ME 352 Final Exam 11 December 2008

Print Your Name: ______________________________

Your Signature: ______________________________

This exam booklet contains

1. This cover sheet.
2. Multiple choice and true/false questions worth 5 points each.
3. Additional questions ranging in value from 10 points to 20 points. The point value increases with the number of the question.
4. A separate sheet for providing your final answer to one of the questions.
5. Code listing for a MATLAB function.

Do not open the exam booklet until you are instructed to do so.

You will have 1 hour and 50 minutes to complete the exam.

You *may not* use a calculator.

Do all problems on the exam.

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1. **[5 points]** Analysis of the effects of roundoff on the numerical solution to a system of n equations in n unknowns leads to the following expression

$$\frac{\|\hat{x} - x\|}{\|\hat{x}\|} \leq \kappa(A)\frac{\|r\|}{\|b\|}$$

where $\hat{x}$ is the numerical solution to $Ax = b$, $\kappa(A)$ is the condition number of A, and $r = b - Ax$. Which of the following statements are true? (Circle the letter for each choice that is true.)

   (a) The equation shows that the sign of the condition number determines the accuracy.

   (b) The equation shows that small residuals are no guarantee that the system was solved accurately.

   (c) The equation is not useful unless (or until) the $\| \ \|$ operator is defined.

   (d) The equation shows that $\kappa(A)$ is always small when the \ operator is used to solve the system.

   (e) None of the above.

2. **[5 points]** Choose the one phrase that best completes the following statement. Given the positive scalar values α and β, the numerical computation of γ with the expression $\gamma = \alpha + \beta$ will have the largest error due to roundoff when

   (a) α is much larger than β.

   (b) α and β are very nearly equal.

   (c) α and β have binary representations with non-terminating bit patterns.

   (d) α and β are approximately equal to ε_m.

   (e) None of the above.

3. **[5 points]** Choose the one phrase that best completes the following statement. Given the positive scalar values α and β, the numerical computation of γ with the expression $\gamma = \alpha - \beta$ will have the largest error due to roundoff when

   (a) α is much larger than β.

   (b) α and β are very nearly equal.

   (c) α and β have binary representations with non-terminating bit patterns.

   (d) α and β are approximately equal to ε_m.

   (e) None of the above.

4. **[5 points] True or False**: The MATLAB backslash operator performs matrix division with a version of Gaussian elimination.

5. **[5 points] True or False**: The MATLAB expressions `B = A*A` and `B = A.*A` produce the same matrix `B` only if `A` is a square matrix, i.e., only when `A` has n rows and n columns.

6. **[5 points]** Choose the one phrase that best completes the following statement. At each iteration of the bisection method, the interval, δ, containing the root

   (a) may or may not be reduced depending on the sign of $f'(x)$.

   (b) is reduced by a factor of δ.

   (c) is reduced by a factor of 2.

   (d) is reduced by an amount that depends on how close the current guess is to the true root.

7. **[5 points]** When applied to an $n \times n$ system of equations, the \ operator performs Gaussian elimination with partial pivoting. Consider the following MATLAB session.

```
>> A = rand(5,5);   b = rand(5,1);   x = A\b;   r = b - A*x;
>> fprintf('  %13.3e\n',r)
     4.163e-016
     8.327e-017
     1.887e-015
     1.332e-015
    -9.992e-016
```

   The `fprintf('  %13.3e\n',r)` statement prints the elements of the `r` vector in scientific notation with one element per line. Choose the one phrase that best completes the following statement.

   The values of the elements of `r` indicate

   (a) that the solution is accurate.

   (b) that there are large errors in the solution.

   (c) nothing, the \ operator always gives small residuals.

   (d) that there are sign errors in the A matrix or b vector because the residuals should always be positive.

8. **[5 points]** The fourth order Runge-Kutta scheme has a truncation error that is $\mathcal{O}(h^4)$. The midpoint scheme has a truncation error that is $\mathcal{O}(h^2)$. Consider the application of the fourth order Runge-Kutta scheme and the midpoint scheme to the solution of an ODE on an interval $0 \leq t \leq T_n$. Which of the following statements are true? (Circle all that apply.)

   (a) The fourth order Runge-Kutta scheme will produce a numerical solution that is approximately twice as accurate as the midpoint scheme at time T_n.

   (b) The fourth order Runge-Kutta scheme will produce a numerical solution that is approximately four times as accurate as the midpoint scheme at the end of each step of size h.

   (c) The fourth order Runge-Kutta scheme and the midpoint scheme take as much effort per time step.

   (d) The relative accuracy of the two schemes cannot be predicted without knowing the the details of the ODE being solved.

   (e) None of the above.

9. **[5 points]** Complete the following statement. In contrast to software such as MATLAB that performs numerical calculations, software that performs symbolic calculations...

   Hint: There is more than one way to complete the statement. Answers that focus on the key technical differences will earn more points.

10. [**10 points**] Given matrix A defined to the right.

(a) Compute $A^T A$

(b) What is A^{-1}?

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

11. **[15 points]** The goal is to set up the numerical solution of

$$\frac{dy}{dt} + 4yt = \sin(3\pi t), \qquad y(0) = 1.$$

with the `odeRK4` function. Refer to the universal cheat sheet for the listing of `odeRK4`.

(a) Write the m-file function that defines the ODE in a form suitable for use with `odeRK4`. *Do not* modify the code in `odeRK4`.

(b) Write the MATLAB statements to call `odeRK4` and obtain the solution for a time interval of 3 and a step size of 0.1.

(c) Write the MATLAB statements to plot the $y(t)$ obtained from the `odeRK4`.

12. **[20 points]** The goal is to find a curve fit of $y = \alpha x + \beta x^2$ to the data in the table to the right.

x	y
-2	-1
0	0.5
1	2

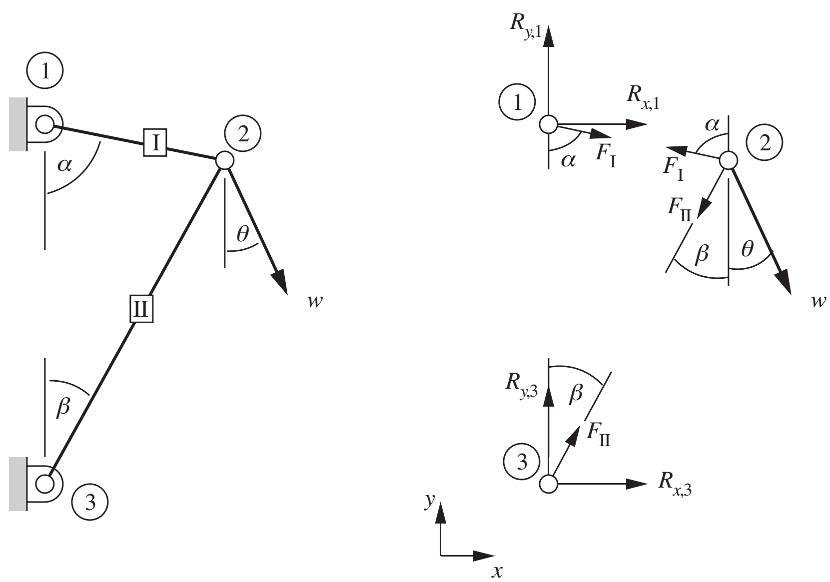
(a) Write the overdetermined system of equations.

(b) Write the normal equations. Show the numerical values, not just the symbols. Show the steps to obtain the normal equations.

(c) When the curve fit coefficients α and β are found, what is the value of the fit function at $x = 0$?

13. **[20 points]** The diagram on the left shows a two-link frame attached to a wall by pinned joints. The joints are numbered 1, 2, and 3. The links are labelled I and II.

A force w is applied to joint 2 at an angle θ relative to the vertical. On the right side of diagram, the force components at the three joints are identified. $R_{x,1}$ and $R_{y,1}$ are the reaction forces at joint 1. $R_{x,3}$ and $R_{y,3}$ are the reaction forces at joint 3. F_{I} and F_{II} are the tension forces in links I and II.



The following equations are obtained from force balances on the three joints.

$$
\begin{aligned}
\text{joint 1:} \quad & R_{x,1} + F_{\text{I}} \sin\alpha &= 0 \\
& R_{y,1} - F_{\text{I}} \cos\alpha &= 0 \\
\text{joint 2:} \quad & -F_{\text{I}} \sin\alpha - F_{\text{II}} \sin\beta + w\sin\theta &= 0 \\
& F_{\text{I}} \cos\alpha - F_{\text{II}} \cos\beta - w\cos\theta &= 0 \\
\text{joint 3:} \quad & R_{x,3} + F_{\text{II}} \sin\beta &= 0 \\
& R_{y,3} + F_{\text{II}} \cos\beta &= 0
\end{aligned}
$$

Assume that α, β, θ and w are given, and rearrange the preceding equations into a 6×6 system in the form $Ax = b$. The unknowns are the four reaction forces and the two tension forces.

Show the intermediate steps to obtain the matrix form of the equations. Enter your final answer into the form on the next sheet.

There is no special mathematical significance to the dashed lines in the form for recording the matrix equation. The dashed lines are merely used to designate boundaries between elements.

Do not solve the system of equations. Merely set it up.

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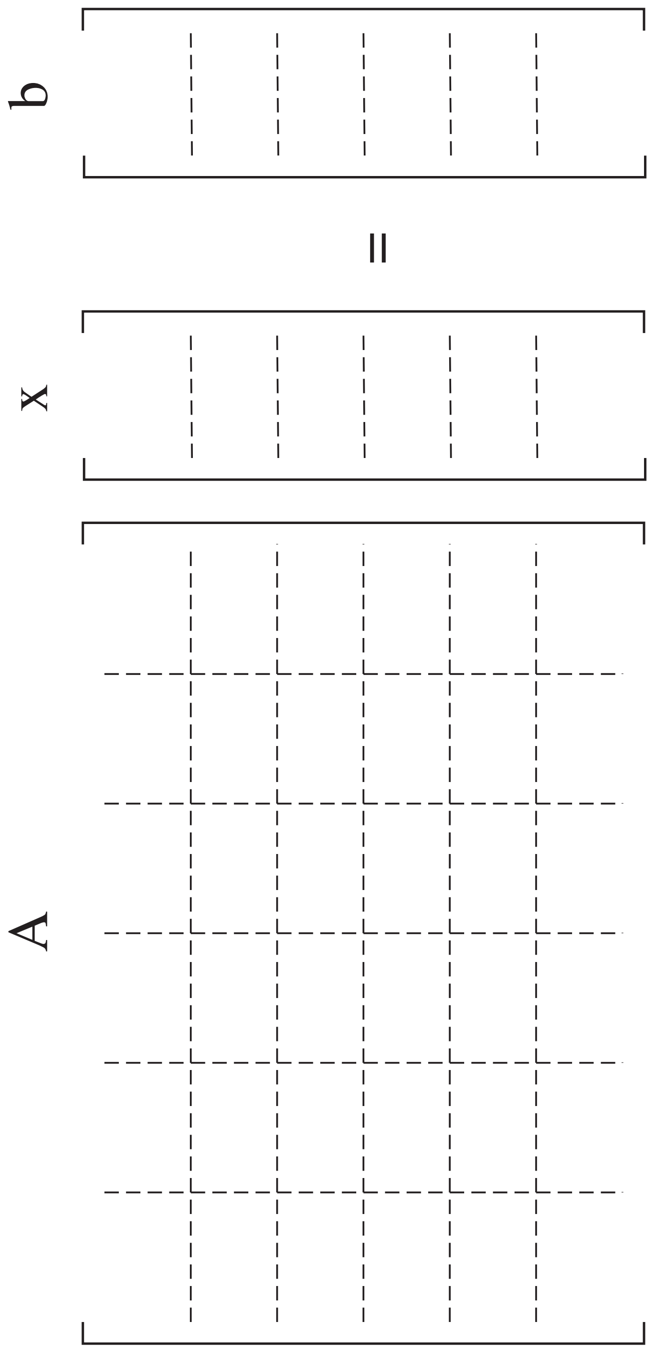
Solution form for problem 13



Universal Cheat Sheet

```
function [t,y] = odeRK4(diffeq,tn,h,y0)
% odeRK4  Fourth order Runge-Kutta method for a single, first order ODE
%
% Synopsis:  [t,y] = odeRK4(fun,tn,h,y0)
%
% Input:     diffeq = (string) name of the m-file that evaluates the right
%                     hand side of the ODE written in standard form
%            tn  = stopping value of the independent variable
%            h   = stepsize for advancing the independent variable
%            y0  = initial condition for the dependent variable
%
% Output:    t = vector of independent variable values:  t(j) = (j-1)*h
%            y = vector of numerical solution values at the t(j)

t = (0:h:tn)';                   %  Column vector of elements with spacing h
n = length(t);                   %  Number of elements in the t vector
y = y0*ones(n,1);                %  Preallocate y for speed
h2 = h/2;  h3 = h/3;  h6 = h/6;  %  Avoid repeated evaluation of constants

%  Begin RK4 integration; j=1 for initial condition
for j=2:n
   k1 = feval(diffeq, t(j-1),    y(j-1)        );
   k2 = feval(diffeq, t(j-1)+h2, y(j-1)+h2*k1  );
   k3 = feval(diffeq, t(j-1)+h2, y(j-1)+h2*k2  );
   k4 = feval(diffeq, t(j-1)+h,  y(j-1)+h*k3   );
   y(j) = y(j-1) + h6*(k1+k4) + h3*(k2+k3);
end
```