

ME 352

Quiz 2

25 November 2008

Print Your Name: _____

Your Signature: _____

This exam booklet contains

1. This cover sheet.
2. Three multiple-choice questions.
3. Three short answer questions.

Do not open the exam booklet until you are instructed to do so.

You will have 25 minutes to complete the quiz.

For problems 1 through 3, choose the one answer that best completes the statement.

1. [5 points] An $n \times n$ matrix is ill-conditioned if
 - (a) the matrix has a rank greater than n .
 - (b) the matrix has diagonal elements that are of order ε_m .
 - (c) an infinite loop occurs when Gaussian elimination is performed on the matrix.
 - (d) two or more rows of the matrix are nearly linearly dependent.
 - (e) the column vectors of the matrix span \mathcal{R}^n .

2. [5 points] Consider two n element column vectors, u and v , and a small, positive scalar value, δ . A mathematical statement that the two vectors are close enough to be considered equal is
 - (a) $u^T v < \delta$.
 - (b) $\|u\| - \|v\| < \delta$.
 - (c) $\|u - v\| < \delta$.
 - (d) $\frac{\|u\|}{\|v\|} < \varepsilon_m$.

3. [5 points] If A is a 5×5 matrix, x and b are 5×1 (column) vectors, and $\text{rank}(A) = 4$, then the solution to $Ax = b$
 - (a) can not exist.
 - (b) may exist, but if it does it will not be unique.
 - (c) requires Gaussian elimination with partial pivoting.
 - (d) will have only four significant digits.

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4. [10 points] Given $A^{-1}b = x$ where

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

What is b ? Show your work.

Chose *one* of the following two problems. Clearly label which of the problems you would like to have graded.

5. [10 points] Suppose that a system defined by $Ax = b$ has a numerical solution \hat{x} . What is the L_2 norm of the residual for the system of equations if

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \hat{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Show your work.

6. [10 points] Given

$$A = \begin{bmatrix} \alpha & 3 & 4 \\ 1 & 0 & \beta \\ 1 & 2 & 3 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad b = \begin{bmatrix} 20 \\ 4 \\ 4 \end{bmatrix}$$

what values of α and β are necessary to satisfy $Ax = b$? Show your work.