if it>=maxit
 warning(sprintf('cd computations not converged after %d iterations',it));
end
```

Listing 1: The nozzleFlow function computes the flow rate through a long radius nozzle using the standard equation for the nozzle discharge coefficient, equation (7). See Equation (3).

```
function mu = airViscosity(T,units)
% airViscosity Dynamic viscosity of air
%
% Synopsis: mu = airViscosity
%
              mu = airViscosity(T)
%
              mu = airViscosity(T,units)
%
% Input: T = (optional) temperature (deg. C) at which properties are
               evaluated. Default: T = 20C. If units='BG' then T is in deg. F
%
%
%
           units = (optional) string to indicate the system of units
                    units = 'SI' for System International (kg-m-s) units,
                    units = 'BG' for British Gravitational (lbm-ft-s) units
%
% Output: mu = dynamic viscosity, kg/(m*s) = Pa*s or lbm/(ft*s)
%
% Ref: Incropera and Dewitt, Introduction to Heat Transfer, 3rd ed., Table A.4
                     \%\, return viscosity at 20 C without evaluating curve fit
if nargin<1
 mu = 1.81316e-05;
 return
end
if nargin==1, units='SI'; end
if nargin>1 & isempty(T), T = 20; end
if units=='BG'
 Tin = (T-32)*5/9; % convert deg F to deg C
else
 Tin = T;
end
if Tin<-23.15 | Tin > 726.85
 error(sprintf('T = %f (deg C) is out of range for viscosity curve fit',Tin));
end
\% --- Store polynomial curve fit coefficients
pm = [ 1.183165303908385553e-14 -2.838455962871034222e-11 ...
4.921892043466875411e-08 1.715849963309301011e-05 ];
\% --- Evaluate the polynomials
mu = polyval(pm,Tin);
if units=='BG', mu = mu*0.67197;
                                      end
```

Listing 2: The **airViscosity** function computes the viscosity of air as a function of temperature.

```
function y = expansionFactor(p,dp,bbeta,k)
% expansionFactor Accounts for compressibility of air in throat
%
% Synopsis: y = expansionFactor(p,dp,bbeta,k)
%
% Input: p = pressure upstream of the nozzle
% dp = pressure drop across the nozzle
% bbeta = ratio of nozzle diameter to plenum diameter
% k = specific heat ratio for the gas
%
% Output: y = expansion factor appearing in equation for flow rate through
long radius nozzle
alpha = (p-dp)/p; % ratio of nozzle exit pressure to approach pressure
t1 = (k-1)/k;
ak = alpha^(2/k);
t2 = (ak/t1) * (1 - alpha^t1)/(1-alpha);
b4 = bbeta^4;
t3 = (1-b4)/(1-b4*ak);
y = sqrt(t2*t3);
```

Listing 3: The expansionFactor function computes a correction to the flow rate due to compressibility. See Equation (4)