Lecture 4: Automatic Truncation of Series for $\sin(x)$

1 Learning objectives

At the end of this class you should be able to...

- give numerical values equivalent to the logical expressions “true” and “false”
- implement a basic “if construct.
- write a MATLAB expression to test convergence of a series

2 Motivation

As $x$ increases, more terms in the series representation of $\sin(x)$ are needed for convergence.

```matlab
>> sn = nTermSine2(pi/3,5);
>> sn = nTermSine2(4*pi/3,5);
>> sn = nTermSine2(4*pi/3,10);
```

We want to add flexibility to the code: terminate the series only when “term” is small compared to “s”.

3 “if...end” Syntax

```matlab
if condition
    block of statements
end
```

The block of statements is executed only if the condition is true

Examples

```matlab
if x<0
    disp('x is negative');
end
```

4 True and False

<table>
<thead>
<tr>
<th>True</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>1</td>
</tr>
<tr>
<td>false</td>
<td>0</td>
</tr>
</tbody>
</table>

| Anything but zero or ” | |

<table>
<thead>
<tr>
<th>Anything but zero or ”</th>
<th></th>
</tr>
</thead>
</table>
Examples

```
>> x = 5;
>> x < 0
ans =
    0
>> (x-5) > 0
ans =
    0
>> (x-3) > 0
ans =
    1
```

5 Automatic Truncation of Series for $\sin(x)$

5.1 Explore Truncation Criteria

Version 1:

```matlab
if term < s/1000
    break;
end
```

what if `term` or `s` is negative?

Version 2:

```matlab
if abs(term) < abs(s)/1000
    break;
end
```

Version 3:

```matlab
if abs(term/s) < 1/1000
    break;
end
```

Version 4: Complete loop

```matlab
tol = 5e-6;
for i=3:2:(2*n-1)
    term = -term*(x^2)/(i*(i-1));
    s = s + term;
    fprintf('%4d %18.13f %8.5f
',i,term,s)
    if abs(term/s) < tol
        break;
    end
end
```
function s = nTermSine3(x,n)
% nTermSine3 Evaluate the n-term series approximation to sin(x)
% Recursive evaluation of terms and check convergence
% Synopsis: s = nTermSine3(x,n)
% Input: x = argument of sine(x)
% n = number of terms in the series
% Output: s = approximation to sin(x) with a maximum of n terms
% of the series. Stop when abs(term/sum) < tol
% where tol = 5e-6

term = x;
s = term; % initialize the sum and the sign of the term

tol = 5e-6;
fprintf(' i term s
');
fprintf(' %4d %18.13f %8.5f
',1,term,s);
for i=3:2:(2*n-1)
    term = -term*(x^2)/(i*(i-1));
s = s + term;
    fprintf(' %4d %18.13f %8.5f
',i,term,s)
    if abs(term/s) < tol
        break;
    end
end

Testing:

>> sn = nTermSine3(4*pi/3,10);
>> sn = nTermSine3(4*pi/3,30);

6 while loop

while condition
    block of statements
end

The block of statements is repeated as long as condition is true.
6.1 Implement \( \sin(x) \) series with a while loop

```matlab
function s = nTermSine4(x)
% nTermSine4 Evaluate series approximation to \( \sin(x) \). Use recursive
% evaluation of terms, while loop and convergence che
% Synopsis: s = nTermSine4(x)
% Input: x = argument of \( \sin(x) \)
% Output: s = approximation to \( \sin(x) \) with a max of n terms. Stop when
% abs(term/sum) < tol, where tol = 5e-6

term = x; s = term; % initialize the sum and the sign of the term
tol = 5e-6;
fprintf('
 i term s
');
fprintf(' %4d %18.13f %8.5f
',1,term,s);
i = 1;
while abs(term/s)>tol
    i = i + 2;
    term = -term*(x^2)/(i*(i-1));
    s = s + term;
    fprintf(' %4d %18.13f %8.5f
',i,term,s)
end

>> sn = nTermSine4(4*pi/3);

Trouble: If convergence criterion is never met, the while loop will repeat indefinitely, i.e. it will be an infinite loop
Solution: Limit the number of times the while loop can be executed.
```

```matlab
function s = nTermSine5(x,maxterms)
% nTermSine5 Evaluate series approximation to \( \sin(x) \). Use recursive
% evaluation of terms, while loop, convergence check and limit
% Synopsis: s = nTermSine5(x,n)
% Input: x = argument of \( \sin(x) \)
% maxterms = maximum number of terms in the series
% Output: s = approximation to \( \sin(x) \) with a max of n terms. Stop when
% abs(term/sum) < tol, where tol = 5e-6

term = x; s = term; % initialize the sum and the sign of the term
tol = 5e-6;
fprintf('
 i term s
');
fprintf(' %4d %18.13f %8.5f
',1,term,s);
i = 1; n=1;
while abs(term/s)>tol && n<maxterms
    i = i + 2; n = n + 1;
    term = -term*(x^2)/(i*(i-1));
    s = s + term;
    fprintf(' %4d %18.13f %8.5f
',i,term,s)
end
```

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6.1 Implement \( \sin(x) \) series with a while loop

\[
\begin{align*}
\text{>> sn} &= \text{nTermSine5}(4\cdot\pi/3,5); \\
\text{>> sn} &= \text{nTermSine5}(4\cdot\pi/3,15);
\end{align*}
\]