EAS 361 | Final Exam | 5 December 2006

with corrections December 5, 2006

Print Your Name: ____

Sign Your Name: ____

This exam booklet contains

1. This cover sheet.
2. A short list of potentially useful formulas and data.
3. Five multiple choice questions that require justification of your choice. Each of these questions is worth 10 points
4. Four short answer questions worth 15 points each.
5. Three longer problems worth 25 points. **Choose only two of these three longer problems.**

The maximum score on the exam is 160 points.

Do not open the exam booklet until you are instructed to do so.

You will have 1 hour and 50 minutes to complete the exam.

Universal Cheat Sheet

Properties of Water at 20 °C:

$$\rho = 999\ \frac{\text{kg}}{\text{m}^3} = 1.94\ \frac{\text{slugs}}{\text{ft}^3} \qquad \mu = 1.12\times10^{-3}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} = 2.34\times10^{-5}\ \frac{\text{lb}_\text{f}\cdot\text{s}}{\text{ft}^2}$$

Properties of Air at 1 atm and 20 °C:

$$\rho = 1.23\ \frac{\text{kg}}{\text{m}^3} = 2.38\times10^{-3}\ \frac{\text{slugs}}{\text{ft}^3} \qquad \mu = 1.79\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} = 3.74\times10^{-7}\ \frac{\text{lb}_\text{f}\cdot\text{s}}{\text{ft}^2}$$

Basic Constants

$$R_u = 8.314\ \frac{\text{kJ}}{\text{kmol}} = 1545\ \frac{\text{ft}\cdot\text{lb}_\text{f}}{\text{lbmol}\cdot\text{R}} \quad R_\text{air} = 286.9\ \frac{\text{J}}{\text{kg}\cdot\text{K}} = 1716\ \frac{\text{ft}\,\text{lb}_\text{f}}{\text{slug}^\circ\text{R}} \quad g_c = 32.174\ \frac{\text{lb}_\text{m}\,\text{ft}}{\text{lb}_\text{f}\,\text{s}^2}$$

Conservation Principles:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\mathcal{V} + \int_{CS} \rho(\mathbf{V}\cdot\hat{\boldsymbol{n}})\, dA$$

$$\sum \mathbf{F} = \frac{\partial}{\partial t}\int_{CV} \mathbf{V}\,\rho\, d\mathcal{V} + \int_{CS} \mathbf{V}\rho(\mathbf{V}\cdot\hat{\boldsymbol{n}})\, dA$$

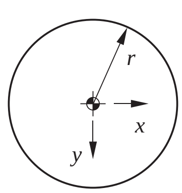
$$\dot{Q}_\text{net,in} + \dot{W}_\text{shaft,in} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, d\mathcal{V} + \int_{CS}\left(h + \frac{V^2}{2} + gz\right)\rho(\mathbf{V}\cdot\hat{\boldsymbol{n}})\, dA$$

$$\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_\text{out} = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_\text{in} + h_s - h_L$$

Miscellaneous:

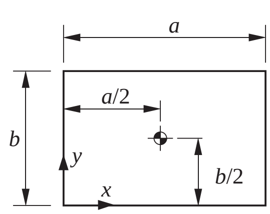
$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

Properties of Sections:



$$I_{xc} = I_{yc} = \frac{\pi r^4}{4}$$

$$I_{xyc} = 0$$



$$I_{xc} = \frac{1}{12}ab^3 \quad I_{yc} = \frac{1}{12}ba^3$$

$$I_{xyc} = 0$$

Properties of Volumes:

Sphere: $V = \dfrac{\pi D^3}{6}, \quad A = \pi D^2$

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Problems 1 through 5 are a combination of multiple choice and reasoning. For each problem, circle the choice that most correctly completes the sentence. In addition to choosing one of the answers, provide a short justification for your choice.

For Problems 1 through 5 the correct choice is worth 5 points and the justification is worth 5 points.

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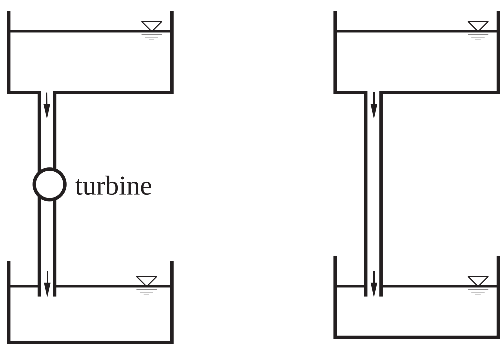
1. [**10 points**] The sketch below shows water flowing from an upper reservoir to a lower reservoir. Applying the energy equation to each of the two arrangements shows that the head loss for the system with the turbine

   a. is greater than the head loss for the system without the turbine.

   b. is less than the head loss for the system without the turbine.

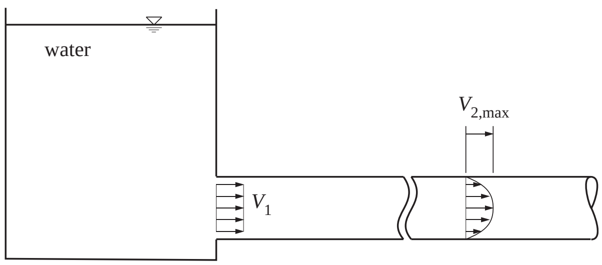
   c. is the same as the head loss for the system without the turbine.

   d. may be greater or less than the head loss for the system without the turbine, depending on the efficiency of the turbine.



2. [**10 points**] In the sketch below, water flows steadily from a very large tank into a horizontal pipe. At station 1 the velocity profile is uniform. At station 2, which is far downstream from station 1 the profile has a maximum on the centerline. If V_1 is the average velocity at station 1, and $V_{2,\max}$ is the *maximum* velocity at station 2, which one of the following statements is true? *Do not* attempt to compare the magnitudes of V_1 and $V_{2,\max}$ based on the length of the arrows in the sketch. The arrows representing the velocity vectors at station 1 and station 2 are *not drawn to scale.*

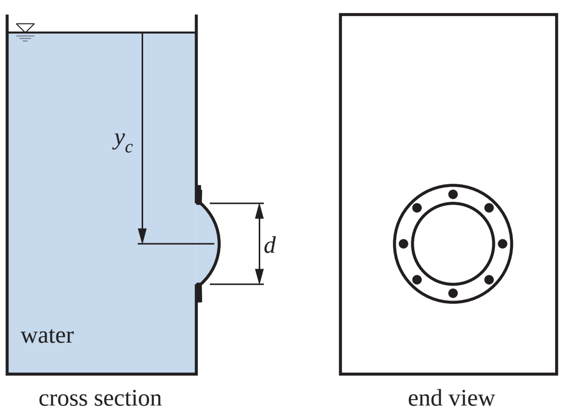
   a. $V_{2,\max} < V_1$
   b. $V_{2,\max} = V_1$
   c. $V_{2,\max} > V_1$
   d. The relative values of V_1 and $V_{2,\max}$ cannot be determined without more information.



3. [**10 points**] A dome shaped bulkhead of diameter, d, is located on the side of a large rectangular tank of water. The geometric center of the bulkhead is a distance, y_c, down from the free surface.

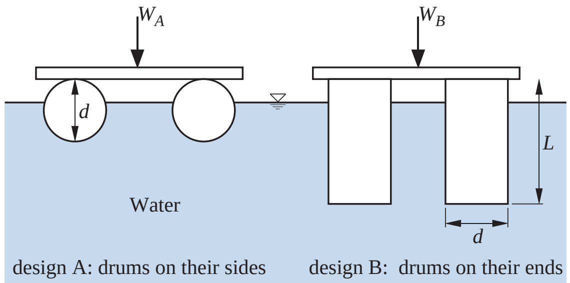
   The center of pressure for the bulkhead is located at

   (a) y_c (b) $y_c + \dfrac{d}{2} - \dfrac{d}{3}$ (c) $y_c + \dfrac{d}{3}$ (d) None of the above.



4. **[10 points]** You have been asked to design a floating deck for swimmers at a lake. Four identical, cylindrical drums (diameter d, and length L) are to be used for floatation. Two designs being considered depicted in the sketch. (Only two of the four drums are visible in each design.) The top surface of the deck is the same for both designs. To provide the deck capable of carrying the maximum weight

   a. choose design A.
   b. choose design B.
   c. choose design A or B, both can carry the same weight.
   d. more information about the material used to construct the deck is needed before deciding which design can carry more weight.



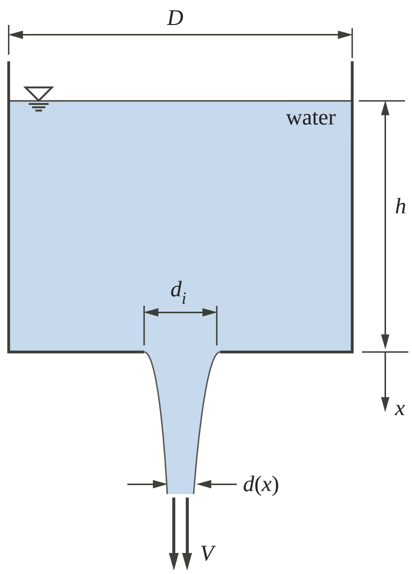
5. **[10 points]** The PSU Human Powered Vehicle (HPV) Team is designing an aerodynamic bicycle that has a fairing to surround the rider and reduce drag. The HPV team wants to use a wind tunnel to measure the drag on a scale model of the fairing. In order to maintain similitude, the Reynolds number $\rho V L/\mu$ for the model tests must be the same as the Reynolds number for the real fairing at the design speed for the bicycle.

   The model will be 1/10 the size of the real fairing. To maintain similitude, the air velocity in the wind tunnel

   a. must be 1/10 of the design speed.
   b. must be 10 times the design speed.
   c. must be equal to the design speed.
   d. cannot be determined from the information given.

Show all your work for the remaining problems. Minimal credit will be given to correct answers that are not justified with appropriate mathematical analysis.

6. [**15 points**] A cylindrical tank of water has a hole of diameter d_i in the bottom. You observe that the stream of water draining from the bottom of the tank gets narrower with x. Why? Use a mathematical formula to show that the $d(x)$ must decrease.



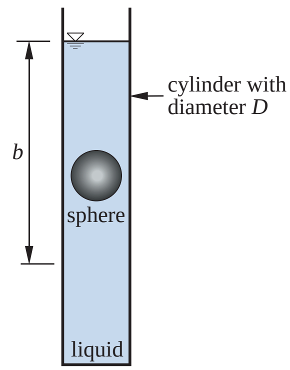
7. [**15 points**] In a book on whitewater rescue techniques, the authors try to demonstrate how easy it is to misjudge the force of flowing water. The table to the right shows how an increase in the water velocity increases the force exerted by the water on the legs of a person standing in a river. Assume that water has the same knee-high depth for all speeds and that the velocity profile is uniform. What value of force should replace the "?" in the table?

velocity (ft/s)	water force lb_f
3	16.8
6	67.2
9	?

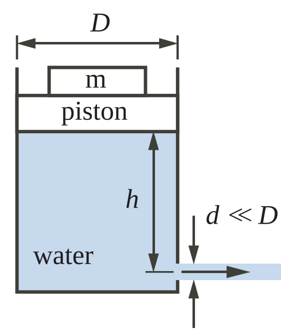
8. [**15 points**] The viscosity, μ, of a liquid can be measured by determining the time, t, it takes for a sphere of diameter, d, to settle slowly through a distance, b, in a vertical cylinder of diameter, D, containing the liquid. The apparatus for making such a measurement is shown in the sketch to the right. Assume that the ball velocity $V = b/t$ is related to the other parameters of the problem by

$$V = f(d, D, \mu, \gamma, W)$$

where μ is the viscosity of the fluid, γ is the specific weight of the fluid, and W is the weight of the ball.
Use repeating variables d, μ, and γ to find a dimensionless group for V.



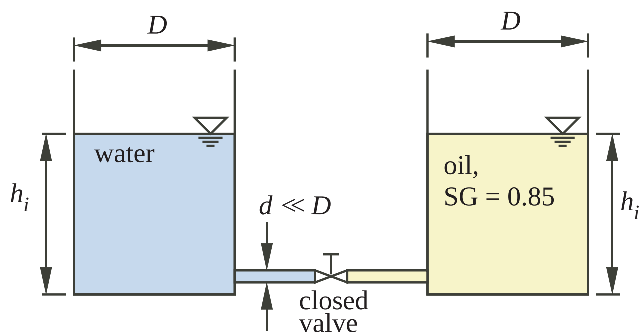
9. [**15 points**] A cylindrical tank of water is fitted with a frictionless piston. A mass m is placed on the top of the piston. There is a hole of diameter d in the side of the tank. For $t < 0$ a plug is placed in the hole in the side of the tank and the mass and piston are held in place by a mechanism not shown in the sketch. At $t = 0$ the plug is removed and the mass and piston are released. Derive a formula for the velocity of the water jet as a function of h.



Choose two of the three problems 10, 11 and 12. Clearly indicate which problem you wish to have graded.

10. [**25 points**] Two identical cylindrical tanks are connected by a pipe with shut-off valve. One tank is filled to a depth h_i with water, and the other tank is filled to a depth h_i with oil having SG = 0.85. Both tanks are open to atmospheric pressure at the top. The diameter of the connecting pipe is much smaller than the diameter of the tanks.

    The valve is opened and the two fluids reach a new static equilibrium. Let h_w be the height of the free surface for the tank that originally contained only water. Let h_o be the height of the free surface for the tank that originally contained only oil. What are the equilibrium values of h_w and h_o when the valve is open? Neglect the volume of fluid in the connecting pipe. Assume h_i is known. The solutions are two simple formulas for h_w and h_o in terms of h_i and SG.



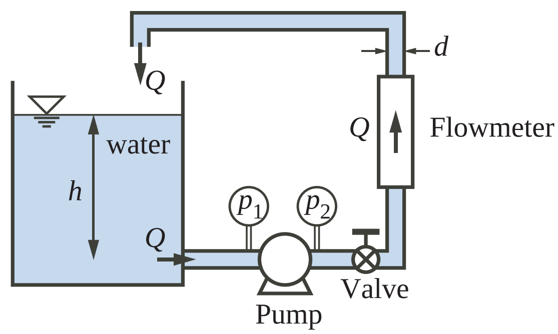
Choose two of the three problems 10, 11 and 12. Clearly indicate which problem you wish to have graded.

11. The schematic depicts a pump test stand that is used to measure the ability of a pump to deliver a volumetric flow rate Q when working against a flow resistance. The table to the right shows the type of data measured during a pump test. The $\sim$ symbol represents a numerical value recorded during the test. All piping in the test stand has the same diameter d.

Valve	Q	p_1	p_2	$\dot{W}_{\text{in}}$
closed	0	$\sim$	$\sim$	$\sim$
1/4 open	$\sim$	$\sim$	$\sim$	$\sim$
1/2 open	$\sim$	$\sim$	$\sim$	$\sim$
3/4 open	$\sim$	$\sim$	$\sim$	$\sim$

a. **[10 points]** Assume that measured data for Q, p_1 and p_2 are available for each setting of the control valve. Derive an expression for the head loss between the downstream side of the pump and the free surface of the tank. Consider only the head loss downstream of the pump.

b. **[15 points]** The $\dot{W}_{\text{in}}$ column is the measured electrical power supplied to the motor that drives the pump. Derive an expression for efficiency $\eta = \dot{W}_{\text{shaft}}/\dot{W}_{\text{in}}$ in terms of the measured data where $\dot{W}_{\text{shaft}}$ is the shaft power actually delivered to the water. Neglect head loss in the pump. (Head loss in the pump reduces the head gain that the pump can produce, so internal head loss is already accounted for in the data.)



Choose two of the three problems 10, 11 and 12. Clearly indicate which problem you wish to have graded.

12. [**25 points**] An upward jet of water from a pipe of diameter d impinges on a circular plate of diameter D. The plate has a mass m, and it is suspended by thin cable attached to a spring scale. The other end of the spring scale is attached to a wall. What flow rate Q is needed so that the spring scale reads zero? Neglect friction in the pulley and the mass of the cable and mass of the spring scale.