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ME 322 Lecture Notes

Lecture #11
Winter 2007.

A Simple Model a Centrifugal Pumps

Performance Model Based on Change in Bulk Angular Momentum MYO § 12.3 12.4

- leads to Euler turbomachine equation

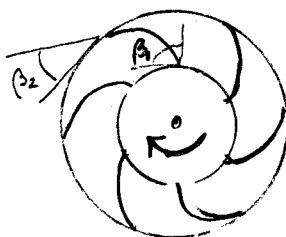
Define \vec{U} = blade speed = ωr a function of radial position on the blade

\vec{V} = absolute fluid velocity

\vec{W} = fluid velocity relative to the blade

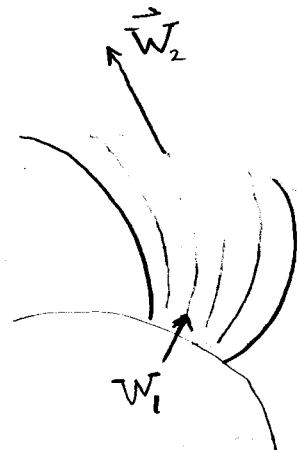
$$\vec{V} = \vec{W} + \vec{U}$$

Simple blade design for a pump impeller



β = blade angle

measured from tangent to inner and outer diameters of the impeller (pump) or rotor (turbine)



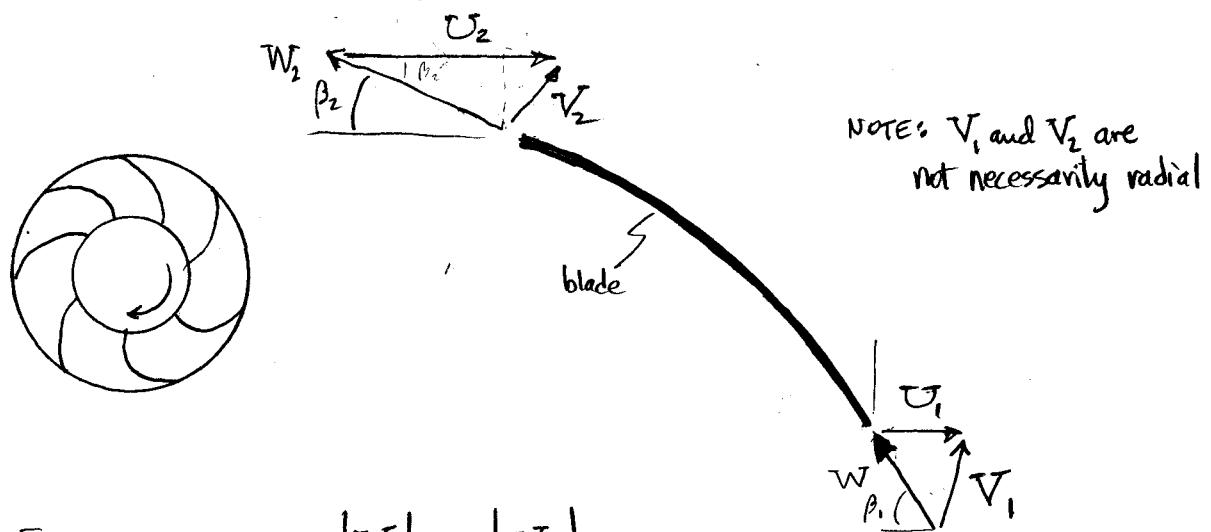
In our simple model the fluid streamlines of the relative velocity field $\vec{W} = \vec{W}(r, \theta, z)$ are parallel to the blade surfaces

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Basic Pump Operation

SEE BETTER DRAWING ON PAGE (4)

need $U_2 > V_2 \cos \beta_2$ because...



$$\text{For a pump: } |V_2| > |V_1|$$

Impeller increases the kinetic energy of the fluid.

The pump casing (volute) collects the fluid and decelerates it, causing a rise in pressure.

Conservation of Angular Momentum for steady flow - see MYO § 5.2.3

$$\sum (\vec{r} \times \vec{F}) = \int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \hat{n} dA$$

torque applied
to fluid in
a control volume

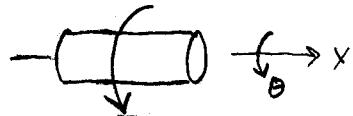
net outflow of angular
momentum of the fluid
across the control surface

- $\sum (\vec{r} \times \vec{F}) > 0$ for a pump \Rightarrow angular momentum of fluid must increase.

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Apply angular momentum conservation

- we've concerned only with axial component (x-dir) of angular momentum



$$\vec{r} \times \vec{F} = T_{\text{shaft}} \hat{e}_x$$

$$\vec{r} \times \vec{V} = r \vec{V}_{\theta} \hat{e}_x$$

For one inlet (station 1) and one outlet (station 2) and uniform flow across C.S.

$$T_{\text{shaft}} = (r_1 V_{\theta,1}) \rho (-V_1) A_1 + (r_2 V_{\theta,2}) \rho (V_2) A_2$$

$$= - \underbrace{\rho V_1 A_1}_{\dot{m}} r_1 V_{\theta,1} + \underbrace{\rho V_2 A_2}_{\dot{m}} r_2 V_{\theta,2}$$

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$$T_{\text{shaft}} = \dot{m} [r_2 V_{\theta,2} - r_1 V_{\theta,1}]$$

MYO Egn 12.2
Euler
turbomachine
Equation

OBSERVE:
 $r_2 V_{\theta,2} > r_1 V_{\theta,1}$ for $T_{\text{shaft}} > 0$
 \therefore pump must increase rV_{θ}

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$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = \text{Shaft power} \quad \left\{ \begin{array}{l} \text{+ve for pump} \\ \text{-ve for turbine} \end{array} \right.$$

$$\Rightarrow \dot{W}_{\text{shaft}} = \dot{m} \left[\underbrace{\omega r_2 V_{\theta,2}}_{U_2} - \underbrace{\omega r_1 V_{\theta,1}}_{U_1} \right]$$

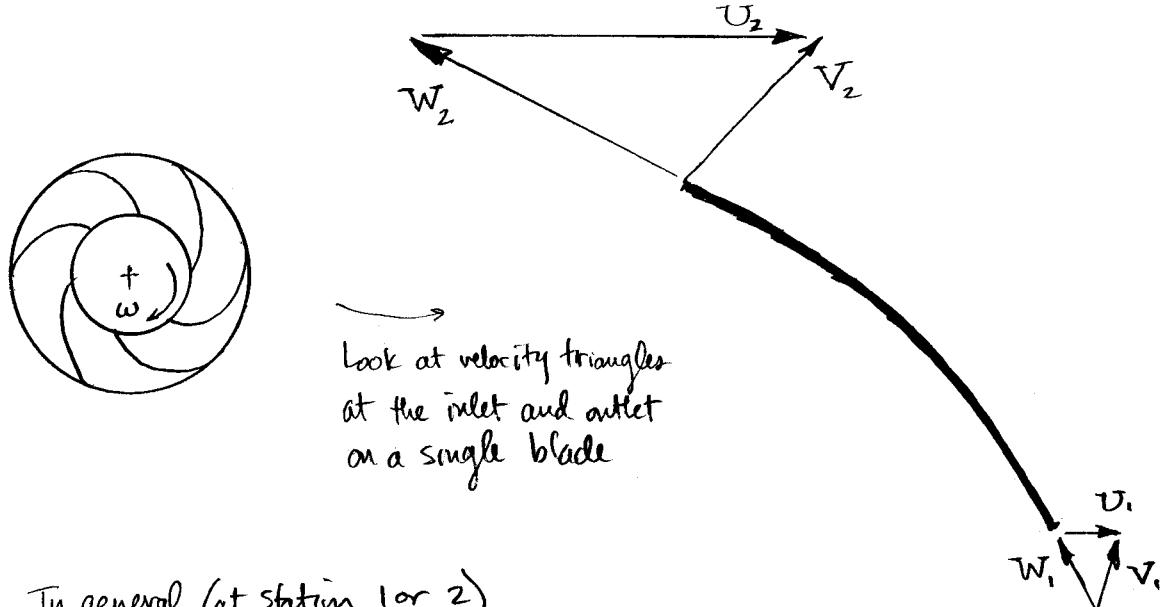
$$\dot{W}_{\text{shaft}} = \dot{m} [U_2 V_{\theta,2} - U_1 V_{\theta,1}]$$

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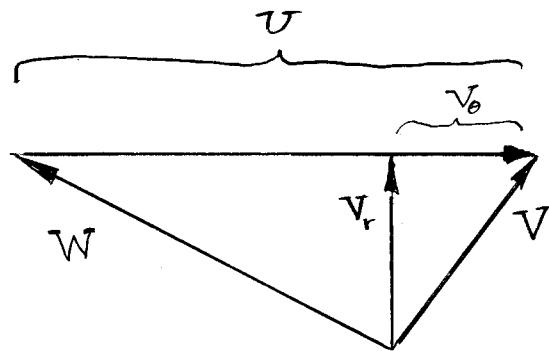
The shaft work per unit mass is

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{m} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \quad (3)$$

Use velocity triangles to obtain an alternative form of the preceding equation



In general (at station 1 or 2)
a velocity triangle looks like:



V_r = radial (outward) component

V_θ = angular component
(parallel to U)

use Pythagorean theorem:

$$V^2 = V_r^2 + V_\theta^2 \quad (*)$$

$$V_r^2 + (U - V_\theta)^2 = W^2 \quad (**)$$

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solve (**) for V_r and substitute into (*)

$$\begin{aligned}\Rightarrow V^2 &= [W^2 - (U - V_\theta)^2] + V_\theta^2 \\ &= W^2 - (U^2 - 2UV_\theta + V_\theta^2) + V_\theta^2 \\ &= W^2 - U^2 + 2UV_\theta\end{aligned}$$

$$\therefore UV_\theta = \frac{1}{2} (V^2 + U^2 - W^2)$$

Now substitute this into equation (3) to get

$$w_{\text{shaft}} = \frac{1}{2} [V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)] \quad (4)$$

MYO eqn(12.8)

look at each pair of terms

$V_2^2 - V_1^2$ = increase in kinetic energy of fluid

$U_2^2 - U_1^2$ = difference in kinetic energy of two particles attached to impeller at inlet and outlet.

$-(W_2^2 - W_1^2)$ = decrease in apparent kinetic energy as measured by relative velocity.

$W_2 < W_1$ because passage area (flow area between blades) increased with radius.

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Convert shaft work expressions to head

$$\underbrace{\dot{W}_{\text{shaft}}}_{= \text{shaft power actually transferred to the fluid. } \dot{W}_{\text{shaft}} \text{ is less than shaft power supplied by the motor}} = \cancel{\rho g Q} h_i = \dot{m} g h_i$$

shaft power actually transferred to the fluid. \dot{W}_{shaft} is less than shaft power supplied by the motor

h_i = ideal head rise

= head rise of fluid if there were no losses in the casing and along the impeller blades

$$\Rightarrow h_i = \frac{\dot{W}_{\text{shaft}}}{\dot{m} g} = \frac{w_{\text{shaft}}}{g}$$

combine with equation (3)

$$h_i = \frac{1}{g} (V_2 V_{\theta 2} - V_1 V_{\theta 1}) \quad (5)$$

Using equation (5) we can obtain a simplified model of pump performance if we neglect swirl at the inlet.

SWIRL or INLET SWIRL

is the angular velocity of the fluid at the eye of the impeller. Most pumps have swirl, and some have guide vanes to control the swirl.

$V_{\theta 1}$ is swirl velocity

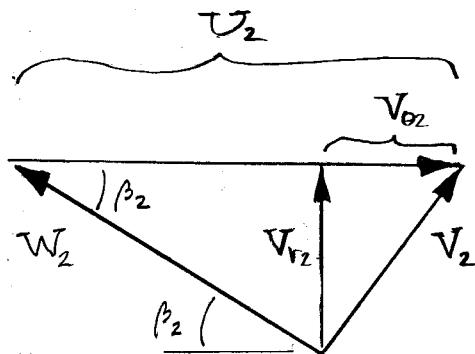
If $V_{\theta 1} = 0$ the inlet is said to have no swirl

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set $\nabla_{\theta 1} = 0$ in equation (5) to get

$$h_i = \frac{U_2 V_{\theta 2}}{g} \quad (7)$$

Look at velocity triangle for the blade exit.



$$\tan \beta_2 = \frac{V_{r2}}{U - V_{\theta 2}} \Rightarrow U - V_{\theta 2} = \frac{V_{r2}}{\tan \beta_2}$$

$$\therefore V_{\theta 2} = U_2 - V_{r2} \cot \beta_2 \quad (8)$$

Substitute into (7)

$$h_i = \frac{1}{g} (U_2^2 - U_2 V_{r2} \cot \beta_2) \quad (9)$$

Introduce volumetric flow rate through the pump

$$Q = (\vec{V} \cdot \hat{n}) A \quad \text{at outlet surface: } \vec{V} \cdot \hat{n} = V_{r2}$$

$$= V_{r2} 2\pi r_2 b_2$$

$$A_2 = 2\pi r_2 b_2$$

$$\Rightarrow V_{r2} = \frac{Q}{2\pi r_2 b_2}$$

b_2 = width of blade
measured into
the page

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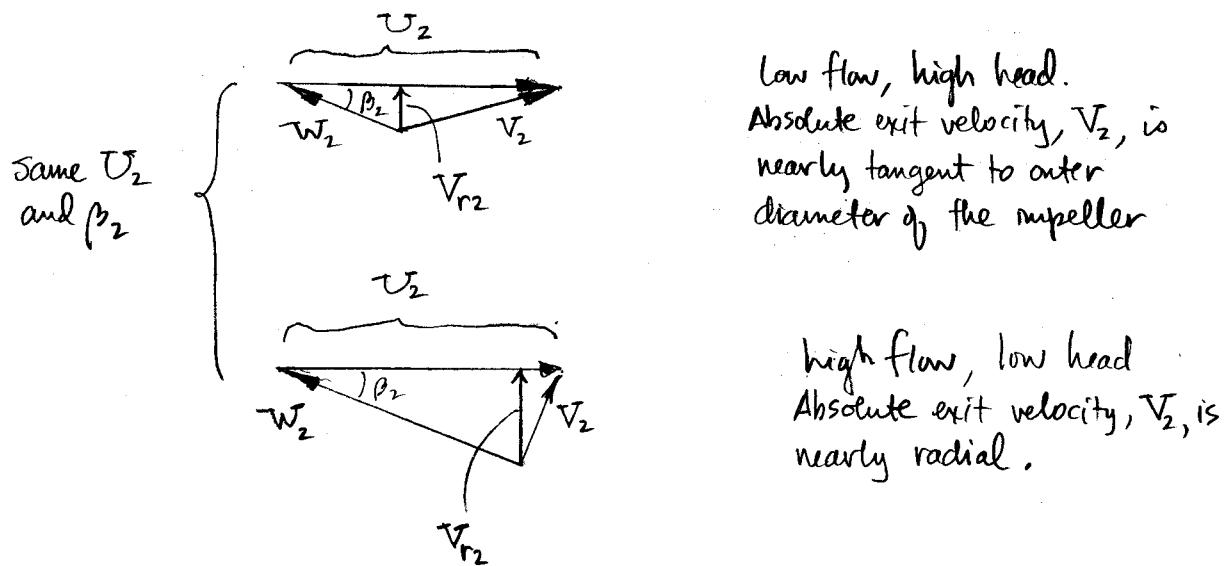
Substitute expression for V_{r2} into equation (9)

$$\Rightarrow h_i = \frac{1}{g} \left[U_2^2 - \frac{U_2 \cot \beta_2}{2\pi r_2 b_2} Q \right] \quad (10)$$

c. MYO, eqn 12.1B
page 696

Basic trends revealed by equation (10)

- head produced by pump increases with U_2 since $U_2 = \omega r_2$ head can be increased by increasing the speed (ω) or radius (r_2) or both.
- The $\frac{U_2^2}{g}$ term represents the head at the no-flow condition ($Q=0$)
- For fixed U_2 and $\cot \beta_2 > 0$ the head decreases with increased flow rate.



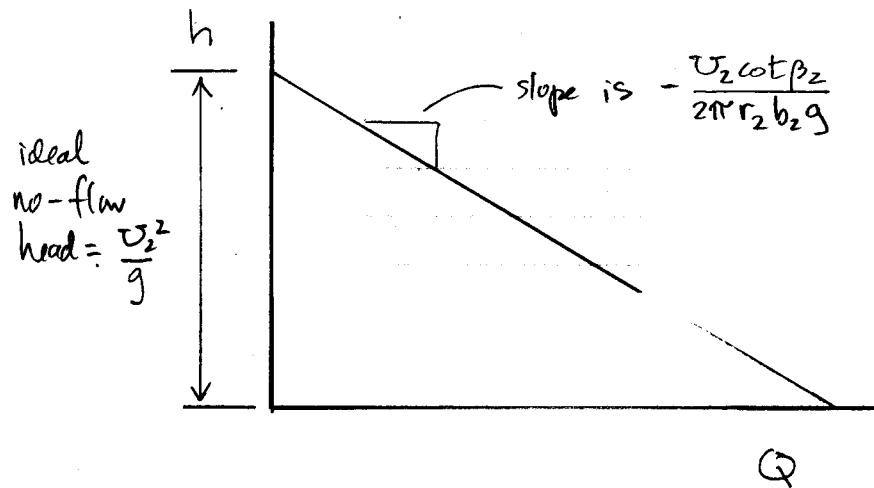
Typical impeller blade angle : $20^\circ < \beta_2 < 25^\circ$ (normal)

according to MYO, p. 696 $15^\circ < \beta_2 < 35^\circ$ (largest range used in practice)

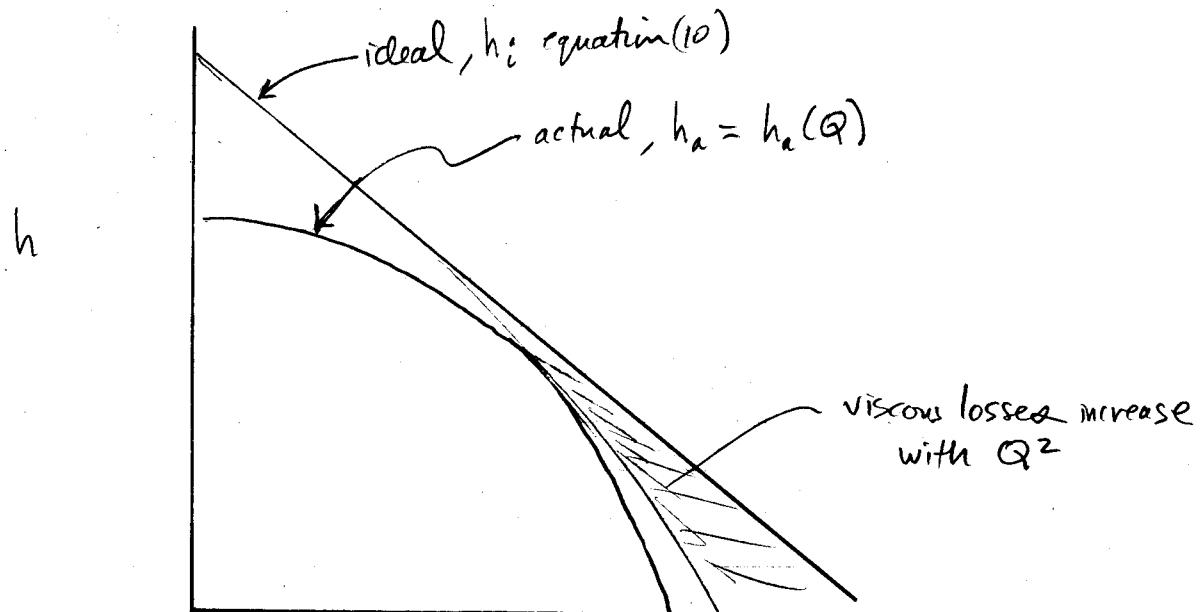
Note: some blowers used in HVAC applications have forward-curved ($\beta > 90^\circ$) vanes. See Table in ASHRAE EQUIPMENT HANDBOOK

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Equation (10) allows us to sketch a pump curve for a centrifugal pump



Actual Performance Curves



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Reasons for Differences between Ideal and Actual Pump Curve

- viscous losses (skin friction) in blade passages
increases as Q^2
- flow separation
 - At inlet the fluid velocity may not be aligned with inlet blade angle
 - Along the blade or impeller surfaces the boundary layer might separate
- 3D effects

In general the flow in blade passages is three-dimensional. For example, helical vortices may develop near corners between blade and impeller. The 3D flow patterns interact with and may increase viscous losses and flow separation.

- leakage
- Clearance between the impeller and casing provides a path for fluid to slip back over blades.

Pump designers must make many trade-offs between performance, cost and reliability. Each pump has a best efficiency point, BEP