ME 510AF — Problem Set 3 — Spring 1997

due 22 April 1997

1. Given that the expression for $\boldsymbol{\nabla}\cdot\mathbf{v}$ in cylindrical coordinates is

$$\boldsymbol{\nabla}\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial v_z}{\partial z}$$

and the symbolic form of the continuity equation is

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\mathbf{u}) = 0$$

write out the continuity equation in cylindrical coordinates.

2. Determine whether the following velocity fields satisfy the continuity equation for incompressible flow. The parameters a and b are scalar constants.

   a. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{x}\, a$
   b. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{x}\, ax$
   c. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{r}\, af(r,t)$
   d. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{x}\, ay + \hat{\mathbf{e}}_\mathbf{y} bx$
   e. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{x}\, 3x^2 - \hat{\mathbf{e}}_\mathbf{y}\, 6xy + \hat{\mathbf{e}}_\mathbf{z}\, 16xy$
   f. $\mathbf{u} = -\hat{\mathbf{e}}_\mathbf{x}\frac{ay}{x^2+y^2} + \hat{\mathbf{e}}_\mathbf{y}\frac{bx}{x^2+y^2}$
   g. $\mathbf{u} = \hat{\mathbf{e}}_\mathbf{x}\,\frac{ax}{t} - \hat{\mathbf{e}}_\mathbf{y}\,\frac{by}{t}$

3. For steady flow of a liquid through a long straight pipe with $u_r = u_\theta = 0$, what, if any, are the constraints on u_z?

4. White, Chapter 2, problem 3

5. The velocity components for a two-dimensional (plane) flow are

$$v_r = \frac{a}{r} + \frac{b}{r^2}\cos\theta \qquad v_\theta = \frac{b}{r^2}\sin\theta$$

where a and b are constants. Determine the corresponding stream function, $\psi = \psi(r,\theta)$. *Hint*: Set $\psi = \psi_0 =$ constant at any convenient location.