ME 510AF Problem Set 2 Spring 1997

due 15 April 1997

1. The isotherms of a two-dimensional temperature field are given by $x^2 + y^2 = C^2$, where C is a constant. What is the direction of the maximum rate of change of temperature at the point $(3, 4)$?

2. A surface is defined by $\varphi(\mathbf{r}) = x_1^2 + x_2^2 + x_3^2 = C^2$ where C is a constant. Find a unit vector that is everywhere perpendicular to the surface.

3. Compute

$$Q = \int_S \mathbf{F} \cdot \hat{\mathbf{n}} \, d\sigma$$

where $\mathbf{F} = \hat{\mathbf{e}}_1 x_3 - \hat{\mathbf{e}}_2 x_2 + \hat{\mathbf{e}}_3 x_1$ and S is that portion of the plane defined by $x_1 + 2x_2 + 2x_3 = 2$, which is bounded by the coordinate axes.

Hints:

- Sketch the surface by considering its intersection with the $x_1 = 0$, $x_2 = 0$, and $x_1 = 0$ planes.
- Compute $\hat{\mathbf{n}}$ from

$$\hat{\mathbf{n}} = \frac{\boldsymbol{\nabla} S}{|\boldsymbol{\nabla} S|}$$

where the surface is defined by a function of the form $S(x_1, x_2, x_3) = 0$.

4. Consider the following flow fields

I. $\mathbf{u} = a(-\hat{\mathbf{e}}_1 \, x_2 + \hat{\mathbf{e}}_2 \, x_1)$

II. $\mathbf{u} = \hat{\mathbf{e}}_1 \, b \, \exp(-x_2^2/\lambda^2)$

where a, b, and λ are arbitrary constants. For each flow field

a. Compute $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{u}$

b. Sketch velocity vectors in the (x_1, x_2) plane. Choose enough points so that you can clearly vizualize the flow.

c. Explain the results of part (a.) in terms of the pictures you drew in part (b.). What would happen to a small, square material volume during the time interval Δt?

5. Compute the fluid acceleration for each of the following flow fields if a and b are scalar constants, $f(t)$ is a scalar function of time alone.

a. $\mathbf{u} = \hat{\mathbf{e}}_1 \, f(t)$

b. $\mathbf{u} = \hat{\mathbf{e}}_1 \, a x_1$

c. $\mathbf{u} = \hat{\mathbf{e}}_1 \, a x_2 + \hat{\mathbf{e}}_2 \, b x_1$

d. $\mathbf{u} = \hat{\mathbf{e}}_1 \dfrac{a x_1}{t} - \hat{\mathbf{e}}_2 \dfrac{b x_2}{t}$ What is the significance of $a = b = 1$?