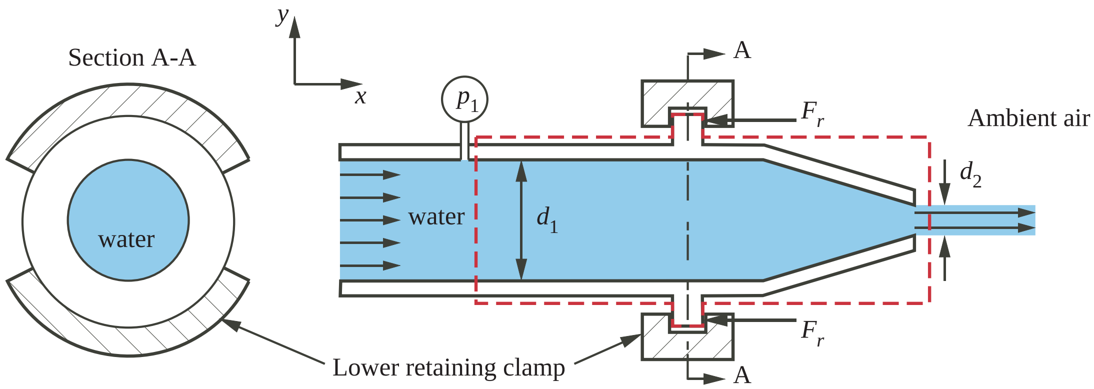
EAS 361 Quiz 3 Solution 28 November 2006

Water from a pipeline flows through the axisymmetric nozzle depicted in the sketch. The jet of water leaving the nozzle flows into the ambient air. A ridge around the external circumpherence of the nozzle provides a surface for attaching two annular retaining clamps that hold the nozzle in place.



1. The volumetric flow rate Q, the dimensions d_1 and d_2, and the reading on the pressure gage p_1 are known. Neglect the weight of the water and derive a single formula for the horizontal force on the retaining ring. The formula should be of the form $F = \ldots$ where all terms on the right hand side are known.

   Solution: Locate the (x, y) coordinate axes as shown in the schematic. Draw a control volume as indicated by the red dashed line. The control volume traces the boundary between the clamps and the external surface of the ridge on the nozzle body. It would also be acceptable to have the control volume cut through the ridge on horizontal planes.

   The control volume exposes the contact force F_r between the clamp and the ridge on the nozzle. The direction of F_r is assumed to be to the left, i.e. in the negative x direction. Two F_r vectors are shown in the sketch: one for the top clamp and one for the bottom clamp.

   Assume that the flow is steady and incompressible. Apply the linear momentum equation in the x direction. (Drop the unsteady term)

$$\sum F_x = \int_{CS} V_x \rho (\mathbf{V} \cdot \hat{\boldsymbol{n}})\, dA$$

   The left hand side (LHS) is the sum of pressure and reaction forces

$$\text{LHS} = p_1 A_1 - p_2 A_2 - 2F_r = p_1 A_1 - 2F_r$$

   where $p_2 = 0$ because the jet enters the ambient and the analysis is carried out in gage pressure units. The factor of two multiplying F_r accounts for the same magnitude of force being applied to the upper and lower clamps.

   Assume the velocity is uniform over the inlet and outlet, and evaluate terms on the right hand side (RHS)

$$\text{RHS} = \rho V_1(-V_1)A_1 + \rho V_2(V_2)A_2$$

   Use $Q = V_1 A_1 = V_2 A_2$ to simplify and collect terms

$$\text{RHS} = \rho Q V_2 - \rho Q V_1 = \rho Q V_1 \left(\frac{V_2}{V_1} - 1 \right)$$

Rearrange $V_1 A_1 = V_2 A_2$ to get $V_2/V_1 = (d_1/d_2)^2$, and

$$\text{RHS} = \rho Q V_1 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right]$$

Set LHS = RHS and solve for F_r

$$p_1 A_1 - 2F_r = \rho Q V_1 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right] \quad \Longrightarrow \quad F_r = \frac{1}{2} \left\{ p_1 A_1 - \rho Q V_1 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right] \right\}$$

Since $Q = V_1 A_1$ we can factor out a common A_1

$$F_r = \frac{A_1}{2} \left\{ p_1 - \rho V_1^2 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right] \right\} = \frac{\pi d_1^2}{8} \left\{ p_1 - \rho V_1^2 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right] \right\}$$

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2. Assume that $p_1 > 0$ (gage) and $d_1 > d_2$ (as shown). Under what conditions would the nozzle move to the right if the clamps were suddenly removed?

   Solution: The solution to problem 1 shows that a positive F_r acts to the left. Therefore, when F_r is positive, i.e. when the right hand side of the expression for F_r evaluates to a positive number, the nozzle will move to the right when the clamp is removed. The nozzle moves to the right when

$$F_r > 0 \quad \text{or} \quad p_1 > \rho V_1^2 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right].$$

   Note that both $p_1 A_1$ and $\rho V_1^2 \left[\left(\frac{d_1}{d_2} \right)^2 - 1 \right]$ are always positive. Also note that according to the momentum balance F_r could be either positive or negative. Further analysis is required to determine whether there are additional constraints on the sign of F_r.

   One case can be easily observed. Assume that $A_2 = 0$, i.e. that the nozzle has no opening. $A_2 = 0$ implies that $V_1 = 0$. Under this restriction the formula for F_r reduces to $F_r = p_1 A_1/2$. The factor of 1/2 accounts for the assumption that F_r acts on both the top and bottom clamps.

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3. The volumetric flow rate Q, the dimensions d_1 and d_2, and the reading on the pressure gage p_1 are known. Derive a formula for the head loss for the flow through the nozzle. The head loss should be of the form $h_L = \ldots$ where all terms on the right hand side are known.

   Solution: Assume the flow is steady, incompressible, with one inlet and one outlet. Assume the velocity profile over the inlet and outlet is uniform. Apply the steady flow energy equation with station 2 as the "out" condition and station 1 as the "in" condition.

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L$$

   Simplify with $z_1 = z_2$, $p_2 = 0$ and $h_s = 0$ (no pump)

$$\Longrightarrow \frac{V_2^2}{2g} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_L$$

Solve for h_L

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

Note: it is more to set $V_2 = 0$ because the fluid leaving the jet will eventually come to rest without doing further work (to reduce the energy wasted and entropy increased). We'll leave $V_2 \neq 0$ for now, but return to consider $V_2 = 0$ in the answer to question 4.

Factor out V_1

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g}\left[1 - \frac{V_2^2}{V_1^2}\right]$$

Use $V_1 A_1 = V_2 A_2 \Longrightarrow V_2/V_1 = (d_1/d_2)^2$ to replace V_2^2/V_1^2

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g}\left[1 - \frac{d_1^4}{d_2^4}\right] = \frac{p_1}{\gamma} - \frac{V_1^2}{2g}\left[\frac{d_1^4}{d_2^4} - 1\right]$$

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4. Is there a value of p_1 that would make the head loss zero? If so, what is that value?

Solution: $d_1/d_2 > 0$ and $p_1 > 0$ so $\left[\dfrac{d_1^4}{d_2^4} - 1\right] > 0$ and $\dfrac{p_1}{\gamma} > 0$.

Therefore, $h_L = 0$ if

$$\frac{p_1}{\gamma} = \frac{V_1^2}{2g}\left[\frac{d_1^4}{d_2^4} - 1\right]$$

But the head loss calculation model used $V_2 \neq 0$, which assumes the fluid stream leaving the nozzle can do useful work. If instead we neglect the kinetic energy of the fluid leaving the nozzle, i.e. if we set $V_2 = 0$ in the expression for h_L we get

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g}$$

Therefore, when the kinetic energy of the exit jet is assumed to be wasted, $h_L > 0$ always (because $p_1 > 0$ and $V_1^2 > 0$). Note that for a physically realistic flow, h_L is never negative.

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