Gas Vapor Mixtures and Air-Conditioning

ME 322 Lecture Slides, Winter 2007

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Overview

Learning Objectives

- Be able to define humidity ratio, and relative humidity.
- Be able to apply mass and energy balances to streams of atmospheric air undergoing heat and moisture transfer.

Definitions

Dry Air
Air with zero water vapor content

Atmospheric Air
or *moist* air
A mixture of dry air and water vapor

The water vapor concentration in atmospheric air can strongly affect human comfort.

For air-conditioning applications, air pressure is nominally 1 atmosphere and air temperature varies in the range $-10 \leq T \leq 50^{\circ}\mathrm{C}$. Under these conditions, dry air and atmospheric air are treated as ideal gases.

Ideal Gas Properties of Atmospheric Air

Use Dalton's Law

$$p = p_a + p_v = \text{pressure of the mixture}$$

$$p_a = \text{partial pressure of dry air}$$

$$p_v = \text{partial pressure of the water vapor}$$

p_v is the *vapor pressure of the water*.

For a given sample (volume) of air at temperature T

$$p_a V = m_a R_a T \qquad\qquad p_v V = m_v R_v T$$

$$\text{and} \quad pV = mRT$$

where R is the gas constant of the mixture.

Absolute Humidity, Specific Humidity, or Humidity Ratio (1)

For a given sample of moist air

$$\omega = \frac{m_v}{m_a} \qquad \frac{\text{kg water vapor}}{\text{kg water dry air}}$$

Apply the ideal gas law

$$\omega = \frac{p_v V / R_v T}{p_a V / R_a T} = \frac{p_v}{p_a}\frac{R_a}{R_v} \qquad \frac{R_a}{R_v} = \frac{\mathcal{R}_u / \mathcal{M}_a}{\mathcal{R}_u / \mathcal{M}_v} = \frac{\mathcal{M}_v}{\mathcal{M}_a}$$

Absolute Humidity, Specific Humidity, or Humidity Ratio (2)

But $\mathcal{M}_v$ and $\mathcal{M}_a$ are constants

$$\frac{\mathcal{M}_v}{\mathcal{M}_a} = \frac{18.015}{28.97} = 0.6219$$

Therefore

$$\boxed{\omega = 0.622\frac{p_v}{p_a}} \quad \text{and} \quad \boxed{\omega = \frac{0.622 p_v}{p - p_v}}$$

Relative Humidity

Relative humidity is a measure of humidity content relative to the maximum attainable moisture content, i.e. saturation.

For a given sample of moist air

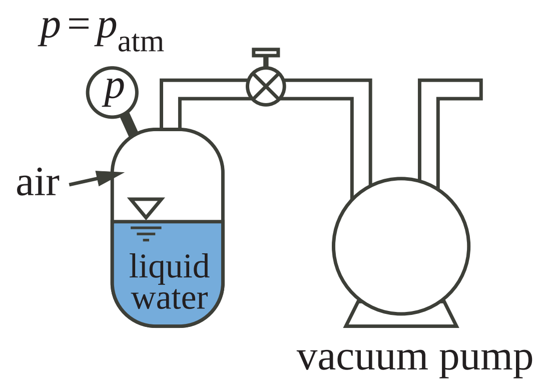
$$\phi = \frac{m_v}{m_{v,\text{sat}}} = \frac{\text{mass of water vapor}}{\text{mass of water vapor when the air is saturated with moisture}}$$

Saturation: condition where no more vapor can be added without condensation.

Saturated air is moist air with the maximum concentration (maximum ω) of water vapor *at that temperature*.

Saturation Thought Experiment (1)

Add water to a tightly sealed tank. Attach a vacuum pump to the tank and remove all of the air.

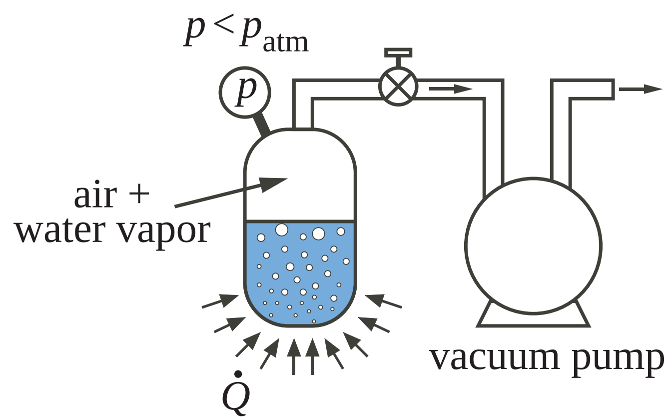


Saturation Thought Experiment (2)

As air is removed, the pressure in the tank decreases and the water begins to evaporate.

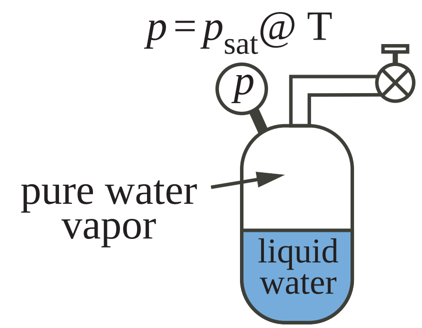
If the vacuum pump is strong enough, the pressure in the tank will drop enough that the liquid will begin boiling. The decrease in pressure inside the tank causes the vapor temperature to decrease, which causes heat to flow from the surroundings into the tank.

Eventually the water vapor displaces all of the air so that the vacuum pump pulls pure water vapor from the tank.



Saturation Thought Experiment (3)

When the gas space in the tank contains no more air ($m_a = 0$, $m_v \neq 0$), close the valve and allow the tank to come into thermal equilibrium with the surroundings at temperature T. The pressure in the tank is the *saturation pressure* for the given temperature, $p = p_{\text{sat}}(T)$.



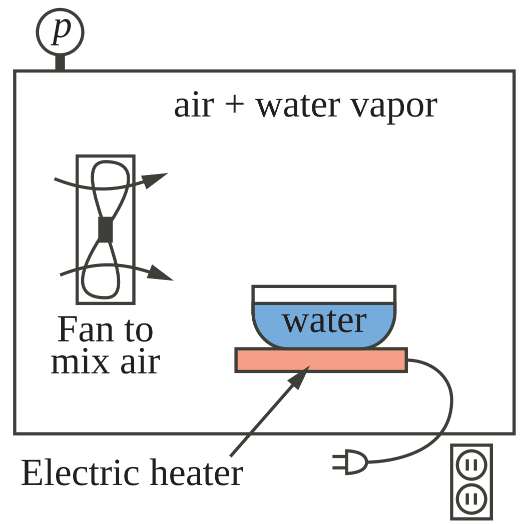
With the valve closed, the tank can be moved to environments at different temperatures. As the temperature of the liquid/vapor mixture increases, more water molecules freely leave the liquid.

p_{sat} increases with temperature

Moist Air Thought Experiment (1)

Build an air tight chamber. Inside the chamber place a box fan for circulation and place a pan of water on an electric heater.

Turn on the fan and wait. The air temperature will rise slightly because the work done by the fan on the air is eventually dissipated. If left long enough, and if the surrounding laboratory space is held at constant temperature, the tank will eventually come into thermal equilibrium with the laboratory.



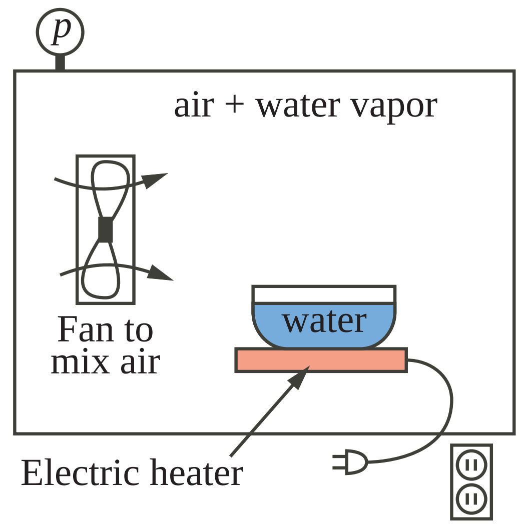
Moist Air Thought Experiment (2)

After a while, the water vapor content of the room will become saturated. Water evaporates from the surface of the pan until p_v in the mixture of dry air and water vapor equals the saturation pressure for water at the room temperature.

Ultimately,

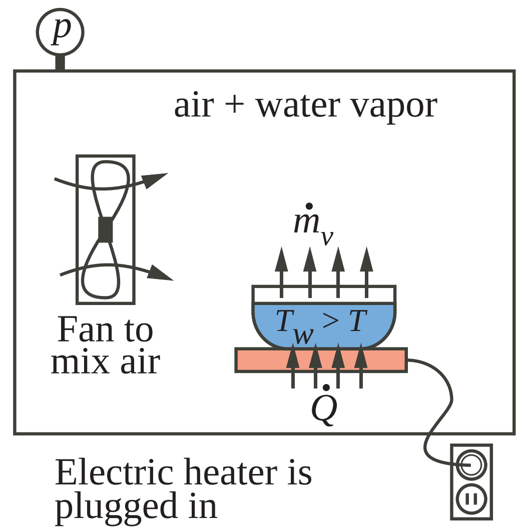
$$p_v = p_{v,\mathrm{sat}}(T)$$

This is the condition of the air in a room containing a swimming pool.



Moist Air Thought Experiment (3)

After the air is saturated, plug in the electric heater. When the water temperature T_w rises the vapor pressure of the water increases, which causes more water vapor to enter the chamber. The excess water vapor returns to a liquid state, usually on a surface that is slightly cooler than other surfaces in the chamber. The excess water vapor *condenses* to a liquid.



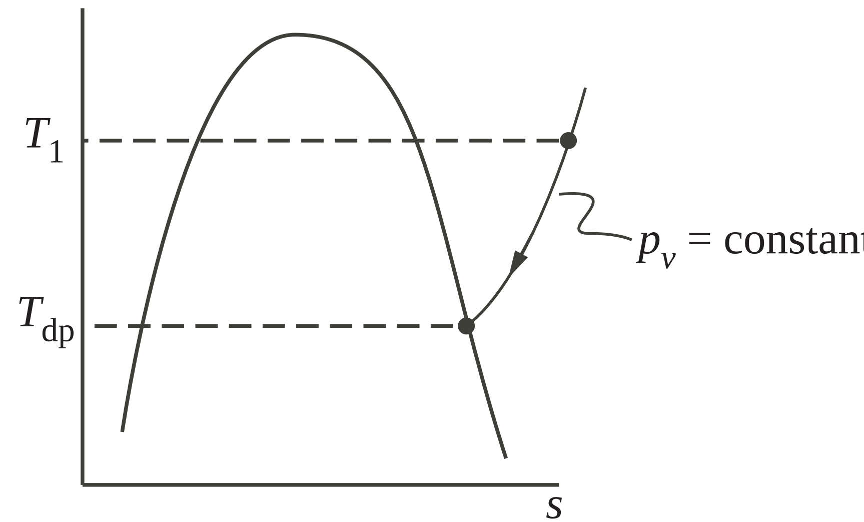
Dew Point

T_{dp} is the temperature at which condensation begins when air is cooled at constant pressure.

T_{dp} is uniquely determined by the moisture content in the air (ω) and the air temperature.

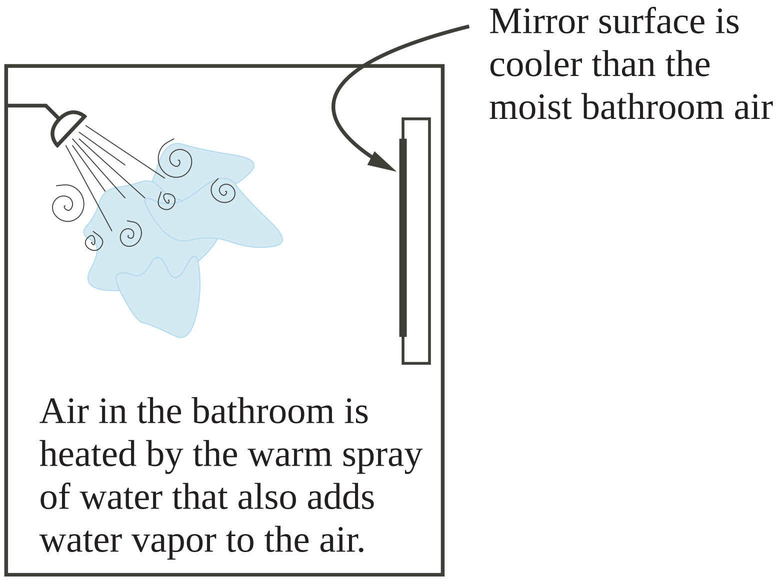
$$p, \omega \rightarrow p_v = f(p, T, \phi)$$

$$\rightarrow T_{\mathrm{dp}} = T_{\mathrm{sat}}(p_v)$$



Dew Point Example: Foggy Bathroom Mirror

If $T_{\mathrm{mirror}} < T_{\mathrm{dp}}$ then water condenses on the mirror



Enthalpy of Moist Air (1)

Moist air is an *Ideal Gas Mixture* of *Dry Air* and *Water Vapor*.

- Dry air is an ideal gas with constant specific heat

$$c_{p,\text{dry air}} = 1005 \, \frac{\text{J}}{\text{kg} \cdot \text{K}} = 0.240 \, \frac{\text{Btu}}{\text{lb}_\text{m} {}^\circ\text{R}}$$

- Enthalpy of water vapor in air is equal to enthalpy of *saturated vapor* at the same temperature

$$h_v \approx h_g(T)$$

- Water vapor is treated as an ideal gas $\Longrightarrow \Delta h_v = c_{p,v}(T) \Delta T$

$$h_g(T) = 2501.3 + 1.82T \qquad (\text{kJ/kg}) \qquad T \text{ in } {}^\circ\text{C}$$

$$h_g(T) = 1061.5 + 0.435T \qquad (\text{Btu/lb}_\text{m}) \qquad T \text{ in } {}^\circ\text{F}$$

Enthalpy of Moist Air (2)

Use mixture rules to compute enthalpy of moist air

$$H = H_a + H_v = m_a h_a + m_v h_g$$

It is convenient to use the *mass of the dry air* as a basis for the specific enthalpy

$$h \equiv \frac{H}{m_a} = h + \frac{m_v}{m_a} h_g \quad \Longrightarrow \quad \boxed{h = h_a + \omega h_g} \qquad \frac{kJ}{\text{kg dry air}}$$

Psychometric Chart