Linear Regression / Least Squares

Exponential and Power Law Relationships

The relationship between quantities is not always linear.

behavior	equation	physical example
		spring: $F = kx$
		radioactive decay: $A = A_o e^{-\lambda t}$
		accelerating body: $y = \frac{1}{2} a t^2$

We have already learned to find m and b for linear relationships. We need to find a way to find m and b for exponential and power law relationships.

Consider the following data:

x (seconds)	y (meters)
2	50
6	450
10	1400
14	2600
18	4200
22	6550
26	8800
30	11750

(a) Plot y versus x in Excel. Add the following axis title: "linear y versus linear x"

(b) Copy the plot from (a) to another part of your worksheet (keep the plot from (a) for future reference).

(c) Select the vertical axis on the copied plot, right click and select "format axis." Select logarithmic scale. Change the title of this plot to "log y versus linear x"

(d) Copy the plot from (a) to another part of your worksheet, keeping the plots from (a) and (c) above.

(e) Use a logarithmic axis for the horizontal axis. Change the title to: "log y versus log x"

Look at the three plots that you created above. Which one results in a straight line?

Look at the shape of the plot to determine the appropriate form of the equation:

plot	form	equation

Considering this table, what equation best describes the data we plotted earlier?

Now, go back to your Excel plots:

(a) On linear-linear plot, add a trendline (select linear, display equation and r^2).

(b) On log-linear plot, add a trendline (select exponential, display equation and r^2).

(c) On log-log plot, add a trendline (select power, display equation and r^2).

Now, look at the three fits. Which one has the highest r^2? What does this mean?????

Math behind linear regression for <u>POWER LAW</u> relationships

math	comment
$y = bx^m$	power law equation
$\log(y) = \log(bx^m)$ $= \log(x^m) + \log(b)$ $= m\log(x) + \log(b)$	Take "log" of both sides. Use these rules: $\log(pq) = \log(p) + \log(q)$ $\log(p^r) = r\log(p)$

$\log y = m \log x + \log b$

- $\log y$: plays part of y
- $\log x$: plays part of x
- $\log b$: plays part of b

X	Y	LOG X	LOG Y	LOGX * LOGY	$(\text{LOG } X)^2$	$(\text{LOG } Y)^2$
2	50	0.3013	1.6990	0.51144	0.09062	2.8865
⋮	⋮	⋮	⋮	⋮	⋮	⋮
		5.8230 Σx_i	18.3528 Σy_i	19.3073 $\Sigma x_i y_i$	6.3876 Σx_i^2	59.1778 Σy_i^2

NOTICE that in our table LOG X & LOG Y have taken the place of X & Y.

Plugging in these sums . . .

$$m = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{(6)(19.3073) - (5.8230)(18.3528)}{(6)(6.3876) - (5.8230)^2}$$

← Remember!

$m = 2.0317 \approx$ $\boxed{2.032 \approx m}$

WATCH OUT! b is MORE Unusual!

It's not $b =$

but $\log b = \dfrac{\Sigma y_i - m\Sigma x_i}{n} = \dfrac{18.3528 - (2.0317)(5.8230)}{6}$

$\log b = 1.0871$

Now $10^{\log b} = b = 10^{1.0871} = \boxed{12.22 = b}$

FINALLY

$Y = bX^m \rightarrow \boxed{Y = 12.22\,X^{2.032}}$

It is a power law EQN!

It plots straight on LOG-LOG axes.

Math behind linear regression for EXPONENTIAL relationships

math	comment
$y = be^{mx}$	power law equation
$\ln(y) = \ln(be^{mx})$ $= \ln(e^{mx}) + \ln(b)$ $= mx\ln(e) + \ln(b)$ $= mx + \ln(b)$	Take the natural log (or *ln*) of both sides (since we don't want to mix *log* with *e*). Logarithm rules (applying to natural log): $\ln(pq) = \ln(p) + \ln(q)$ $\ln(p^r) = r\ln(p)$
When fitting exponentials, remember that *ln(y)* replaces y and *ln(b)* replaces b in the least squares regression formulas!!!!	

Class Example – Air Pressure Versus Elevation:

The air that we breathe exerts a significant amount of pressure on objects at sea level (101.3 kPa or 14.7 psi). As we go higher into the atmosphere, the pressure decreases and eventually reaches zero in outer space where there is no air. Pressure measurements were recorded as a function of elevation above sea level resulting in the following table (km = kilometers and kPa = kilopascals):

altitude (km)	pressure (kPa)
0.1	99.5
5	50.7
8	33.8
16	10.1
33	1.0
47	0.1

(a) Hand plot pressure versus elevation using linear – linear scales.
(b) Hand plot pressure versus elevation using log-linear scales.
(c) Hand plot pressure versus elevation using log-log scales.
(d) Based on your hand plots, what type of function is most appropriate (linear, exponential, power law)????
(e) Create an Excel spreadsheet, and create a plot of the data along with a trendline showing r^2 for linear, exponential and power law functions. Which has the highest r^2?
(f) If you have time, manually compute m and b for this function using Excel.

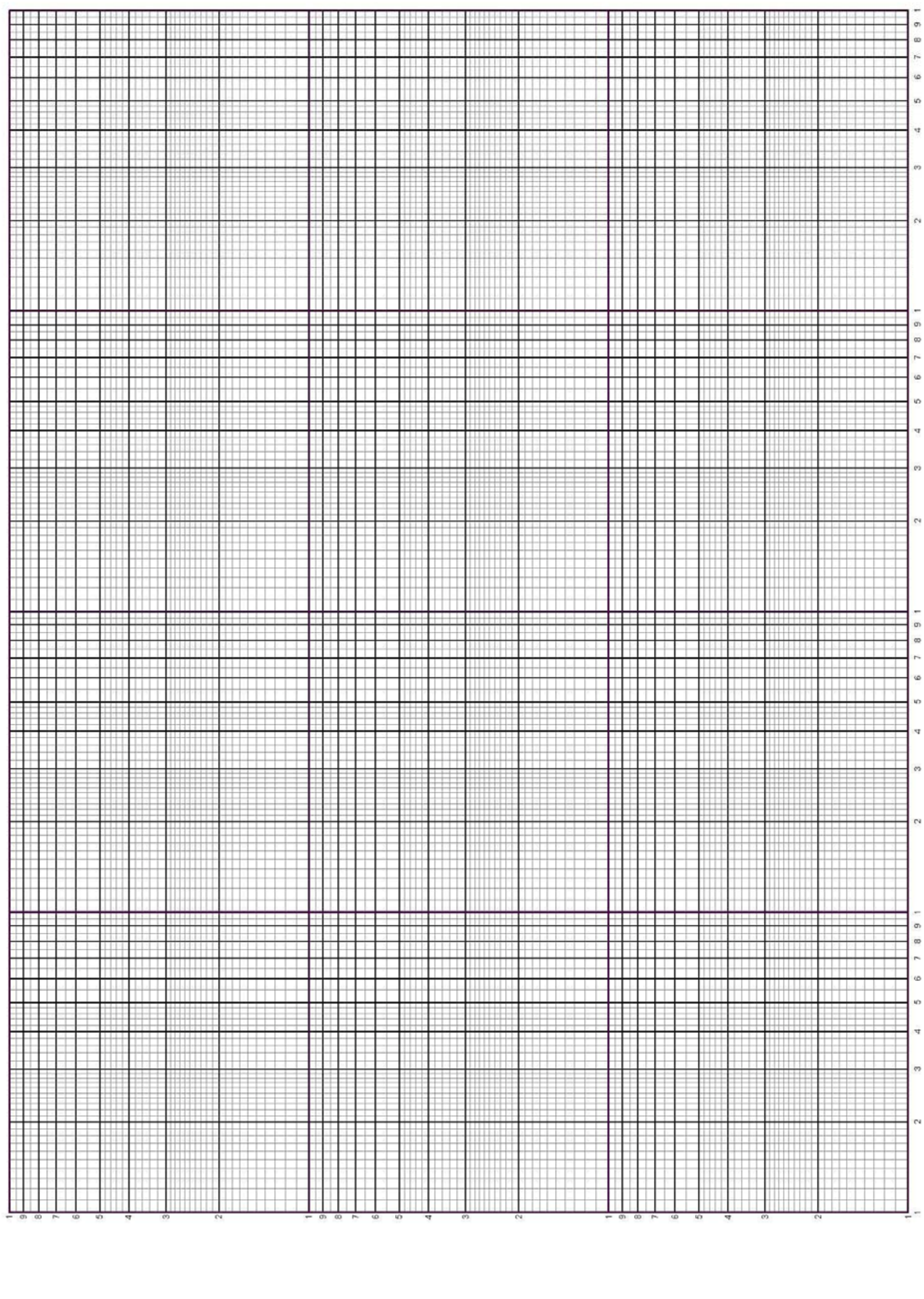
Reminder of equations:

$$m = \frac{n\sum x_i \ln(y_i) - \sum x_i \sum \ln(y_i)}{n\sum x_i^2 - (\sum x_i)^2} \qquad \ln(b) = \frac{\sum \ln(y_i) - m\sum x_i}{n} \qquad b = e^{\ln(b)}$$

Cartesian Graph Paper

SemiLogarithmic Graph Paper

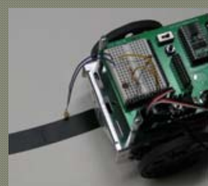
Notes: (1) powers of 10 on log axis (2) zero is not allowed on log axis



Log-Log Graph Paper

Soldering Safely

Freshman Engineering
Louisiana Tech University



Objective

- The objective of this tutorial is to prepare you for safe and successful soldering.
- Soldering is an integral skill in freshman engineering education.
- Learning to solder will help you in future engineering courses.

What is soldering?

- Soldering in the context of this class is the joining of metals through the heating of an easily melted, metallic material (solder).
- Soldering is useful for extending the length of circuit component wires and connecting circuit components together.
(Here 2 resistors are joined in series.)

Soldering Hazards

- Because the tip of a soldering iron can get up to 750 degrees F (this varies depending on the model), soldering is a fire hazard.
- Not only is soldering a fire hazard, but fumes associated with soldering are dangerous, especially to asthmatics.
- Materials used in soldering may contain lead or other toxins.
- Because of these hazards, it is important to practice safety first.

Soldering Equipment

- Equipment needed for extending the leads on the photoresistors is shown below.

Safety Step 1 - Location

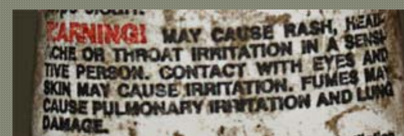
- Location is the most important decision when it comes to soldering.
- You should only solder in workshops and places with safety equipment nearby.
- Do not solder in rooms with carpeting or in your dorm rooms!

Step 2- Safety equipment

- Make sure that you are aware of the location of ALL safety equipment.
- The most important pieces of equipment for soldering are a first aid kit and a fire extinguisher.
- Respect your workshop and follow safety procedures. You may not think you need eye protection, but you often share a workshop with other students.

Step 3- Read all warning lables

- Reading all warning labels when you first use something is a good practice in engineering.
- Soldering chemicals, equipment, and flux may have information on how you should treat their fumes.
- Some soldering chemicals are lead based, and these should be handled with special precaution. When using lead products, wash your hands and take ventilation precautions.



Materials

- Now that you are informed about the safety precautions, get to know the materials you will be working with:
  - The Soldering Iron – This tool's tip heats up to melt the solder.
  - The Solder – The solder you will use will likely be silver-based. It has a low melting point.
  - The Flux – The flux is a chemical cleaner that prepares surfaces for soldering.

How to solder

- First, set up your workstation. Always keep your soldering iron on its proper mount when it is not in use.
- The soldering iron may take several minutes to heat up. Do not place your hands near the iron to feel if it is hot.

Prepare your wires

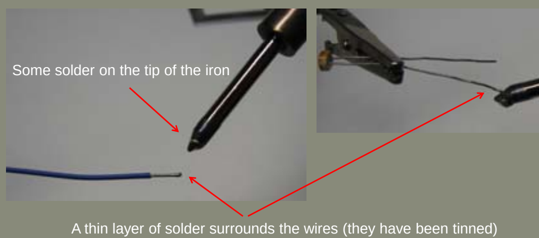
- For the photoresistor extensions, cut two 4-inch pieces of wire, and strip approximately ¼ inch of insulation from each end.

Applying flux and tinning the soldering iron

- First, apply the flux to both workpieces that you are connecting.
- We will be attaching extensions to photoresistors.
- After applying the flux, place the workpieces together.
- "Tin" the tip of the soldering iron with a small amount of solder, as shown in the figure.

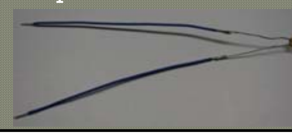
Pre-tinning the wires

- When joining two wires, it is helpful to tin the parts of both wires that will be soldered together; a vise is handy for holding wires. Just touch the tip of the soldering iron to the wire ends (flux already applied).

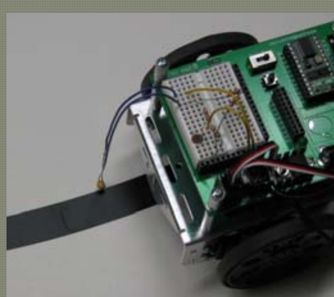


Soldering the joint

- Press the tinned soldering iron to the pre-tinned workpieces, thus applying heat.
- Add enough solder to make a strong joint. You won't need to add much (if any) since you have pre-tinned the wire ends.
- The finished product should look like this:



Boe-Bot with photoresistor extension



Clean up your workstation

- Whenever you are completely finished soldering, follow your workshop's cleanup protocol.
  - Turn off the soldering iron and let it sit for at least a half-hour so that it can cool.
  - Package and seal all of your flux.
  - Allow the room to ventilate.