

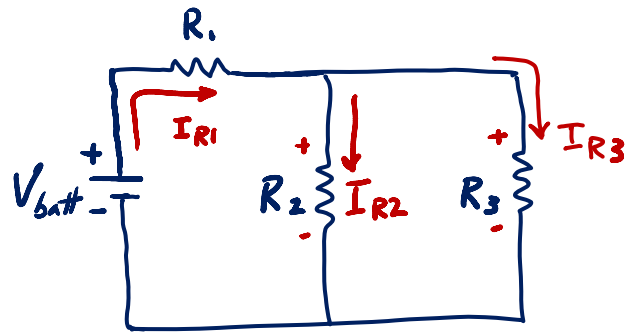
## KIRCHOFF'S VOLTAGE LAW: EXAMPLE 2

**GIVEN:** Consider the circuit shown, where

$$R_1 = 12\Omega \quad R_2 = 4\Omega \quad R_3 = 12\Omega \quad V_1 = 12V$$

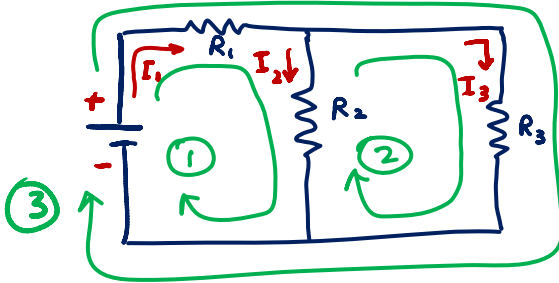
**REQUIRED:**

- The current through  $R_1$ .
- The current through  $R_2$ .
- The current through  $R_3$ .



**SOLUTION:**

(a) First, we identify the loops in the circuit. As shown below, we can choose any two of the three loops.



We use the assumed direction of current to determine whether voltage increases or decreases across a resistor.

**Kirchoff's Voltage Law:** The sum of the voltage drops around any closed loop is zero.

Applying KVL to Loop 1:

$$V_{\text{batt}} - V_{R1} - V_{R2} = 0$$

$$V_{\text{batt}} = V_{R1} + V_{R2} = I_{R1} \cdot R_1 + I_{R2} \cdot R_2$$

Unfortunately, we have one equation with two unknowns ( $I_{R1}$  and  $I_{R2}$ ). Note that  $I_{R1} \neq I_{R2}$  because  $I_{R1}$  splits between two resistors ( $R_2$  and  $R_3$ ). Students often do not recognize that the current SPLITS in this way.

Find  $R_{\text{eq}}$  for the circuit so that we can find  $I_{R1}$ :

$$R_{\text{eq}} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = 12\Omega + \frac{1}{\frac{1}{4\Omega} + \frac{1}{12\Omega}} = 12\Omega + \frac{12\Omega}{4} = 15\Omega$$

Apply Ohm's Law:

$$V_{\text{batt}} = I_{R1} \cdot R_{\text{eq}} \rightarrow I_{R1} = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{12V}{15\Omega} = \underline{\underline{0.8A}}$$

(b) Now that we know the current through  $R_1$ , we can use KVL around Loop 1 (as shown earlier):

$$\left. \begin{aligned} V_{\text{batt}} - V_{R1} - V_{R2} &= 0 \\ V_{\text{batt}} &= V_{R1} + V_{R2} = I_{R1} \cdot R_1 + I_{R2} \cdot R_2 \end{aligned} \right\} \begin{aligned} & \text{12V} \quad \text{0.8A} \quad \text{12}\Omega \\ & \text{4}\Omega \end{aligned} \quad \underline{\underline{I_{R2} = 0.6A}}$$

(c) Apply KVL to Loop 3:

$$V_{\text{batt}} - V_{R1} - V_{R3} = 0$$

$$I_{R3} \cdot R_3 = 12V - I_{R1} \cdot R_1$$

$$I_{R3} = \frac{1}{12\Omega} [12V - 0.8A \cdot 12\Omega] = \frac{2.4V}{12\Omega} = \underline{\underline{0.2A}}$$

DISCUSSION:

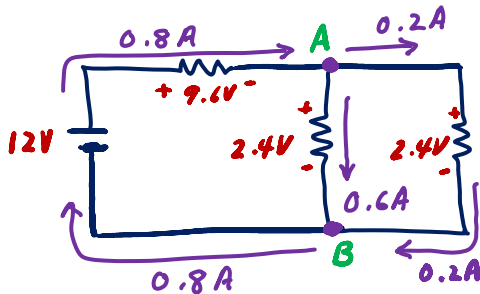
- We can apply KVL to Loop 2 to check our work:

$$V_{R2} - V_{R3} = 0$$

$$I_{R2} \cdot R_2 - I_{R3} \cdot R_3 = 0$$

$$2.4V - 2.4V = 0 \leftarrow \text{Loop 2 checks out!}$$

- Look at a voltage and current balance:



Notice that the voltages around all three loops sum to zero.

LOOP 1:  $12V - 9.6V - 2.4V = 0$  ✓  
 LOOP 2:  $2.4V - 2.4V = 0$  ✓  
 LOOP 3:  $12V - 9.6V - 2.4V = 0$  ✓

Notice that the currents entering and leaving nodes A and B balance

NODE A  $\left\{ \begin{array}{l} I_{in} = I_{out} \\ 0.8A = 0.2A + 0.6A \end{array} \right.$  ✓

- Notice that when the current splits at node A, more current goes through  $R_2$  ( $4\Omega$ ) than through  $R_3$  ( $12\Omega$ ). Electricity takes the path of least resistance.

