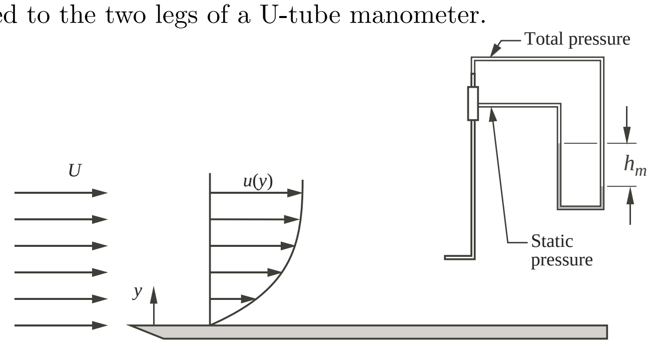
Given: Tabulated data from pitot probe measurements in a boundary layer. Dynamic pressure measurements are made with a water-filled, U-tube manometer.

y (mm)	h_m (mm)
0.0	0.0
2.1	10.6
4.3	21.1
6.4	25.6
10.7	32.5
15.0	36.9
19.3	39.4
23.6	40.5
26.8	41.0
29.3	41.0
32.7	41.0

Schematic: The sketch depicts a pitot probe apparatus for measuring velocity in a boundary layer. The total and dynamic pressure taps of the pitot probe are attached to the two legs of a U-tube manometer.



Find:

δ_{99}, the boundary layer thickness
δ^*, the displacement thickness
θ, the momentum thickness

Assumptions: Assume that the given data corresponds to a boundary layer velocity profile. No other assumtions are necessary.

Properties: Specific weight of water: $\gamma_w = 999\,\text{N/m}^3$, Density of air at standard conditions: $\rho = 1.23\,\text{kg/m}^3$

Analysis: The first step of the analysis is to convert the manometer readings $h(y)$ to velocity values $u(y)$. After that, the definitions of δ_{99}, δ^*, and θ are applied to the $u(y)$ data.

The dynamic pressure of the free stream is

$$\Delta p_{\text{dyn}} = \frac{1}{2}\rho u^2 \tag{$\star$}$$

where ρ is the density of the air, and u is the velocity in the x direction. The pressure measured by the manometer is

$$\Delta p = \gamma_m h_m \tag{$\star\star$}$$

Use Equation ($\star$) and Equation ($\star\star$) to eliminate the Δp gives

$$\frac{1}{2}\rho u^2 = \gamma_m h_m$$

Solve for u to get

$$u = \sqrt{\frac{2\gamma_m h_m}{\rho}}. \tag{$\star\star\star$}$$

Applying Equation ($\star\star\star$) to the given data yields the $u(y)$ and $u(y)/U$ values in the following table.

y (mm)	h_m (mm)	u (m/s)	u/U
0.0	0.0	0.00	0.000
2.1	10.6	4.15	0.508
4.3	21.1	5.85	0.717
6.4	25.6	6.45	0.790
10.7	32.5	7.27	0.890
15.0	36.9	7.74	0.949
19.3	39.4	8.00	0.980
23.6	40.5	8.11	0.994
26.8	41.0	8.16	1.000
29.3	41.0	8.16	1.000
32.7	41.0	8.16	1.000

The data shows that δ_{99} is somewhere between $y = 19.3$ mm and $y = 23.6$ mm. Use linear interpolation to find δ_{99}

$$\delta_{99} = 19.3 + \frac{0.990 - 0.980}{0.994 - 0.980}(23.6 - 19.3) = 22.37$$

Therefore $\boxed{\delta_{99}\text{=22.4 mm}}$.

The displacement thickness is

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

and the momentum thickness is

$$\theta = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

These formulas must be evaluated numerically. Numerical integration using the trapezoid rule is easy to perform with the built-in MATLAB function called `trapz`. Assume that the `u` and `y` vectors contain $u(y)$ and y, respectively. Assume the free stream velocity is stored in the variable `U`. The values of δ_{99} and θ are obtained with the following two statements.

```
delStar = trapz(y,1-u/U);
theta = trapz(y,(u/U).*(1-u/U));
```

The `MY09_13` function performs all calculations necessary to statisfy the assignment. Running `MY09_13` produces the following text output and the plot in Figure 1.

```
>> MY09_13

    y (mm)      h (mm)    u (m/s)       u/U
     0.0         0.0        0.00       0.000
     2.1        10.6        4.15       0.508
     4.3        21.1        5.85       0.717
     6.4        25.6        6.45       0.790
    10.7        32.5        7.27       0.890
    15.0        36.9        7.74       0.949
    19.3        39.4        8.00       0.980
    23.6        40.5        8.11       0.994
    26.8        41.0        8.16       1.000
    29.3        41.0        8.16       1.000
    32.7        41.0        8.16       1.000

delta99 =  22.37 (mm)
delta*  =   4.19 (mm)
theta =     2.24 (mm)
```

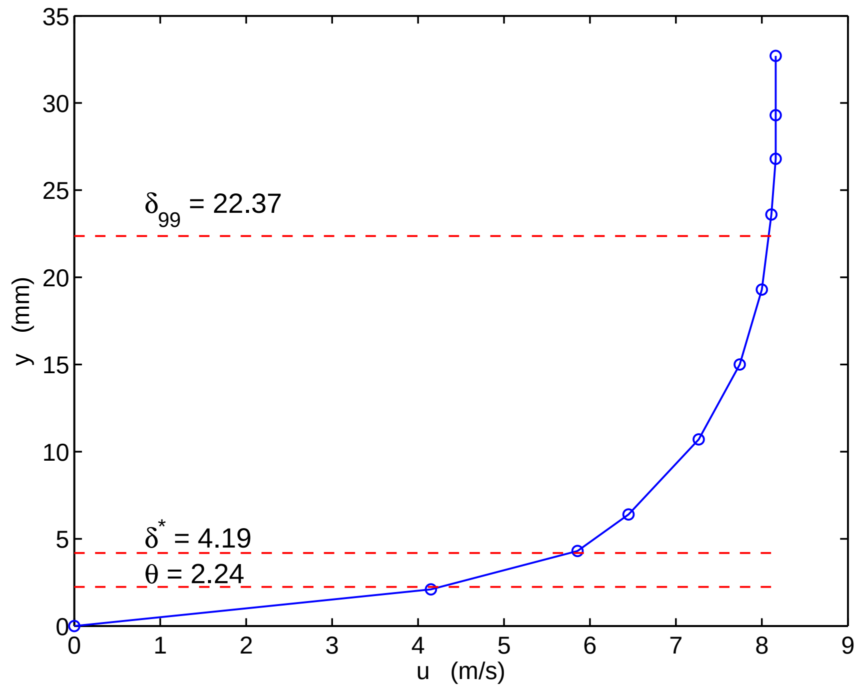


Figure 1: Velocity profile and computed values of δ_{99}, δ^* and θ.

```
function MYO9_13
% MYO9_13  Solution to problem 9.13 in Munson, Young, and Okiishi

% --- Given data
%       y = position  (mm)
%       h = manometer reading, (mm H2O)
y = [0   2.1   4.3   6.4  10.7  15.0  19.3  23.6  26.8  29.3  32.7];
h = [0  10.6  21.1  25.6  32.5  36.9  39.4  40.5  41.0  41.0  41.0];

gammaw = 999;     %  specific weight of water,  N/m^3
rhoa = 1.23;      %  density of air

% --- Convert manometer readings to velocities
%         0.5*rhoa*u^2 = gammaw*h => u = sqrt(2*gammaw*h/rhoa)
u = sqrt(2*gammaw*(h/1000)/rhoa);   %  h/1000 is manometer reading in meters
U = max(u);                         %  Free stream velocity
plot(u,y,'o-');   xlabel('u  (m/s)');   ylabel('y (mm)');

% --- Print a nice table of data
fprintf('\n   y (mm)      h (mm)    u (m/s)       u/U\n');
for i=1:length(y)
  fprintf('%7.1f     %7.1f   %9.2f   %9.3f\n',y(i),h(i),u(i),u(i)/U);
end

% --- Find del99 by interpolation in the velocity data
iU = min(find(u==U));          %  iU is index of first element with u(y) = U
del99 = interp1(u(1:iU)/U,y(1:iU),0.99); %  Find y value where u/U = 0.99

% --- use numerical integration to compute delStar and theta
delStar = trapz(y,1-u/U);       %  Integral of (1-u/U) w.r.t. y
theta = trapz(y,(u/U).*(1-u/U));   %  Integral of (u/U)*(1-u/U) w.r.t. y
fprintf('\ndelta99 = %6.2f (mm)\ndelta*  = %6.2f (mm)\ntheta   = %6.2f (mm)\n',...
        del99,delStar,theta);

% --- Add horizontal lines at y = delta, delStar, and theta
hold on
plot([0 U],del99*[1 1],'r--');             %  horizontal line at y = del99
theText = sprintf('\\delta_{99} = %4.2f',del99);
text(0.1*U,1.05*del99,theText,'FontSize',16);    %  label the line

% --- Add horizontal line at y =
uds = interp1(y,u,delStar);              %  interpolate to find u(y)(delStar)
plot([0 U],[delStar delStar],'r--');    %  line at y = delStar
theText = sprintf('\\delta^* = %4.2f',delStar);
text(0.1*U,1.2*delStar,theText,'FontSize',16);   %  and label the line

ut = interp1(y,u,theta);                     %  interpolate to find u(theta)
plot([0 U],[theta theta],'r--');             %  line at y = theta
theText = sprintf('\\theta = %4.2f',theta);
text(0.1*U,1.35*theta,theText,'FontSize',16);        %  and label the line
hold off
xlabel('u   (m/s)');   ylabel('y   (mm)');
```