

Compressible Flow Through Orifices

Example: gas flow through a valve

Given upstream stagnation conditions and downstream pressure, what is the flow rate?

The maximum flow occurs when $M=1$ at the smallest flow area - the throat.

$$\dot{m}_{\max} = \rho^* A^* V^* = \text{conditions where } M=1$$

A^* = throat area if $\underline{M=1}$ at the throat

$$= \rho^* V^* A^*$$

$$= \rho^* A^*$$

$$\therefore \boxed{\dot{m}_{\max} = A^* P_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)}} \quad \text{for choked flow (M=1) in the throat}$$

$$\text{for air with } k=1.4 \quad \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)} = 0.685$$

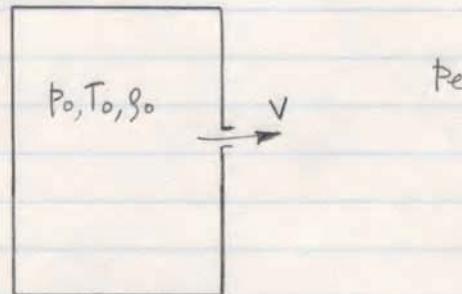
$$\boxed{\dot{m}_{\max, \text{air}} = 0.685 A^* P_0 \sqrt{\frac{k}{RT_0}}}$$

P_0, T_0 are conditions in the upstream plenum.

What is the flow rate when the flow is not choked?

To answer this question we'll derive yet another expression for the flow rate.

Consider a large tank of quiescent gas. The tank has a small opening through which the gas is being discharged to another chamber at pressure p_e . We assume that the flow up to the orifice is isentropic.



The mass flow rate through the orifice is

$$\dot{m} = \dot{S}_t A_t V_t \quad (0-1)$$

where subscript "t" indicates properties at the throat, i.e. the smallest cross-sectional area in the orifice

An expression for V_t can be obtained from the steady flow energy equation

$$h_0 = h + \frac{V^2}{2}$$

Solve for V and substitute $h = c_p T$

$$\begin{aligned} V &= \sqrt{2(h_0 - h)} = \sqrt{2c_p(T_0 - T)} \\ &= \sqrt{2c_p T_0 \left(1 - \frac{T}{T_0}\right)} \end{aligned}$$

use the isentropic relationship $\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{(k-1)/k}$

$$\Rightarrow V = \sqrt{2C_p T_0 \left[1 - \left(\frac{P}{P_0}\right)^{(k-1)/k} \right]}$$

Apply this formula to the orifice and substitute into equation (O-1)

$$\Rightarrow m = \rho_t A_t \sqrt{2C_p T_0 \left[1 - \left(\frac{P_t}{P_0}\right)^{(k-1)/k} \right]}$$

The density of the fluid in the throat is related to the pressure by the isentropic relationship

$$\frac{\rho_t}{\rho_0} = \left(\frac{P_t}{P_0}\right)^{1/k}$$

$$\Rightarrow m = \rho_0 \left(\frac{P_t}{P_0}\right)^{1/k} A_t \sqrt{2C_p T_0 \left[1 - \left(\frac{P_t}{P_0}\right)^{(k-1)/k} \right]}$$

$$= \rho_0 A_t \sqrt{2C_p T_0 \left(\frac{P_t}{P_0}\right)^{2/k} \left[1 - \left(\frac{P_t}{P_0}\right)^{(k-1)/k} \right]}$$

$$= \rho_0 A_t \sqrt{2C_p T_0 \left[\left(\frac{P_t}{P_0}\right)^{2/k} - \left(\frac{P_t}{P_0}\right)^{(k+1)/k} \right]}$$

use the ideal gas relationships

$$P_0 = \frac{P_0}{RT_0} \quad C_p = \frac{kR}{k-1}$$

$$\Rightarrow \beta_0 \sqrt{2c_p T_0} = \frac{P_0}{R T_0} \sqrt{\frac{2k R T_0}{k-1}}$$

$$= \frac{P_0}{\sqrt{T_0}} \sqrt{\frac{2k}{R(k-1)}}$$

$$\therefore \dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{2k}{R(k-1)} \left[\left(\frac{P_t}{P_0} \right)^{\frac{2}{k}} - \left(\frac{P_t}{P_0} \right)^{\frac{(k+1)}{k}} \right]} \quad (O-2)$$

This formula is in a nearly usable form. Three additional considerations are necessary before it is used in engineering calculations.

1. What is the gas pressure in the throat?
2. What is the flow area of the throat?
3. How does this formula relate to the choked flow formula

Gas Pressure in the throat of an Orifice

The flow downstream of the orifice is a jet.

It is standard practice to assume that the pressure in the jet is equal to the pressure in surroundings

$\therefore P_t = P_e$ may be used for engineering calculations

See Shames § 11.16 for further discussion. In the case of discharge from an orifice $M \leq 1$ in the throat.

Flow area in the throat

Consider the two cases of orifice geometry sketched below



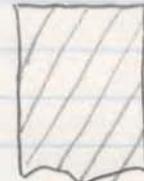
$$\downarrow A_1$$



rounded inlet



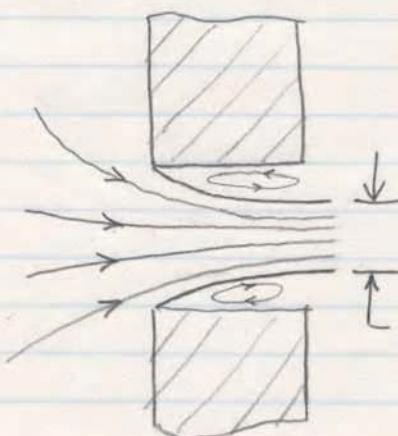
$$\downarrow A_2$$



sharp-edged
orifice

Geometrically $A_1 = A_2$

For the same pressure difference across the orifices there will be more flow through the smooth orifice than through the sharp orifice. The difference is caused by flow separation in the sharp-edged orifice. The streamlines of flow in the sharp-edged orifice are sketched below.



The effective flow area is reduced by separation at the leading edge of the sharp orifice

effective flow area

Boundary layer growth on the smooth orifice will also reduce the effective flow area, but the reduction will be less than that occurring on the sharp-edged orifice.

The standard engineering approach is to introduce a flow coefficient to account for the reduction of flow area

$$A_{\text{eff}} = C A_t$$

$$0 \leq C \leq 1$$

A_t = geometric cross-sectional area of the throat.

Flow coefficients depend on the flow rate and the geometry of the orifice. In most situations the flow coefficient will exceed 0.5. In the limit as the flow rate goes to zero the flow coefficient approaches 1.0.

As an aid for design calculations, manufacturers provide flow coefficient data in tables and formulae.

The Choked Flow Limit

The flow in the throat cannot exceed $M=1$
When $M=1$ the flow rate is (neglecting the flow coefficient)

$$\dot{m}_{\max} = \frac{p_0 A_t}{\sqrt{T_0}} \sqrt{\frac{k}{R}} \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)}$$

This limiting case must apply to equation (0-2)

Consider what happens to the flow rate as the external pressure p_e is varied for a fixed p_0

When $p_e = p_0$ there is no flow

As p_e is decreased the flow rate increases until the flow in the throat is choked.

Let p_{cr} be the (critical) pressure, i.e. the external pressure that results in choked flow in the orifice.

As p_e is decreased below p_{cr} the flow rate through the orifice is unaffected.

What is the value of p_{cr} ?

The pressure in the throat is related to the upstream stagnation pressure by

$$\frac{p}{p_0} = \left[\frac{1}{1 + \frac{k-1}{2} M^2} \right]^{k/(k-1)}$$

The throat pressure equals the critical pressure when the flow in the throat is choked

$$M=1 \Rightarrow \frac{p_{cr}}{p_0} = \left(\frac{1}{1 + \frac{k-1}{2}} \right)^{k/(k-1)} = \left(\frac{2}{k+1} \right)^{k/(k-1)}$$

As before, we assume that (for flow through the orifice) the pressure in throat equals the downstream (ambient) pressure

$$\therefore \frac{P_{e,cr}}{P_0} = \left(\frac{2}{k+1} \right)^{1/(k-1)}$$

the flow is choked whenever

$$\frac{P_e}{P_0} \leq \frac{P_{e,cr}}{P_0}$$

Note that $\frac{P_{e,cr}}{P_0}$ only depends on the thermodynamic properties of the gas.

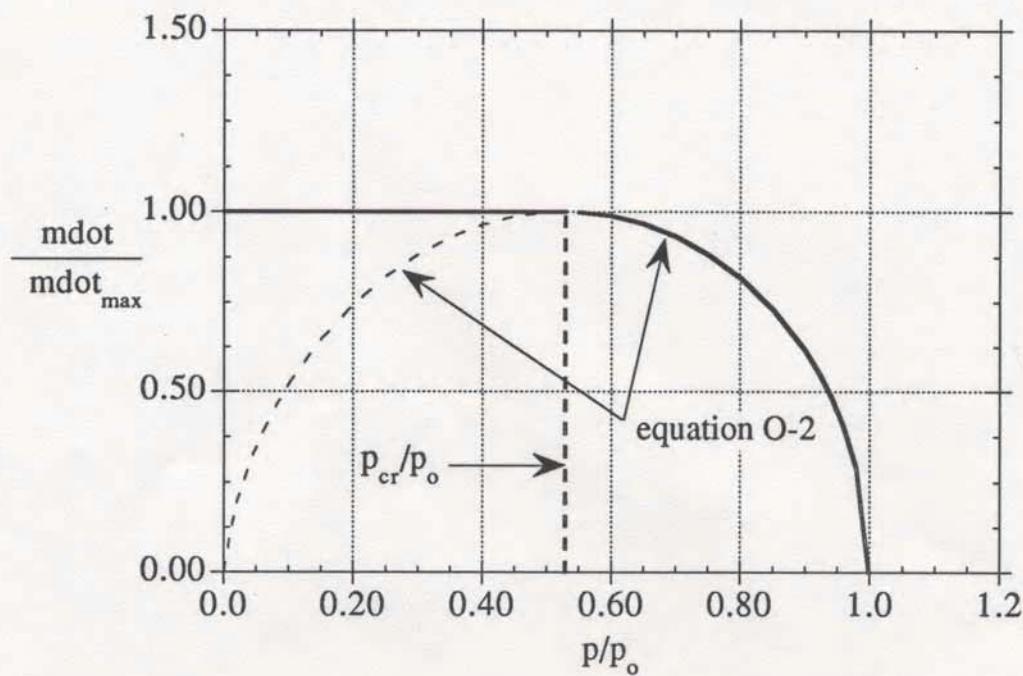
For air, with $k = 1.4$

$$\left(\frac{P_{e,cr}}{P_0} \right)_{\text{air}} = \left(\frac{2}{2.4} \right)^{1.4/0.4} = 0.528$$

The plot on the follow page shows the relationships between the flow rate formulas as a function of P/P_0 .

CO-9

Dimensionless flow rate for compressible discharge
of air from a large tank through an orifice



Summary of Orifice Flow Rate Calculations

For the gas under consideration, compute

$$\frac{p_{cr}}{p_o} = \left(\frac{2}{k+1} \right)^{\frac{k}{(k-1)}}$$

If $\frac{p_e}{p_o} < \frac{p_{cr}}{p_o}$ then the flow is choked. Compute the mass flow rate from

$$\dot{m} = \dot{m}_{max} = \frac{p_o A_{eff}}{\sqrt{T_o}} \sqrt{\frac{k}{R}} \left(\frac{2}{k+1} \right)^{\frac{k+1}{2(k-1)}}$$

Otherwise $\frac{p_{cr}}{p_o} < \frac{p_e}{p_o} \leq 1$ and the mass flow rate is given by

$$\dot{m} = \frac{p_o A_{eff}}{\sqrt{T_o}} \sqrt{\frac{2k}{R(k-1)}} \left[\left(\frac{p_e}{p_o} \right)^{\frac{2}{k}} - \left(\frac{p_e}{p_o} \right)^{\frac{k+1}{k}} \right]$$

where

p_o, T_o are upstream stagnation conditions

p_e is the external (downstream) pressure

A_{eff} is the effective flow area of the orifice

k is the specific heat ratio of the gas

R is the gas constant

The effective flow area is

$$A_{eff} = CA_t$$

where

C is the flow coefficient

A_t is the geometric area of the throat