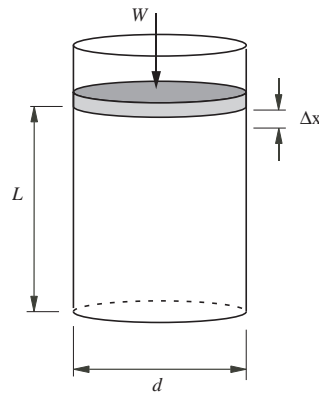1. A two-liter cylindrical tank, 10 cm in diameter, has a piston that fits perfectly. The piston does not leak, and there is no friction between the piston and walls of the tank.

   Suppose the tank is filled with water that is initially at 1 atm of pressure. How much additional weight must be placed on the piston to move the piston 1 cm?

2. Suppose the tank in the preceding exerciseis filled with air that is initially at 1 atm of pressure. How much additional weight must be placed on the piston to move the piston 1 cm?

3. Calculate the speed of sound in air, and in Hydrogen at 70°F.

4. Calculate the speed of sound in air at the altitudes in the following table. Assume the specific heat ratio is $k = 1.4$.

Altitude (m)	Temperature (°C)	Sound Speed (m/s)
0	15	
1000	8.5	
5000	−17.5	
10000	−49.9	
20000	−56.5	

5. What is the stagnation temperature and stagnation pressure on the nose of a reentry vehicle moving at a Mach number of 7 at an altitude of 200,000 ft.? At 200,000 ft. the ambient temperaure is 457°Rand the pressure is 0.58×10^{-2} inch Hg.

6. An ideal gas ($k = 1.4$, $R = 100 \, \text{ft} \cdot \text{lb}_\text{f}/\text{lb}_\text{m}/°\text{R}$) is supplied to a converging nozzle at low velocity at at $100 \, \text{psia}$ and $540 °\text{F}$. The nozzle discharges to atmospheric pressure, $14.7 \, \text{psia}$. Assuming frictionless, adiabatic flow, and a mass flow rate of 2,$\text{lb}_\text{m}/\text{s}$, calculate

    a. The pressure in the exit plane in psia.
    b. The velocity in the exit plane in ft/s.
    c. The cross-sectional area of the exit plan in in^2.

7. Air flowing in a duct has a pressure of $20 \, \text{psia}$, a Mach number of 0.6, and a flow rate of $0.5 \, \text{lb}_\text{m}/\text{s}$. The cross sectional area of the duct is $1.5 \, \text{in}^2$.

    a. Compute the stagnation temperature of the stream.
    b. What is the maximum percent reduction in area that could be introduced without reducing the flow rate of the stream?
    c. For the maximum reduction from part (b), find the velocity and pressure at the minimum area. Assume that the flow is adiabatic and friction is negligible.

Solution

1. Apply the definition of bulk modulus to finite changes in p and $\mathcal{V}$

$$E_v = -\mathcal{V}\frac{dp}{d\mathcal{V}} \approx \mathcal{V}\frac{\Delta p}{\Delta \mathcal{V}} \quad \implies \quad \Delta p = E_v\frac{-\Delta \mathcal{V}}{\mathcal{V}} \tag{1}$$

where $-\Delta\mathcal{V}$ means that a positive pressure change occurs when $\Delta\mathcal{V}$ is negative.

Use simple geometry of the cylinder

$$\mathcal{V} = LA \quad \text{and} \quad \Delta\mathcal{V} = -L\Delta x \quad \implies \quad \frac{-\Delta\mathcal{V}}{\mathcal{V}} = \frac{+\Delta x}{L} \tag{2}$$

where $A = \frac{\pi}{4}D^2$ is the area of the cylinder.

Combine Equation (1) and Equation (2)

$$\Delta p = E_v\frac{\Delta x}{L} \tag{3}$$

From the definition of pressure, $\Delta p = (W - mg)/A$, where W is the added weight and m is the mass of the piston. Define $W' = W - mg$ as the change in weight necessary to compress the liguid, then

$$\Delta p = \frac{W'}{A} \quad \implies \quad W' = A\Delta p = E_v A\frac{\Delta x}{L} \tag{4}$$

From the given geometric data

$$L = \frac{\mathcal{V}}{A} = \frac{(2\ell)\left(1000\,\frac{\text{cm}^3}{\ell}\right)}{\frac{\pi}{4}(0.10\,\text{m})^2} = 0.25\,\text{m}$$

Substitute numerical values into Equation (4). From Table 1.6 in Munson, Young and Okiishi (inside book cover), $E_v = 2.15\times10^9\ \text{N/m}^2$

$$W' = \left(2.15 \times 10^9\ \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}(0.10\,\text{m})^2\right)\frac{0.010\,\text{m}}{0.25\,\text{m}} = 675,000\ \text{N}$$

If a small car weighs $1.5\,\text{ton} = 1.5 \times 2000\,\text{lb}_\text{f} \times (1\,\text{lb}_\text{f}/4.448\,\text{N}) = 674\,\text{N}$, then the weight of 1000 cars is required to compress the liquid by one cm!

2. Use the ideal gas law to find the relationship between changes in pressure and changes in volume.

$$p\mathcal{V} = mRT \quad \Longrightarrow \quad p = \frac{mRT}{\mathcal{V}}$$

$$\Longrightarrow \quad \frac{p_2}{p_1} = \frac{m_2 RT_2/\mathcal{V}_2}{m_1 RT_1/\mathcal{V}_1} = \frac{T_2}{T_1}\frac{\mathcal{V}_1}{\mathcal{V}_2}$$

Assume that the compression process is quasistatic and thermal equilibrium with the environment is maintained. Then $T_1 = T_2$ and

$$\frac{p_2}{p_1} = \frac{\mathcal{V}_1}{\mathcal{V}_2} \quad \Longrightarrow \quad p_2 = p_1\frac{\mathcal{V}_1}{\mathcal{V}_2}$$

From the geometry

$$\mathcal{V}_1 = LA \quad \text{and} \quad \mathcal{V}_2 = (L - \Delta x)A \quad \Longrightarrow \quad p_2 = p_1\frac{L}{L - \Delta x}$$

From definition of pressure, $W = p_2 A$. Combining the preceding equations gives

$$p_2 = \frac{W}{A} = p_1\frac{L}{L - \Delta x} \quad \Longrightarrow \quad W = p_1 A\frac{L}{L - \Delta x}$$

Substitute numerical values

$$W = \left(101325\,\frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}(0.10\,\text{m})^2\right)\frac{0.25\,\text{m}}{0.25\,\text{m} - 0.01\,\text{m}} = 828\,N$$

828 N is the weight of

$$m = \frac{W}{g} = \frac{828\,\text{N}}{9.8\,\text{m/s}^2} = 85\,\text{kg}.$$

which is the mass of a large adult male.

3. Evaluate $c = \sqrt{kRT}$. Use absolute temperatures and g_c to get correct units.

Air: $\quad R = \dfrac{\mathcal{R}_u}{\mathcal{M}} = \dfrac{1545 \, \frac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\text{·mol°R}}}{28.97 \, \frac{\text{lb}_\text{m}}{\text{lb}_\text{m}\text{·mol}}} = 53.33 \, \dfrac{\text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{°R}}$

$\qquad k = 1.4$

$\qquad c = \sqrt{kRT}$

$$= \sqrt{(1.4)\left(53.33 \, \frac{\text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{°R}}\right)(70 + 460 \, \text{°R})\left(\frac{32.174 \, \text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{s}^2}\right)}$$

$\qquad \therefore \quad \boxed{c = 1130 \, \dfrac{\text{ft}}{\text{s}}}$

H_2: $\quad R = \dfrac{\mathcal{R}_u}{\mathcal{M}} = \dfrac{1545 \, \frac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\text{·mol°R}}}{2.018 \, \frac{\text{lb}_\text{m}}{\text{lb}_\text{m}\text{·mol}}} = 765.6 \, \dfrac{\text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{°R}}$

$\qquad k = 1.4 \quad$ coincidentally same as air

$\qquad c = \sqrt{kRT}$

$$= \sqrt{(1.4)\left(765.6 \, \frac{\text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{°R}}\right)(70 + 460 \, \text{°R})\left(\frac{32.174 \, \text{ft · lb}_\text{f}}{\text{lb}_\text{m}\text{s}^2}\right)}$$

$\qquad \therefore \quad \boxed{c = 4275 \, \dfrac{\text{ft}}{\text{s}}}$

4. Use $c = \sqrt{kRT}$ with $k = 1.4$, $R = \dfrac{8315 \frac{\text{J}}{\text{kg·mol K}}}{28.97 \frac{\text{kg}}{\text{kg·mol}}} = 287 \frac{\text{J}}{\text{kg·K}}$

At $1000\,\text{m}$:

$$c = \sqrt{kRT}$$

$$= \sqrt{(1.4)\left(287\,\frac{\text{J}}{\text{kg} \cdot \text{K}}\right)(8.5 + 273.15\,\text{K})}$$

$$= 336\,\sqrt{\frac{\text{J}}{\text{kg}}} = 336\,\sqrt{\frac{\text{N} \cdot \text{m}}{\text{kg}}} = 336\,\sqrt{\frac{\frac{\text{kg·m}}{\text{s}^2} \cdot \text{m}}{\text{kg}}}$$

$$\therefore \quad \boxed{c = 336\,\frac{\text{m}}{\text{s}}}$$

Repeat calculations to fill in the table.

Altitude (m)	Temperature (°C)	Sound Speed (m/s)
0	15	340
1000	8.5	336
5000	−17.5	320
10000	−49.9	300
20000	−56.5	295

5. Apply formulas for stagnation pressure and temperature. Use absolute pressures and temperatures. Given pressure and temperature are ambient conditions and are the *static* pressure and temperature in the formulas for stagnation temperature and pressure.

Given:

$$\mathrm{Ma} = 7, \qquad T = 457\,^{\circ}\mathrm{R},$$

$$p = \left(0.58 \times 10^{-2}\,\mathrm{inch\,Hg}\right) \left(\frac{14.7\,\mathrm{psia}}{29.92\,\mathrm{inch\,Hg}}\right) = 0.00285\,\mathrm{psia}$$

Compute stagnation temperature. $k = 1.4$ for air.

$$T_o = T\left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right) = 457\,^{\circ}\mathrm{R}\left(1 + \frac{0.4}{2}\,7^2\right) = 457\,^{\circ}\mathrm{R}(10.8)$$

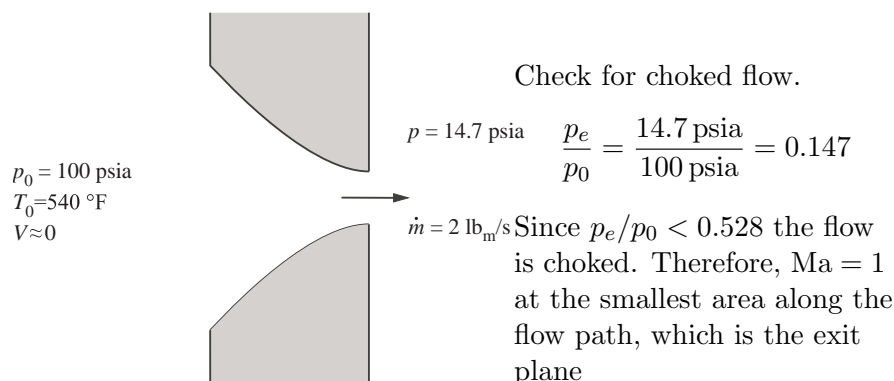$$\therefore \quad \boxed{T_o = 4940\,^{\circ}\mathrm{R}} \qquad \text{Hot!}$$

Compute stagnation pressure.

$$p_o = p\left(\frac{T_o}{T}\right)^{k/(k-1)} = (0.00285\,\mathrm{psia})\,(10.8)^{1.4/0.4}$$

$$\therefore \quad \boxed{p_o = 11.83\,\mathrm{psia}}$$

Note: The given pressure could not be a gage pressure because that would imply that $p \approx 14.7\,\mathrm{psia}$ at 200,000 ft.

6. Assume pressure at the exit plane is equal to the pressure of the surroundings. Use subscript "e" to designate conditions at the exit plane.

$p_0 = 100$ psia
$T_0 = 540$ °F
$V \approx 0$

$p = 14.7$ psia

$\dot{m} = 2$ lb$_\text{m}$/s

Check for choked flow.

$$\frac{p_e}{p_0} = \frac{14.7\,\text{psia}}{100\,\text{psia}} = 0.147$$

Since $p_e/p_0 < 0.528$ the flow is choked. Therefore, Ma $= 1$ at the smallest area along the flow path, which is the exit plane

a. Ma $= 1$ $\implies$ $p = p^*$ at the exit.

$$p_e = p^* = 0.528 p_0 \quad \text{(for } k = 1.4\text{)}$$
$$= 0.528(100\,\text{psia})$$
$$\therefore \quad \boxed{p_e = 52.8\,\text{psia}}$$

b. Ma $= 1$ at the exit means $V_e = c = \sqrt{kRT}$ and $T = T^*$. Compute T^* and then V.

$$\frac{T^*}{T_0} = 0.833 \quad \implies \quad T^* = (0.833)(540 + 460\,°\text{R}) = 833\,°\text{R}$$

Recall that $R = 100\,\text{ft} \cdot \text{lb}_\text{f}/\text{lb}_\text{m}/°\text{R}$ for the gas (not air)

$$V_e = \sqrt{(1.4)\left(100\,\frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}°\text{R}}\right)(833\,°\text{R})\left(\frac{32.174\,\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}\text{s}^2}\right)} \quad \therefore \quad \boxed{V_e = 1937\,\frac{\text{ft}}{\text{s}}}$$

c. $\dot{m} = \rho_e V_e A_e = 2\,\text{lb}_\text{m}/\text{s}$. Use ideal gas law to compute ρ_e and then solve for A_e.

$$A_e = \frac{\dot{m}}{\rho_e A_e}, \qquad \rho_e = \frac{p_e}{R T_e} \qquad \implies \qquad A_e = \frac{\dot{m} R T_e}{p_e V_e}$$

p_e is known from the solution to part (a). T_e and V_e values are

known from solution to part (b).

$$A_e = \frac{\left(2\,\frac{\text{lb}_\text{m}}{\text{s}}\right)\left(100\,\frac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\,^\circ\text{R}}\right)(833\,^\circ\text{R})}{\left(52.8\,\frac{\text{lb}_\text{f}}{\text{in}^2}\,\frac{144\,\text{in}^2}{\text{ft}^2}\right)\left(1937\,\frac{\text{ft}}{\text{s}}\right)} = \left(0.01131\,\text{ft}^2\right)\left(\frac{144\,\text{in}^2}{\text{ft}^2}\right)$$

$$\therefore \quad \boxed{A_e = 1.63\,\text{in}^2}$$

d. $\dot{m} = \rho_e V_e A_e = 2\,\text{lb}_\text{m}/\text{s}$. Use ideal gas law to compute ρ_e and then solve for A_e.

$$A_e = \frac{\dot{m}}{\rho_e A_e}, \qquad \rho_e = \frac{p_e}{RT_e} \qquad \Longrightarrow \qquad A_e = \frac{\dot{m}RT_e}{p_e V_e}$$

p_e is known from the solution to part (a). T_e and V_e values are known from solution to part (b).

$$A_e = \frac{\left(2\,\frac{\text{lb}_\text{m}}{\text{s}}\right)\left(100\,\frac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\,^\circ\text{R}}\right)(833\,^\circ\text{R})}{\left(52.8\,\frac{\text{lb}_\text{f}}{\text{in}^2}\,\frac{144\,\text{in}^2}{\text{ft}^2}\right)\left(1937\,\frac{\text{ft}}{\text{s}}\right)} = \left(0.01131\,\text{ft}^2\right)\left(\frac{144\,\text{in}^2}{\text{ft}^2}\right)$$

$$\therefore \quad \boxed{A_e = 1.63\,\text{in}^2}$$

7. **Known:** $k = 1.4$, Ma $= 0.6$, $\dot{m} = 0.5\,\text{lb}_\text{m}/\text{s}$, $A = 1.5\,\text{in}^2$, $p = 20\,\text{psia}$.

   a. We need to find T before we can compute T_0. Start by calculating p_0.

$$\frac{p}{p_0} = \left[\frac{1}{1 + \dfrac{k-1}{2}\,\text{Ma}^2}\right]^{k/(k-1)} = \left[\frac{1}{1 + \dfrac{0.4}{2}(0.6)^2}\right]^{1.4/0.4} = 0.784$$

$$\implies \quad p_0 = \frac{p}{0.784} = \frac{20}{0.784} \qquad \therefore \quad \boxed{p_0 = 25.51\,\text{psia}}$$

Since $\dot{m}$ is given the formula for the *maximum* flow rate is useful

$$\dot{m}_{\text{max,air}} = \frac{0.6847 p_0 A^*}{\sqrt{RT_0}} \qquad\qquad (\star)$$
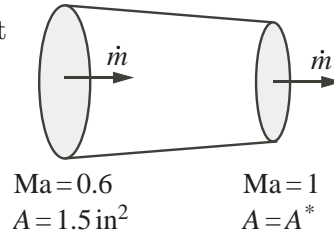
Both $\dot{m}_{\text{max,air}}$ and A^* are unknown. However, since A and Ma are known we can compute A^* from

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2}\text{Ma}^2\right)\right]^{(k+1)/[2(k-1)]} = \frac{1}{0.6}\left[\frac{2}{2.4}\left(1 + \frac{0.4}{2}(0.6)^2\right)\right]^{2.4/0.8}$$

$$\therefore \quad \boxed{\frac{A}{A^*} = 1.1882} \qquad \text{and} \qquad \boxed{A^* = \frac{1.5\,\text{in}^2}{1.1882} = 1.262\,\text{in}^2}$$

But since $\dot{m}$ does not vary along the duct

$$\dot{m}_{\text{max,air}} = \dot{m} = 0.5\,\frac{\text{lb}_\text{m}}{\text{s}}$$

Ma $= 0.6$ Ma $= 1$
$A = 1.5\,\text{in}^2$ $A = A^*$

Solve Equation $(\star)$ for T_0

$$\sqrt{T_0} = \frac{0.6847 p_0 A^*}{\dot{m}_{\text{max,air}}\sqrt{R}} \qquad \implies \qquad T_0 = \left[\frac{0.6847 p_0 A^*}{\dot{m}_{\text{max,air}}}\right]^2 \frac{1}{R}$$

Everything on the right hand side is known. A factor of g_c is needed for the units to work out

$$T_0 = \left[\frac{0.6847\left(25.51\,\frac{\text{lb}_\text{f}}{\text{in}^2}\right)\left(1.262\,\text{in}^2\right)}{0.5\,\frac{\text{lb}_\text{m}}{\text{s}}}\right]^2 \frac{32.174\,\frac{\text{lb}_\text{m}\text{ft}}{\text{lb}_\text{f}\text{s}^2}}{53.33\,\frac{\text{ft}\cdot\text{lb}_\text{f}}{\text{lb}_\text{m}\,{}^\circ\text{R}}} \quad \therefore \boxed{T_0 = 521\,{}^\circ\text{R}}$$

b. A^* is the minimum possible area if the mass flow rate is to remain unchanged. The actual area is A. Therefore the largest percent reduction in area is

$$100 \times \frac{A - A^*}{A} = 100 \times \frac{1.5 - 1.262}{1.5} = 16 \, \text{percent}$$

c. Find V and p at $A = A^*$

$$V^* = \sqrt{kRT^*} = \sqrt{kRT_0 \frac{T^*}{T_0}}$$

$$= \sqrt{(1.4)\left(53.33 \, \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m \, ^\circ\text{R}}\right)(521 \, ^\circ\text{R})(0.8333)\left(\frac{32.174 \, \text{ft} \cdot \text{lb}_f}{\text{lb}_m \text{s}^2}\right)}$$

$$\therefore \quad \boxed{V^* = 1021 \, \frac{\text{ft}}{\text{s}}}$$

$$\frac{p^*}{p_0} = 0.5283 \quad \Longrightarrow \quad p^* = (0.5283)(25.51 \text{psia}) \quad \therefore \quad \boxed{p^* = 13.5 \, \text{psia}}$$