ME 510AF — Problem Set 1 — Spring 1997

due 8 April 1997

1. For each of the following tensor expressions, expand all of the terms, then simplify, *or* state that no simplification is possible, *or* state the expression is invalid. Give your reason(s) for declaring an expression invalid. Assume that all vectors are three-dimensional.

   (a) $a_m b_m$

   (b) $a_n b_n$

   (c) $a_k b_k c_k$

   (d) $y_j = c_{ij} x_i$

   (e) $z_j = c_{ij} c_{ik} y_k$

   (f) δ_{ij}

   (g) $a_i b_j = c_{ij}$

   (h) $a_r b_s = c_{mn}$

   (i) $\partial_i x_i$

   (j) $\partial_k x_j$

2. Add consistent missing subscripts to the following expressions. The missing subscripts are designated by an "o" subscript.

   (a) $a_i b_j = c_{oo}$

   (b) $B_{ij} = c_{mi} c_{nj} A_{mo}$

   (c) $a_m b_{mn} + c_k d_{ko}$

   (d) $\epsilon_{ijk}\, a_j b_k = c_o$

3. Decompose the tensor B_{ij} to obtain it symmetric and anti-symmetric parts for

$$B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 3 & 2 \\ 0 & -2 & 4 \end{pmatrix}$$

4. Given the decomposition of an arbitrary tensor T_{ij} into its symmetric and anti-symmetric parts

$$S_{ij} = \frac{1}{2}\big(T_{ij} + T_{ji}\big) \qquad A_{ij} = \frac{1}{2}\big(T_{ij} - T_{ji}\big)$$

   show that $S_{ij}\, A_{ij} = 0$ always.

5. The isotherms of a two-dimensional temperature field are given by $x^2 + y^2 = C^2$, where C is a constant. What is the direction of the maximum rate of change of temperature at the point $(3, 4)$?

6. A surface is defined by $\varphi(\mathbf{r}) = x_1^2 + x_2^2 + x_3^2 = C^2$ where C is a constant. Find a unit vector that is everywhere perpendicular to the surface.