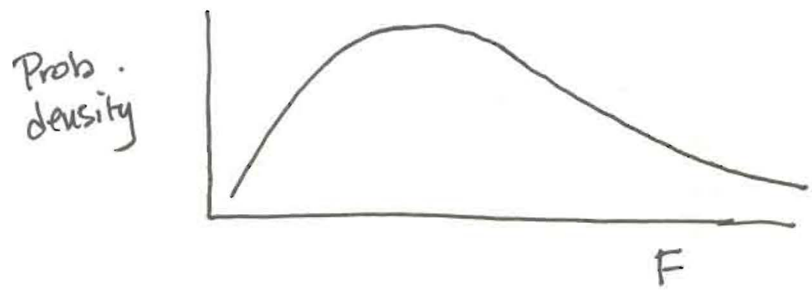
F test for comparing variances

[Textbook pp. 422-425]

$$F = \frac{\sigma_1^2}{\sigma_2^2}$$

R. A. Fischer

see Table A.7



$P = f(\nu_1, \nu_2)$

ν_1 = d.o.f. in numerator σ_1^2
ν_2 = d.o.f. in denominator σ_2^2

WARNING: F test is not robust

results of the F-test are sensitive to non-normality in the underlying populations

t test is robust

Basic Procedure

(1) $H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

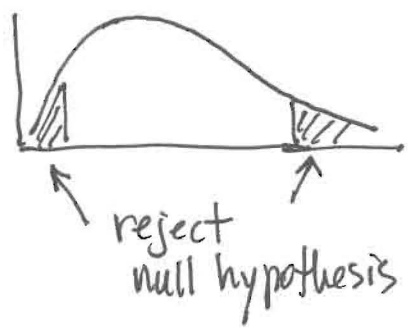
(2) Choose significance level, α

(3) Obtain samples

n_1 from population 1

n_2 from population 2

(4) Identify region of rejection



(5) compute F

(6) compare F to upper and lower critical F values

Determining critical values

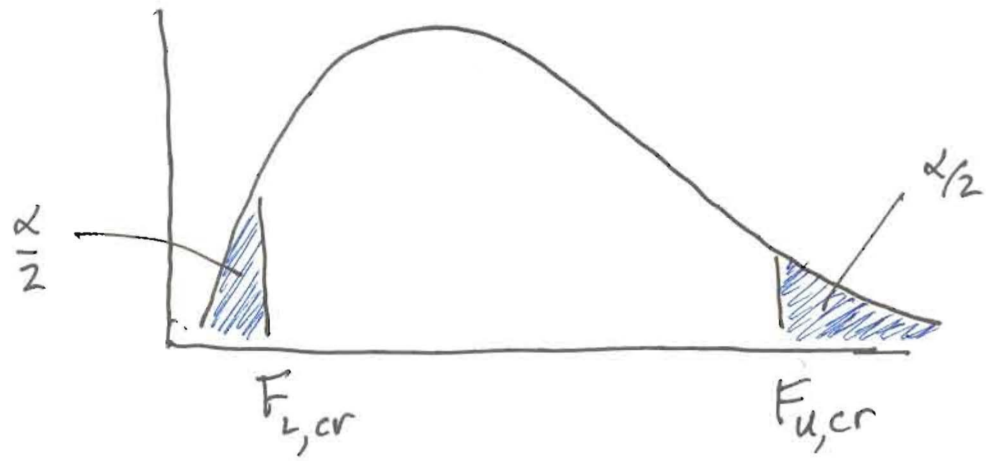


Table A-7 gives values of $F_{u,cr}$ @ α

Mathematical property of F:

$$F_L(\alpha, \nu_1, \nu_2) = \frac{1}{F_u(\alpha, \nu_2, \nu_1)}$$

switch order

Example: Non-destructive evaluation
from textbook. Data in crack.txt
Description on p. 392
Compare means on pp. 415-417
Compare variances on pp. 423-425

Summary Data from Table 9.5 p. 416

No Flaw	Flawed	crack.txt
$n_1 = 18$	$n_2 = 40$	
$\bar{x}_1 = 0.0359$	$\bar{x}_2 = 0.0946$ inches	
$s_1^2 = 0.00481$	$s_2^2 = 0.007059$	
$s_1 = 0.0218$	$s_2 = 0.0840$	

Is there evidence of significant difference between σ_1^2 and σ_2^2?

Before doing the calculations... Why?

Do we really care about the variability of the crack size?

For this data the value of the F test is in helping us know which t test to use.

- pooled variance t test $(\sigma_1^2 = \sigma_2^2)$

or

- separate variance t test $(\sigma_1^2 \neq \sigma_2^2$

On with the calculations

(1) $H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$ } two-tailed test ⇒ two regions of rejection, each contains $\frac{\alpha}{2}$ of the distribution

(2) Choose significance level

$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025 \Rightarrow$ use Table A.7, p A-12

(3) Obtain samples – data are given

$$n_1 = 18 \quad \Rightarrow \quad \nu_1 = 18-1 = 17$$
$$n_2 = 40 \quad \Rightarrow \quad \nu_2 = 40-1 = 39$$

(4) $\nu_1 = 17$ and $\nu_2 = 39$ are not in Table A.7

⇒ use $\nu_1 = 15$ and $\nu_2 = 40$ as close enough for hand calculation of $F_{u,cr}$

later we'll compare with results from Excel & MINITAB

From Table A.7 $F(\underset{\alpha/2}{0.025}, \underset{\nu_1 \text{ approx}}{15}, \underset{\nu_2 \text{ approx}}{40}) = 2.18$

Use $F_{L,cr}(\alpha, \nu_1, \nu_2) = \dfrac{1}{F_{u,cr}(\alpha, \nu_2, \nu_1)}$

Note that the table has ν_2 in finer increments

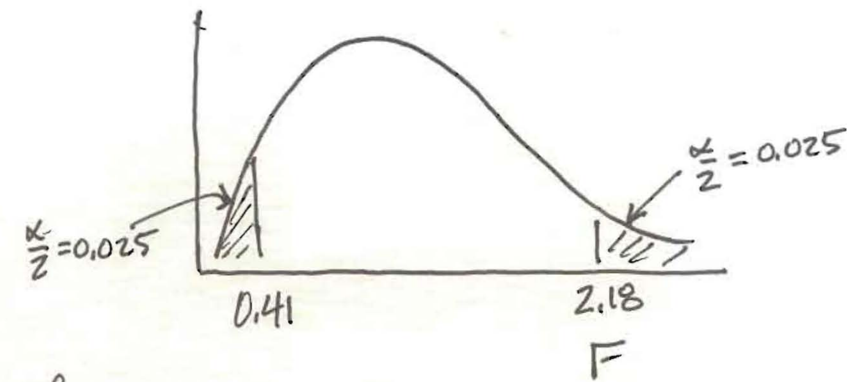
$F_u(0.025, 40, 17) = 2.44$ no need to use $\nu_1 = 15$ here

$\therefore F_{L,cr}(0.025, 17, 40) = \dfrac{1}{2.44} = 0.4095 = 0.41$

So... from the tables

$F_{u,cr} = 2.18$

$F_{L,cr} = 0.41$



Compare with values from
software with ability to compute
F for and α, ν_1, and ν_2, e.g. Excel's FINV function

$$\mathrm{FINV}(0.025, 17, 39) = 2.139$$

$$\mathrm{FINV}(0.025, 39, 17) = 0.4087$$

(5) Compute F from the sample

$$F_{obs} = \frac{0.000481}{0.007059} = 0.068$$

$\therefore F_{obs} < F_{L,cr}$ $(0.068 < 0.41)$

The observed F is outside the region of acceptance

$\therefore$ we have statistical evidence to reject the null hypothesis

$\Rightarrow$ There is evidence that $\sigma_1^2 \neq \sigma_2^2$ and we should use the t test for unequal variances.

Alternative approach using p-values.

(1) $H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

(2) Choose signifance level

(3) Obtain samples

(4) Compute F_{obs} from samples

(5) Use software (or very tedious manual look-up and interpolation) to find the

p-value = probability that F_{obs} could occur for normally distributed random variable

If p-value $< \frac{\alpha}{2}$ reject the null hypothesis

F-test example from Levine, Ramsey and Smidt

pp. 392, 415-417, 423 - 424

Problem description on p. 392
Test of means on pp. 415-417
Test of variances on pp. 423-424

Summary data from Table 9.5 on page 416

	No Flaw (1)	Flawed (2)
n	18	40
Xbar	0.0359	0.0946
variance	0.000481	0.007059
std. dev	0.0218	0.084

F test with approximate degrees of freedom

nu1	15	approximates nu1 = 17
nu2	40	approximates nu2 = 39
alpha	0.05	significance of F-test
FU_crit	2.1819	= FINV(alpha/2, nu1_approx, nu2_approx)
FL_crit	0.3868	= 1 / FINV(alpha/2, nu2_approx, nu1_approx)
FL_crit_alt	0.4095	= 1 / FINV(alpha/2, 40, 17)

F test with exact degrees of freedom

nu1	17	
nu2	39	
alpha	0.05	significance of F-test
FU_crit	2.1388	= FINV(alpha/2, nu1_, nu2_)
FL_crit	0.4087	= 1 / FINV(alpha/2, nu2_, nu1_)