1. Write the following system of equations in standard form

$$\begin{aligned} 25(x_1 - x_2) + 15x_3 + x_4 - 7 &= 0 \\ x_2 + x_3 &= 1 \\ x_1 - 5x_2 - 3x_3 - x_4 + 16 &= 0 \\ \frac{x_1}{x_4} &= 2 \end{aligned}$$

   Use exploratory calculations in MATLAB to determine whether this system can be solved. If a solution exists, find one. If the solution exists is it unique?

2. Solve the following system

$$\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 25 \end{bmatrix}$$

3. Solve the following system

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 25 \end{bmatrix}$$

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4. Solve the following system using Gaussian elimination by hand

$$\begin{bmatrix} 3 & 0 & 1 \\ -3 & 1 & -1 \\ 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

5. Solve the following system using Gaussian elimination by hand

$$\begin{bmatrix} 3 & -1 & 1 \\ -3 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

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6. (G.W. Stewart, *Afternotes on Numerical Analysis*, 1996, SIAM, p. 115) Given an "exact" vector, x, and an approximation $\hat{x}$

$$x = \begin{bmatrix} 1.0000 \\ 0.0100 \\ 0.0001 \end{bmatrix} \qquad \hat{x} = \begin{bmatrix} 1.0002 \\ 0.0103 \\ 0.0002 \end{bmatrix}$$

Compute

(a) $\|\hat{x} - x\|_1$

(b) $\|\hat{x} - x\|_2$

(c) $\|\hat{x} - x\|_\infty$

Given that x is exact, compute the relative, component-wise errors, i.e.

$$\hat{e}_{\mathrm{rel},i} = \frac{\hat{x}_i - x_i}{x_i}$$

Comment on how the norm of the error relates to the component-wise errors.

7. Consider the following system (due to Wilkinson, cf. Higham's testmatrix toolbox)

$$\begin{bmatrix} 1 \times 10^{-10} & 0.9 & -0.4 \\ 0 & 0.9 & -0.4 \\ 0 & 0 & 1 \times 10^{-10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) Obtain the *symbolic* solution by hand, i.e., perform all computations to unlimited precision. What is the x vector?

(b) Solve this system using MATLAB.

(c) Why are the two solutions different?

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8. Suppose that a 4×4 matrix A has $\kappa_2 \sim 1 \times 10^{12}$, and the solution to the system $Ax = b$ is

$$x = \begin{bmatrix} 1.042 \\ 0.0304 \\ 10.045 \\ 0.002052 \end{bmatrix}$$

   (a) How many digits of the solution vector can you trust?

   (b) Which element of x is the least trustworthy? How many digits of that value are likely to be correct?

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