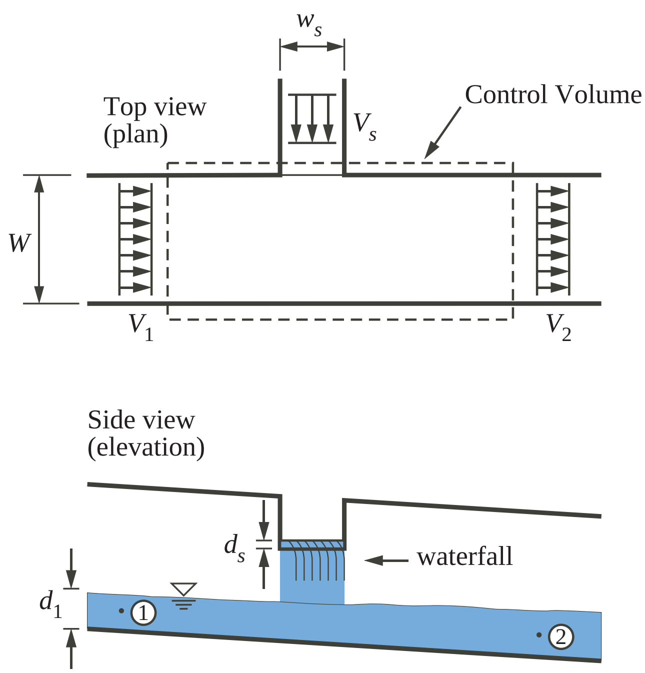
EAS 361 Quiz 2 Solution 16 November 2006

The sketch depicts a concrete drainage ditch with perfectly straight walls and a perfectly flat (but sloped) bottom. Water in the ditch flows freely downhill. The top surface of the water is open to the atmosphere. At the upstream station 1, the average velocity is V_1 and the water depth is d_1. Flow from a side channel enters the drainage ditch with average velocity V_s and depth d_s.



You have paid a consultant to perform some measurements on the water velocity in the ditch. The consultant reports that the velocity at station 2 is the same as station 1, i.e. $V_2 = V_1$. A co-worker tells you that because of the flow from the side channel, V_2 cannot be equal to V_1.

1. [**5 points**] Is there a plausible explanation for $V_2 = V_1$ when $V_s \neq 0$? In other words is it *physically possible* that V_2 can be equal to V_1 when $V_s \neq 0$? (Your answer should be either “yes” or “no”). Assume that the velocity profiles are uniform everywhere in this flow.

   Solution: **Yes**, $V_2 = V_1$ is plausible.

2. [**15 points**] If you answered “yes” to question 1, then explain how the consultant’s measurements can be consistent with the laws of fluid mechanics. If you answered “no” then why is there no chance that the consultant’s measurement could be correct?

   Solution: Water is incompressible. Assume the flow is steady. Apply mass conservation to the control volume in the sketch. The instructions say to assume that velocity profiles are uniform.

$$\int_{CS} \rho(\vec{V} \cdot \hat{n})\, dA = \rho\left[(-V_1)Wd_1 + (-V_s)w_s d_s + (V_2)Wd_2\right] = 0$$

   Since ρ cannot be zero, the previous equation simplifies to

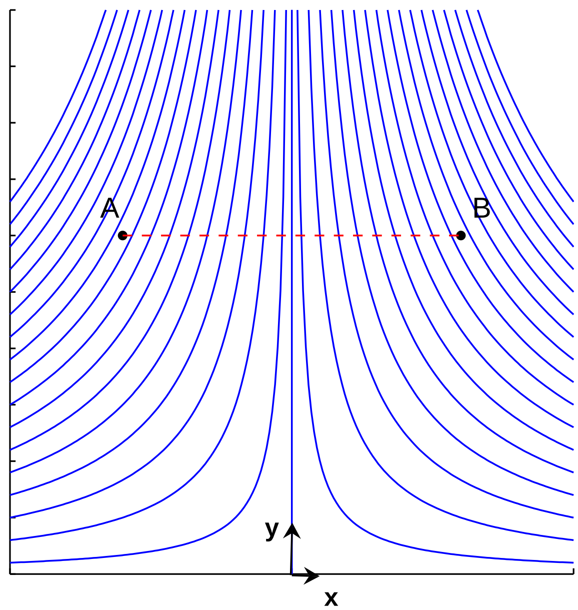
$$V_1 W d_1 + V_s w_s d_s = V_2 W d_2$$

Now let $V_1 = V_2$ and solve for V_2

$$V_1 = V_2 \Longrightarrow V_s w_s d_s = V_2(Wd_2 - Wd_1) \Longrightarrow V_2 = \frac{V_s w_s d_s}{W(d_2 - d_1)}$$

As long as $d_2 > d_1$ the preceding equation makes sense. In other words, if $V_2 = V_1$ then the outlet depth has to be greater than the inlet depth.

◊



3. **[20 points]** The diagram shows the streamlines of a stagnation point flow. The velocity field is $\vec{V} = \hat{\boldsymbol{\imath}}u + \hat{\boldsymbol{\jmath}}v$ where $u = 2x$, and $v = -2y$. Assume that the flow field has a depth b into the page. Given the formulas for u and v, what is the formula for computing the *downward volumetric flow rate* between point A and point B? Let points A and B have the coordinates (x_A, y_A) and (x_B, y_B), respectively. Your answer should start with a basic expression for (or definition of) the volumetric flow rate and end with a formula that does not have the symbols u or v.

Solution: The volumetric flow rate is

$$Q = \int_{\text{area}} \vec{V} \cdot \hat{\boldsymbol{n}} dA$$

On the surface A–B, $|\vec{V} \cdot \hat{\boldsymbol{n}}| = v(y_A) = -2y_A = \text{constant}$. We'll work the sign out in the end because there is no outward normal to a surface that does not surround a volume. Evaluate Q with $dA = b\,dx$.

$$Q = \int_{x_A}^{x_B} (-2y_A) b dx = -2y_A b \int_{x_A}^{x_B} dx = -2y_A b(x_B - x_A).$$

The value of Q is negative because v is negative. The problem statement asks for the downward flow rate so the (positive) value of the downward flow rate is

$$Q_{\text{down}} = +2y_A b(x_B - x_A).$$