Volumetric Flow Rate Measurement

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Overview

1. Motivation
   - System curves and loss coefficients
   - Fan curves
   - Control of flow for heat transfer experiments

2. Methods of flow rate measurement

3. Flow bench
   - Primary components and principle of operation
   - Fan curve measurement
   - Loss coefficient measurement

Steady Flow Energy Equation

Steady flow energy equation

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_1 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_2 + h_{\text{loss}} - h_{\text{fan}} \tag{1}$$

where

p	is the static pressure,
ρ	is the fluid density,
V	is the average velocity at a cross-section,
z	is the elevation relative to a datum,
h_{loss}	is the head loss due to friction, kinetic energy dissipation, etc.,
h_{fan}	is head gain from a fan (pump)

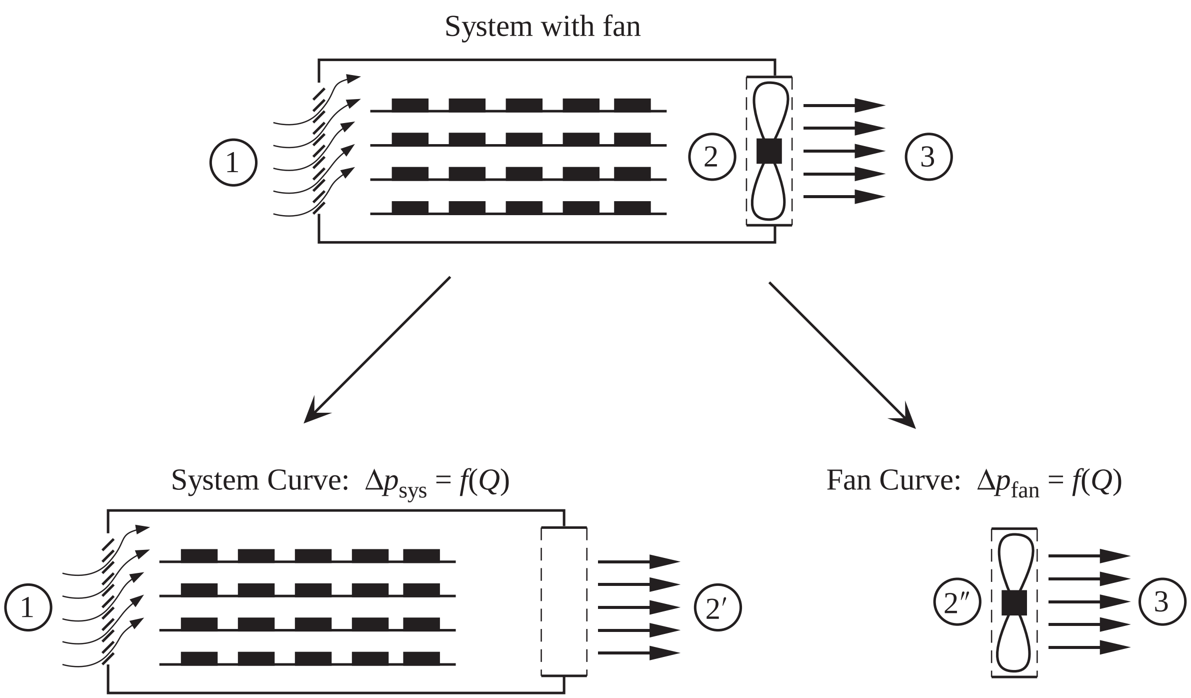
Steady Flow Energy Equation

For air-cooled systems, gravitational effects are negligible, so the energy equation can be rewritten as

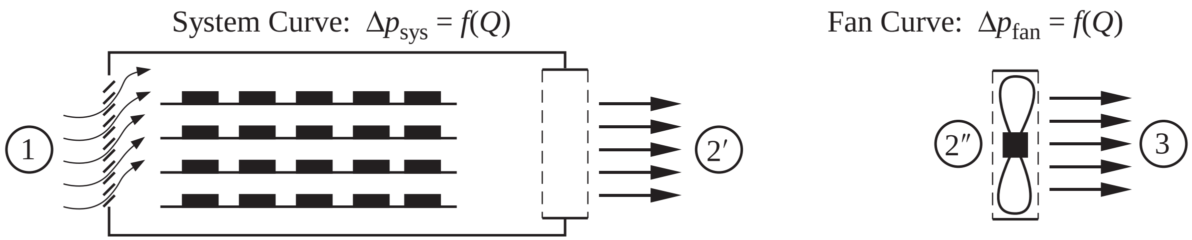
$$\frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} = h_{\text{loss}} - h_{\text{fan}} \tag{2}$$

System Curve (1)

To define the system curve for an electronics enclosure, we need to separate the fan from the system.



System Curve (2)



On the left, station $2'$ is the exit pressure necessary to draw the a given flow rate through the system when the fan is not powered. On the right, station $2''$ is the upstream condition for the fan during a fan test, i.e., when the fan is not connected to a system.

System Curve (3)

Apply energy equation between stations 1 and 3:

$$p_1 = p_3 \quad \Longrightarrow \quad h_{\mathrm{loss}} = h_{\mathrm{fan}}$$

In words: The head input by the fan is matched by the overall head loss for the system.

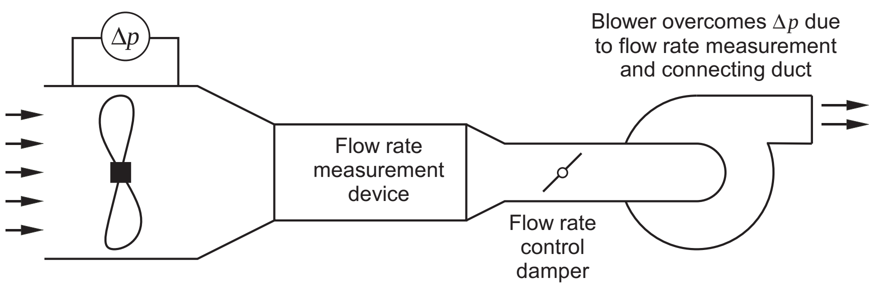
Apply energy equation between stations 1 and $2'$

$$\frac{p_1 - p_{2'}}{\rho g} = h_{\mathrm{sys}} \tag{3}$$

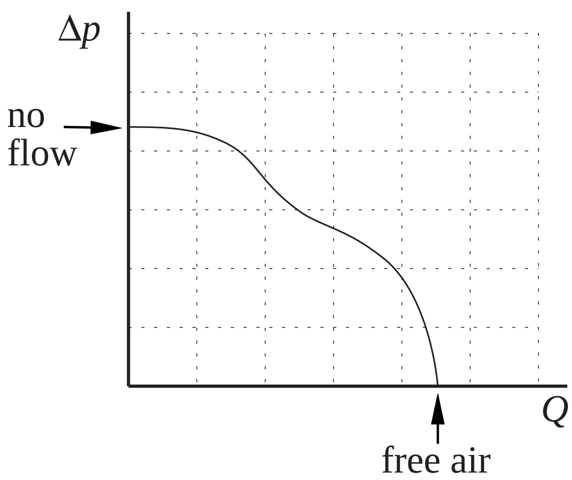
This *defines* the head loss for the system

Fan Curve (1)

Conceptual measurement apparatus:



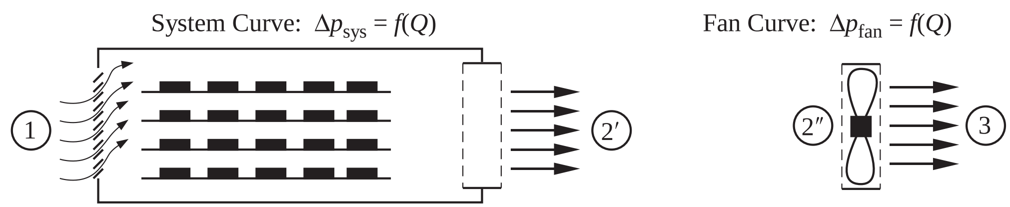
Typical result (a fan curve):



Δp is the *pressure rise* across the fan.

Q is the volumetric flow rate through the fan

Fan Curve (2)



Apply steady flow energy equation between stations $2''$ and 3:

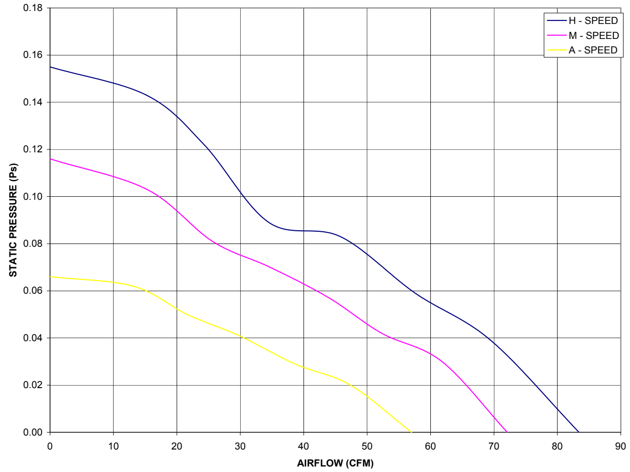
$$p_3 - p_{2''} = \rho g h_{\text{fan}}$$

The pressure *rise* across the fan corresponds to a gain in head. The head gain is matched by head losses elsewhere in the system.

Fan Curve (3)

For vendor-specific information see (as of April 2006)

`http://www.comairrotron.com/engineering_notes.asp`



Methods of Flow Rate Measurement

Measurement of fan curves and system curves requires

- Control of the flow rate
- Measurement of the flow rate
- Measurement of Δp across the device under test (DUT)

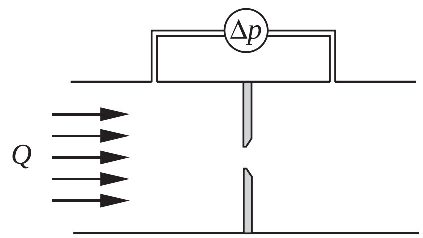
Measuring the Δp across the DUT is relatively simple.

Methods of flow rate measurement

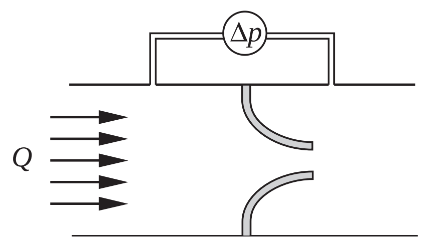
- Velocity profile measurement
- Laminar flow meters
- Rotameters
- Turbine flow meters
- Obstruction flow meters

Obstruction Flow Meters

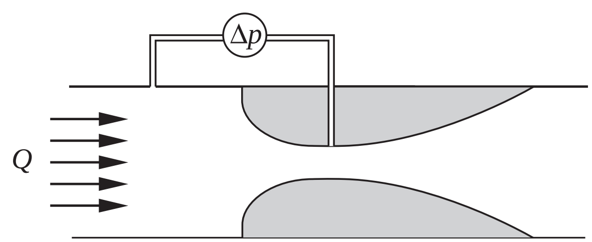
Sharp-Edged
Orifice



Long Radius
Nozzle

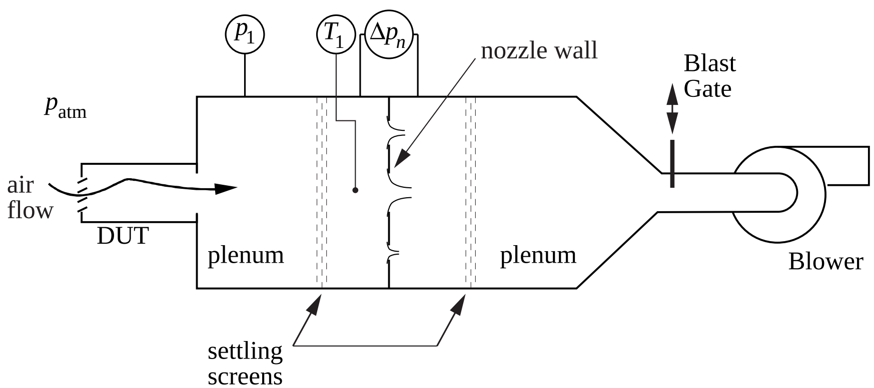


Venturi



Flow Bench

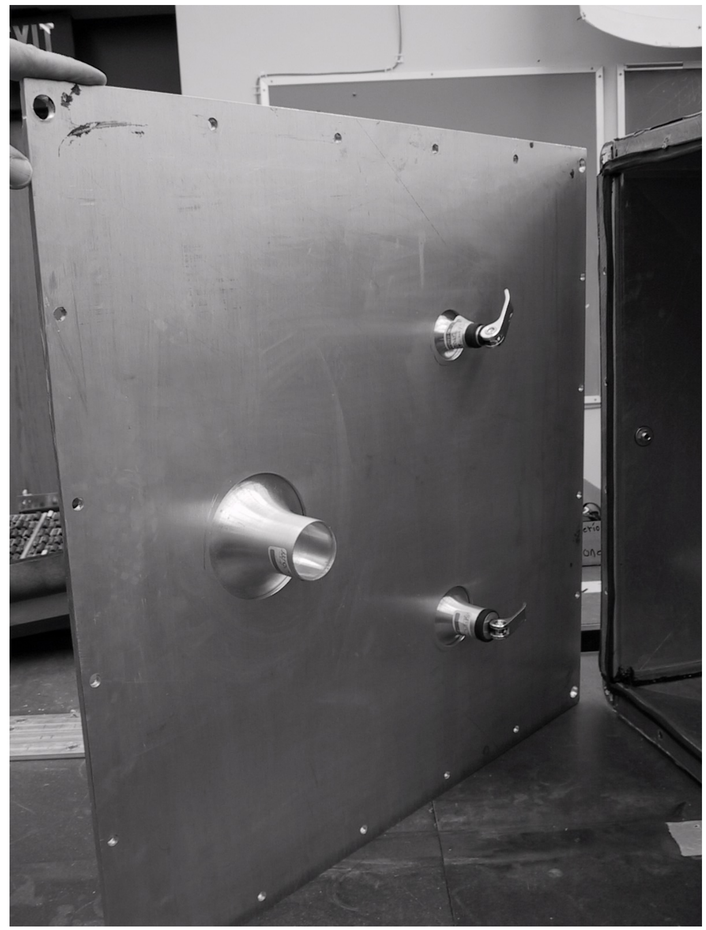
A flow bench is a device for providing a controlled and measurable flow rate to or from a device under test (DUT).



- Two plenums
- Nozzle wall
- Flow control damper (blast gate)
- Blower
- Instrumentation

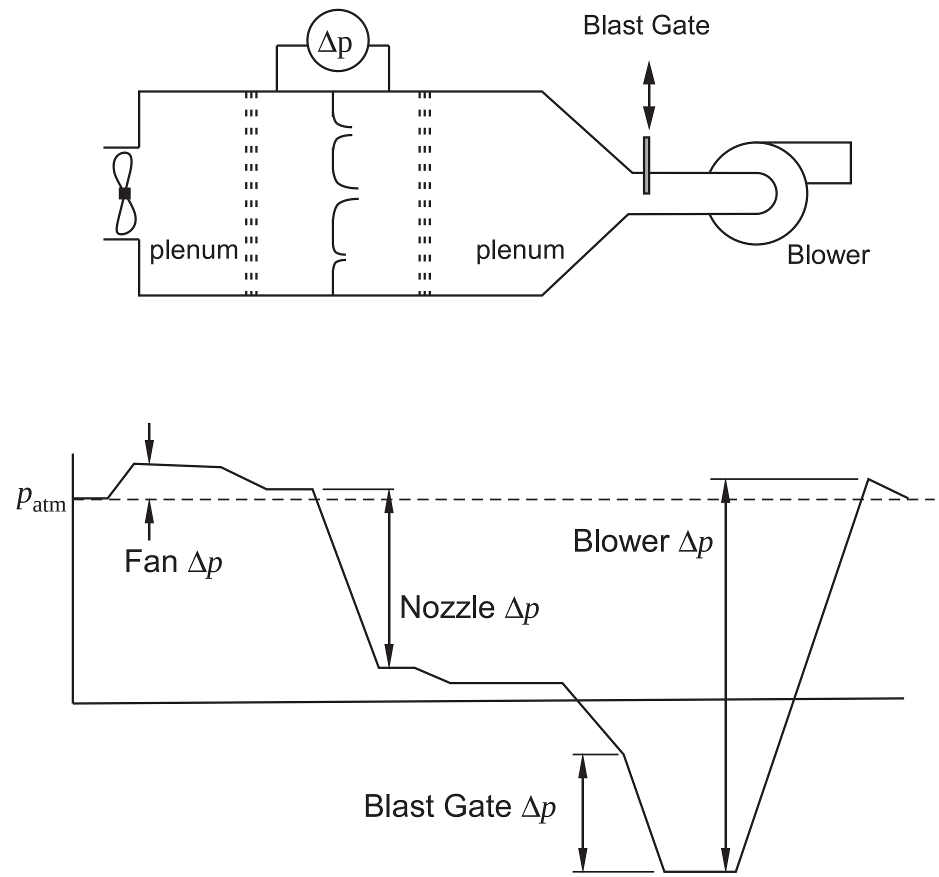
Nozzle Wall

Three nozzles are mounted in an aluminum sheet that separates the two plenums.
Using custom made rubber stoppers, the nozzles can be operated one at a time, or in parallel combinations.



Role of the Blower

- Blower overcomes Δp due to pressure losses
- Largest pressure drops are due to nozzles and blast gate



Control of Flow Rate

During fan curve measurement

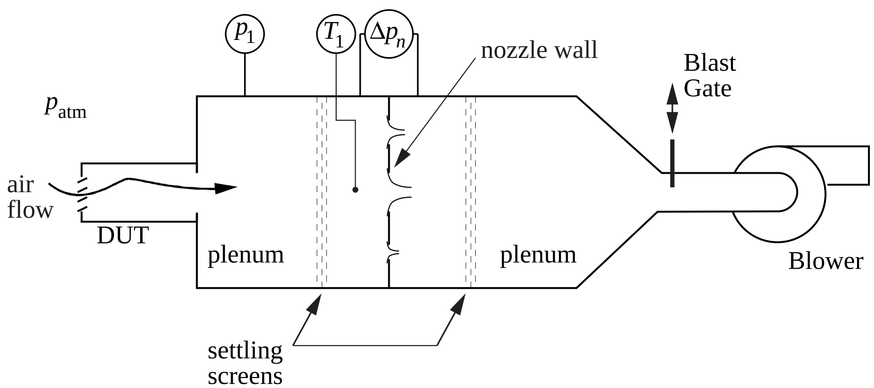
- Blast gate controls system flow rate.
- Flow rate determines fan flow rate.
- At a given flow rate, the fan produces a fixed pressure rise.

During system curve measurement

- Blast gate controls system flow rate.
- Flow rate determines pressure drop through DUT.

System Curve or Loss Coefficient Measurement

Locate the Device Under Test (DUT) at inlet of flow bench



System curve:

$$\Delta p_{sys} = p_{atm} - p_1 = f(Q) \quad (4)$$

Δp_{sys} is pressure drop necessary to overcome flow resistance of DUT,

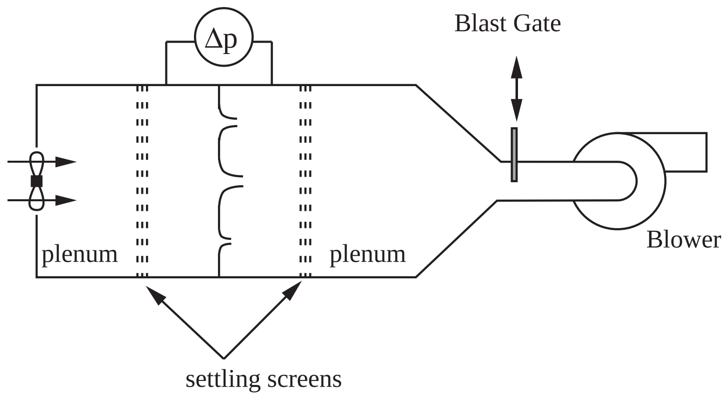
p_{atm} is local ambient pressure,

p_1 is pressure in upstream plenum,

Q is volumetric flow rate measured by nozzle(s).

Fan Curve Measurement

Locate the fan at inlet of flow bench



Fan curve:

$$\Delta p_{\mathrm{fan}} = p_1 - p_{\mathrm{atm}} = f(Q) \quad (5)$$

Δp_{fan}	is pressure *rise* provided by fan,
p_{atm}	is local ambient pressure,
p_1	is pressure in upstream plenum,
Q	is volumetric flow rate measured by nozzle(s).

Note that when the DUT is a fan, $p_1 > p_{\mathrm{atm}}$, and when the DUT is an electronic enclosure, $p_{\mathrm{atm}} > p_1$.

Flow Bench Instrumentation

To compute flow rate, measure

- Pressure in plenum 1
- Temperature in plenum 1
- Pressure drop across the nozzle

To characterize the DUT, measure the pressure drop between ambient and plenum 1. In practice we measure

p_{atm}	with barometer
T_1	with thermocouple upstream of nozzle
$p_{\mathrm{atm}} - p_1$	with inclined manometer or pressure transducer
$p_1 - p_2$	with manometer, pressure gage, or pressure transducer

Data Reduction (1)

Compute

$$p_1 = p_{\mathrm{atm}} - (p_{\mathrm{atm}} - p_1)$$

$$Q = C_d A_n Y \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \tag{6}$$

C_d	is the nozzle discharge coefficient,
A_n	is the area of the nozzle throat,
Y	expansion factor to account for compressibility,
d_t	is the throat diameter,
Δp	is the measured pressure drop across the nozzle,
ρ	is the fluid density upstream of the nozzle
$\beta = d_t/D$	is the contraction ratio,
D	is the diameter of the upstream duct.

Data Reduction (2)

The nozzles are built to ASME/ANSI specification, but are not individually calibrated. Use the generic equation for the discharge coefficient

$$C_d = 0.9986 - \frac{7.006}{\sqrt{\mathrm{Re}_t}} + \frac{134.6}{\mathrm{Re}_t} \tag{7}$$

$$\mathrm{Re}_t = \frac{V_t d_t}{\nu} = \frac{4Q}{\pi d_t \nu} \tag{8}$$

$V_t = Q/A_n$	is the velocity in the throat,
ρ	is the fluid density,
μ	is the fluid viscosity evaluated at the pressure and temperature upstream of the nozzle.

Data Reduction (3)

The analytical expression for the expansion factor is

$$Y = \left[\frac{\gamma}{\gamma - 1}\alpha^{2/\gamma}\frac{1 - \alpha^{(\gamma-1)/\gamma}}{1 - \alpha}\right]^{1/2} \left[\frac{1 - \beta^4}{1 - \beta^4\alpha^{2/\gamma}}\right]^{1/2} \tag{9}$$

where $\gamma = c_p/c_v$ and

$$\alpha = \frac{p - \Delta p}{p}$$

$0 \leq Y \leq 1$. As $\Delta p \rightarrow 0$, $Y \rightarrow 1$.

Data Reduction (4)

An iterative procedure is required to compute Q for each measured Δp:

Initialize:

Compute and store $K_Q = A_n Y \sqrt{\dfrac{2\Delta p}{\rho(1-\beta^4)}}$

Guess a value of C_d, say $C_d = 0.98$

Iterate:

1. Compute $Q = C_d K_Q$
2. Compute Re_t from equation (8)
3. Compute C_d from equation (7)
4. If the new C_d is "close enough" to the old C_d, stop.
   Otherwise, return to step (1)

Data Reduction (5)

```
function Q = nozzleFlow(d,D,dp,p,T)
% nozzleFlow  Volumetric flow rate of air through a long radius nozzle.

% --- Evaluate fluid properties and other constants
mu = airViscosity(T);        %  kinematic viscosity
rho = p/(287*(T+273.15));    %  air density from ideal gas law
bbeta = d/D;
y = expansionFactor(p,dp,bbeta,1.4);
area = 0.25*pi*d^2;
qcon = area*y*sqrt(2*dp/(rho*(1-bbeta^4)));  rcon = rho*d/(area*mu);

% --- Initialize and loop until cd converges
tol = 5e-6;  it = 0;  maxit = 25;   cdold = 0;  cd = 0.9;
while abs(cdold-cd)>tol && it<maxit
  cdold = cd;
  Q = cd*qcon;
  Re = rcon*Q;
  cd = 0.9986 - 7.006/sqrt(Re) + 134.6/Re;
  it = it + 1;
end
if it>=maxit, error('No convergence after %d iterations',it);  end
```

Least Squares Fit to System Curve (1)

The energy equation for the system with an unpowered fan is

$$\frac{\Delta p_{\mathrm{sys}}}{\rho g} = h_{\mathrm{loss}} \tag{10}$$

Recall that for pipe systems, minor losses are represented by

$$h_{\mathrm{minor}} = K\frac{V^2}{2g}$$

where K is the so-called minor loss coefficient.

By analogy we assume that the loss through the system will also vary as the square of the average velocity.

$$h_{\mathrm{loss}} = K\frac{V^2}{2g} = K\frac{Q^2}{2gA^2} \tag{11}$$

where A is the effective cross-sectional area of the system.

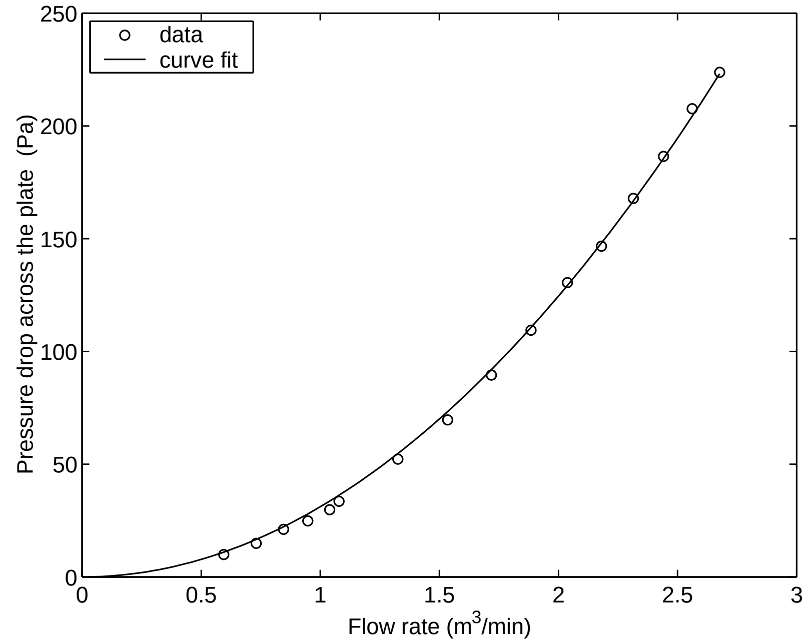
Least Squares Fit to System Curve (2)

Substitute $\Delta p_{\text{sys}} = \rho g h_{\text{loss}}$ from Equation (10) into Equation (11)

$$\Delta p_{\text{sys}} = CQ^2 \qquad (12)$$

where $C = \rho K/(2A^2)$ is assumed to be a constant for the system.

Typical loss coefficient data



Least Squares Fit to System Curve (3)

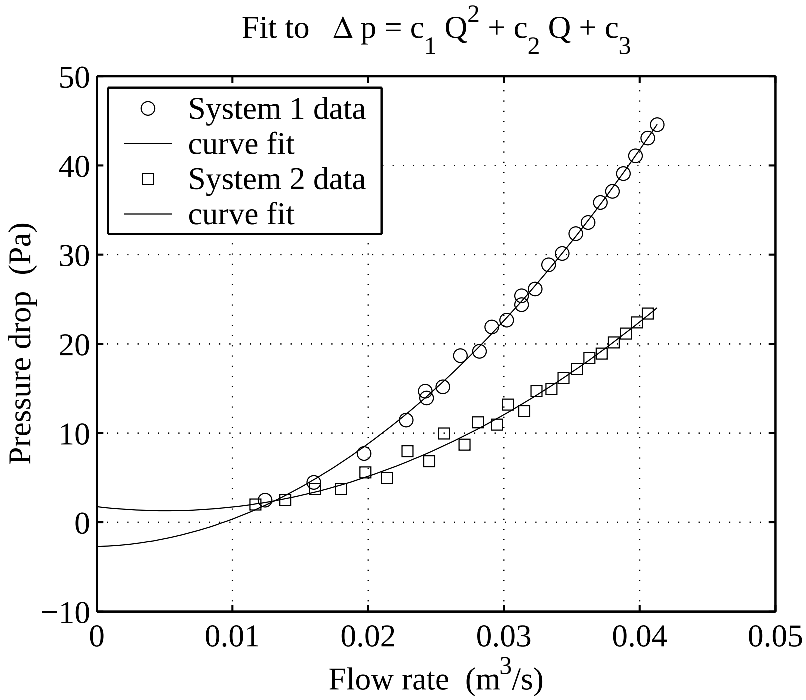
To obtain the system curve, measure a series of $(\Delta p_{\text{sys}}, Q)$ pairs. A least squares curve fit is then used to find C.

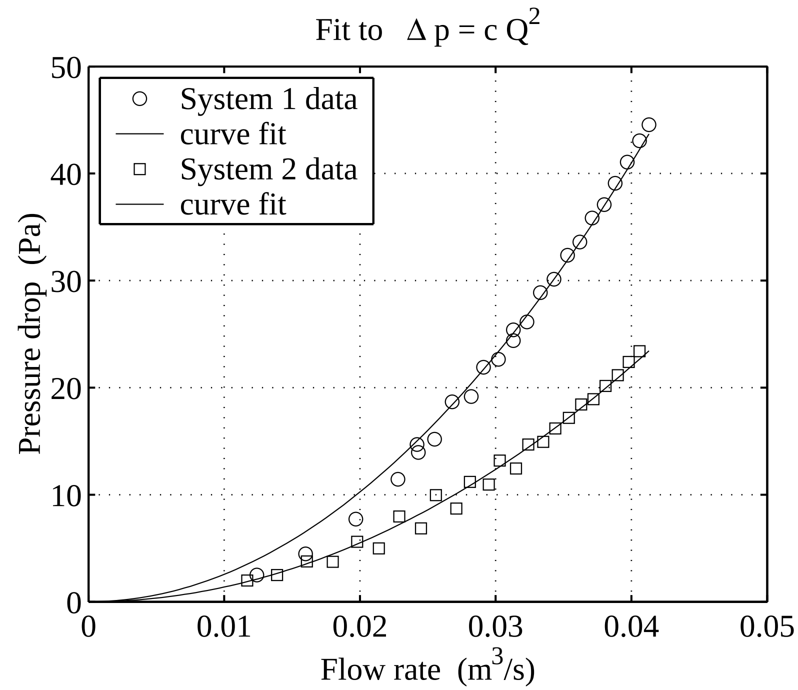
Software tools (e.g. MATLAB or spreadsheets) have built-in procedures for performing least squares fit to polynomials. Such a tool would require a curve fit of the form

$$\Delta p_{\text{sys}} = c_1 Q^2 + c_2 Q + c_3$$

But Equation (12) does not have the constant or a term linear in Q. Fortunately it is very easy to derive a simple formula that uses the least squares principle to obtain C from measured data

Least Squares Fit to System Curve (4)





Least Squares Fit to System Curve (5)

Given a set of m data pairs$(\Delta p_1, Q_1)$, $(\Delta p_2, Q_2)$, . . . $(\Delta p_m, Q_m)$, write the *matrix* equation

$$\begin{bmatrix} Q_1^2 \\ Q_2^2 \\ \vdots \\ Q_m^2 \end{bmatrix} C = \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_m \end{bmatrix} \tag{13}$$

Equation (13) is an overdetermined system for the one unknown value C. Multiply both sides by $(Q_1^2, Q_2^2, \ldots, Q_m^2)^T$

$$\begin{bmatrix} Q_1^2 & Q_2^2 & \cdots & Q_m^2 \end{bmatrix} \begin{bmatrix} Q_1^2 \\ Q_2^2 \\ \vdots \\ Q_m^2 \end{bmatrix} C = \begin{bmatrix} Q_1^2 & Q_2^2 & \cdots & Q_m^2 \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_m \end{bmatrix} \tag{14}$$

Least Squares Fit to System Curve (6)

Equation (14) is the normal equation for the over determined system in Equation (13). Solving Equation (14) gives the value of C that is the least squares solution to Equation (13).

Evaluating the inner products in Equation (14) gives

$$\left(\sum_{i=1}^{m} Q_i^4\right) C = \left(\sum_{i=1}^{m} Q_i^2 \Delta p_i\right) \tag{15}$$

This is just a scalar equation involving the one unknown value C. Solving for C gives

$$C = \frac{\sum_{i=1}^{m} Q_i^2 \Delta p_i}{\sum_{i=1}^{m} Q_i^4} \tag{16}$$

Therefore, given pairs of $(\Delta p, Q)$ data from a flow loss measurement, Equation (16) provides a simple computational formula for obtaining the C that is the least squares fit of the data to Equation (12).