ME 510AF Problem Set 5 Spring 1997

due 15 May 1997

1. Starting with the following general form of momentum equations

$$\rho\frac{\partial u_j}{\partial t} + \rho u_i\frac{\partial u_j}{\partial x_i} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\lambda\frac{\partial u_k}{\partial x_k}\right) + \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)\right] + f_j$$

   a. Write out the y direction momentum equation for incompressible, variable viscosity flow.
   b. Simplify the equation obtained in part (a) under the assumption of steady flow with uniform μ.

2. In equation (3–2), White gives the vector form of the momentum equations with a modified pressure gradient term defined by equation (3–2a)

$$\nabla\hat{p} = \nabla p - \rho\mathbf{g}$$

   This substitution is only valid for incompressible flow. Describe in words the signficance of this substitution. Consult Panton § 10.5 for a justification. (*Hint*: I'm asking you to paraphrase the most important points made by Panton.)

3. White, § 3–2.1 reports the exact solution for Couette flow between parallel plates. Use the following procedure to obtain his results.

   a. Draw a sketch of the flow that identifies a coordinate system and the appropriate physical dimensions.
   b. State any assumptions needed in the analysis.
   c. Write out the steady, incompressible form of the continuity and momentum equations in two dimensions (x and y).
   d. Write out the boundary conditions.
   e. Cross out terms in the governing equations that are zero, and in words very briefly justify neglecting these terms.
   f. Convert the governing partial differential equations to ordinary differential equations, and explain (briefly) why this is appropriate.
   g. Solve for $u(y)$ and present the results in dimensionless form.

   Note: Only obtain the solution to the flow field. You do not need to solve the energy equation to obtain the temperature field.

4. Using same steps as in the preceding problem, solve problem 3–5 in White.