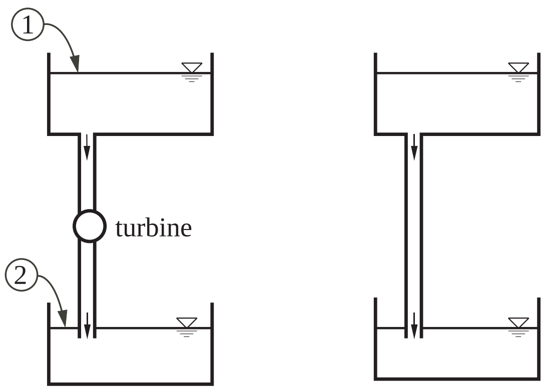
1. The sketch below shows water flowing from an upper reservoir to a lower reservoir. Applying the energy equation to each of the two arrangements shows that the head loss for the system with the turbine **is less than the head loss for the system without the turbine.**



Reason: Apply the energy equation.

$$\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_{\text{out}} = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_{\text{in}} + h_s - h_L$$

The inlet is station 1 on the upper free surface. The outlet is station 2 on the lower free surface. At the free surfaces: $p_1 = 0$, $V_1 = 0$, $p_2 = 0$, $V_2 = 0$. The energy equation simplifies to

$$z_2 = z_1 + h_s - h_L \qquad \Longrightarrow \qquad h_{L,\text{T}} = h_s + z_1 - z_2 \qquad \text{with the turbine}$$

Without the turbine $h_s = 0$ so

$$h_{L,\text{NT}} = z_1 - z_2 \qquad \text{without turbine}$$

For a turbine $h_s < 0$ because work is being done *on* the fluid. Therefore $h_{L,\text{T}} < h_{L,\text{NT}}$

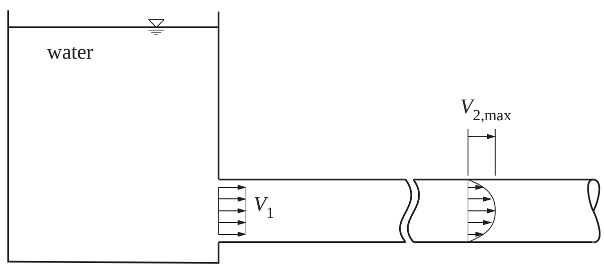
2. In the sketch below, water flows steadily from a very large tank into a horizontal pipe. At station 1 the velocity profile is uniform. At station 2, which is far downstream from station 1 the profile has a maximum on the centerline. If V_1 is the average velocity at station 1, and $V_{2,\text{max}}$ is the *maximum* velocity at station 2, which one of the following statements is true? *Do not* attempt to compare the magnitudes of V_1 and $V_{2,\text{max}}$ based on the length of the arrows in the sketch. The arrows representing the velocity vectors at station 1 and station 2 are *not drawn to scale.*

   Solution: $V_{2,\text{max}} > V_1$

   Reason: Apply the following facts:

   - The velocity profile at station 1 is uniform. Therefore, the average velocity at station 1 is V_1.
   - The problem statement indicates that the flow is steady. Since the flow is steady and water is incompressible, the average velocity is the same at station 1 and station 2. Therefore, the average velocity at station 2 is also V_1.
   - The diagram indicates that the velocity at station 2 is not uniform: the velocity is zero at the wall and a maximum in the center of the pipe. The average velocity at station 2 must be between zero and the maximum velocity at station 2, i.e. $0 \le V_{2,\text{ave}} \le V_{2,\text{max}}$.

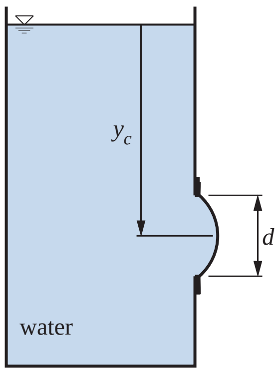
   Therefore, $V_{2,\text{max}} > V_1$ because $V_{2,\text{max}} > V_{2,\text{ave}}$ and $V_{2,\text{ave}} = V_{1,\text{ave}} = V_1$.



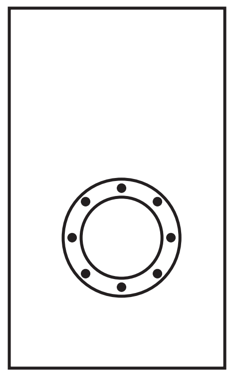
3. A dome shaped bulkhead of diameter, d, is located on the side of a large rectangular tank of water. The geometric center of the bulkhead is a distance, y_c, down from the free surface.

   The center of pressure for the bulkhead is located at

   (a) y_c (b) $y_c + \frac{d}{2} - \frac{d}{3}$ (c) $y_c + \frac{d}{3}$ (d) None of the above.



cross section



end view

Reason: The center of pressure on the bulkhead is at

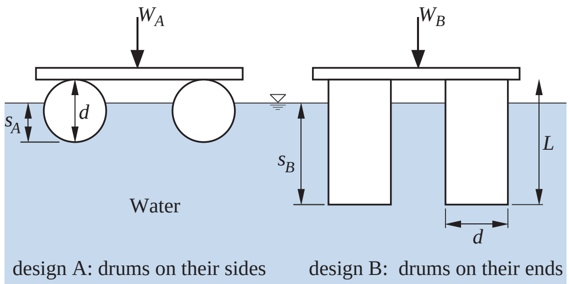
$$y_{\text{CP}} = y_c + \frac{I_{xc}}{y_c A}$$

where y_c is the depth of the centroid, I_{xc} is the moment of inertia of the bulkhead about a horizontal axis through the centroid, and $A = \pi r^2$ is the area of the bulkhead. From the universal cheat sheet $I_{xc} = \pi r^4/4$ so

$$y_{\text{CP}} = y_c + \frac{\pi r^4/4}{y_c \pi r^2} = y_c + \frac{r^2}{4y_c} = y_c + \frac{d^2}{16y_c}$$

where $r = d/2$.

4. You have been asked to design a floating deck for swimmers at a lake. Four identical, cylindrical drums (diameter d, and length L) are to be used for floatation. Two designs being considered depicted in the sketch. (Only two of the four drums are visible for each design in the sketch.) The top surface of the deck is the same for both designs. To provide the deck capable of carrying the maximum weight **choose design A or B, both can carry the same weight.**



Reason: The maximum weight is carried when the drums are completely submerged. Since the total volume of the drums is the same for both designs, the *maximum* load is the same for both designs because the maximum submergible volume is the same.

Design A is better because it is more stable, but that is a different design criteria.

The two designs have different relationships between the applied load and the depth of submergence. Let s_A and s_B be the depths to which the two tanks are submerged. The submerged volume of the tanks in design B increases linearly with s_B. The submerged volume of the tanks in design A increases nonlinearly with s_A.

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5. The PSU Human Powered Vehicle (HPV) Team is designing an aerodynamic bicycle that has a fairing to surround the rider and reduce drag. The HPV team wants to use a wind tunnel to measure the drag on a scale model of the fairing. In order to maintain similitude, the Reynolds number $\rho VL/\mu$ for the model tests must be the same as the Reynolds number for the real fairing at the design speed for the bicycle.

The model will be 1/10 the size of the real fairing. To maintain similitude, the air velocity in the wind tunnel **must be 10 times the design speed**.

Reason: For similitude $\mathrm{Re}_{\mathrm{model}} = \mathrm{Re}_{\mathrm{full\ scale}}$. Let f.s. designate the full scale fairing.

$$\frac{\rho V_{\mathrm{model}} L_{\mathrm{model}}}{\mu} = \frac{\rho V_{\mathrm{f.s}} L_{\mathrm{f.s.}}}{\mu} \quad \Longrightarrow \quad \frac{V_{\mathrm{model}}}{V_{\mathrm{f.s.}}} = \frac{L_{\mathrm{f.s.}}}{L_{\mathrm{model}}} = 10$$

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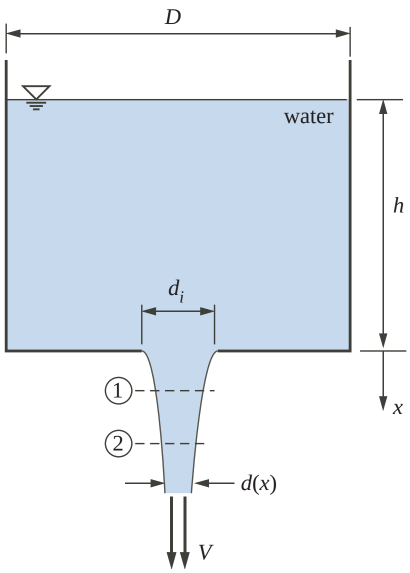
6. A cylindrical tank of water has a hole of diameter d_i in the bottom. You observe that the stream of water draining from the bottom of the tank gets narrower with x. Why? Use a mathematical formula to show that the $d(x)$ must decrease.



Solution: Consider the velocity and diameter of the free falling stream at station 1 and station 2. The fluid accelerates as it falls downward under the influence of gravity, therefore $V_2 > V_1$.

For steady, incompressible flow, ass conservation in the stream requires $V_2 A_2 = V_1 A_1$ where A is the cross sectional area of the stream. Assume that the streams are round

$$V_2 \frac{\pi d_2^2}{4} = V_1 \frac{\pi d_1 2^2}{4} \qquad \Longrightarrow \qquad d_2 = d_1 \sqrt{\frac{V_1}{V_2}}$$

Therefore if V increases with x, then d must decrease with x.

So, to answer the question "Why?": The diameter decreases with x because the velocity increases with x.

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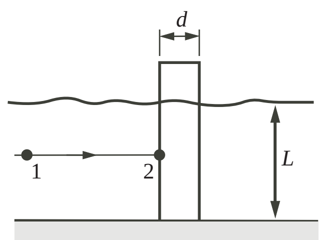
7. In a book on whitewater rescue techniques, the authors try to demonstrate how easy it is to misjudge the force of flowing water. The table to the right shows how an increase in the water velocity increases the force exerted by the water on the legs of a person standing in a river. Assume that water has the same knee-high depth for all speeds and that the velocity profile is uniform. What value of force should replace the "?" in the table?

velocity (ft/s)	water force lb_f
3	16.8
6	67.2
9	?

Solution: The relationship between the force on the leg and the velocity of the water is obtained by a simple scaling argument. The force of the the water increases with the pressure on the front of the leg. By estimating how the pressure on the front of the leg varies with velocity, it is possible to estimate how the force on the leg varies with velocity.

The following sketch depicts a leg as a cylinder of diameter d submerged to a depth L in a stream with on-coming water velocity V.



The force on the front of the leg depends on the pressure and the frontal area of the leg

$$F \sim pA = pLd$$

where L is the depth of the stream and d is the diameter of the leg.

Apply the Bernoulli equation along the stagnation streamline approaching the leg.

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

At the stagnation point $V_2 = 0$ so

$$p_2 - p_1 = \frac{1}{2}\rho V_1^2$$

From this we conclude that the pressure on the front of the leg is proportional to the square of the velocity of the free-flowing water. Therefore,

$$F \sim pLd \sim \frac{1}{2}\rho V^2 LD.$$

and we conclude that F scales varies with V^2

For two different water velocities V_a and V_b we can take the ratio of forces

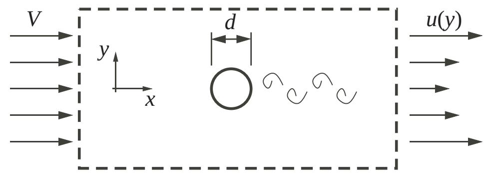
$$\frac{F_a}{F_b} = \frac{V_a^2}{V_b^2} \quad \text{or} \quad F_a = F_b \frac{V_a^2}{V_b^2}$$

The ratio eliminates the need to know L or d. Using the data in the table with $V_a = 9$ ft/s, $V_b = 6$ ft/s, and $F_b = 67.2\,\text{lb}_\text{f}$ gives

$$\boxed{F_a = (67.2\,\text{lb}_\text{f})\frac{9^2}{6^2} = 151\,\text{lb}_\text{f}}$$

Why can't the momentum integral equation be used?

The momentum integral requires information about the velocity entering and leaving a control volume around the leg. The following sketch is a top view of a "leg" of diameter d in a uniform on-coming flow with velocity V.



The dashed line represents a control volume around the leg. The control volume must be sufficiently large in the y direction that water does not leave and enter through the top and bottom boundaries, i.e. the streamlines at the top and bottom boundaries of the control volume must be parallel to the top and bottom edges of the control volume.

If the water flow exerts a force on the leg, then either the velocity or pressure (or both) must change from the inlet to the outlet. Since the river is an open channel, there will be no pressure difference from the inlet to the outlet. We neglect changes in elevation. Therefore, if the fluid exerts a force on the leg, there must be a change in the velocity profile from the inlet to the outlet. Since the problem statement gave no information about the velocity profile, the momentum integral equation is not useful for solving this problem.

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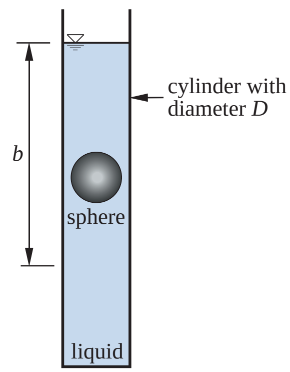
8. The viscosity, μ, of a liquid can be measured by determining the time, t, it takes for a sphere of diameter, d, to settle slowly through a distance, b, in a vertical cylinder of diameter, D, containing the liquid. The apparatus for making such a measurement is shown in the sketch to the right. Assume that the ball velocity $V = b/t$ is related to the other parameters of the problem by

$$V = f(d, D, \mu, \gamma, W)$$

where μ is the viscosity of the fluid, γ is the specific weight of the fluid, and W is the weight of the ball. Use repeating variables d, μ, and γ to find a dimensionless group for V.



Solution: Apply the systematic method for forming a dimensionless group involving V and the repeating variables d, μ, and γ.

First, identify the dimensions of the repeating variables.

$$[d] = \mathrm{L}$$
$$[\mu] = \frac{\mathrm{M}}{\mathrm{LT}}$$
$$[\gamma] = \frac{\mathrm{F}}{\mathrm{L}^3} = \frac{\mathrm{ML/T}^2}{\mathrm{L}^3} = \frac{\mathrm{M}}{\mathrm{L}^2\mathrm{T}^2}$$

Next, form the product of the non-repeating variable V with the repeating variables raised to the undetermined powers.

$$\Pi = V d^a \mu^b \gamma^c$$

where a, b, and c are integers to be determined.

The Pi group must be dimensionless, so

$$\left[V d^a \mu^b \gamma^c\right] = \mathrm{M}^0\mathrm{L}^0\mathrm{T}^0$$

$$\Longrightarrow \left(\frac{\mathrm{L}}{\mathrm{T}}\right)(\mathrm{L})^a\left(\frac{\mathrm{M}}{\mathrm{LT}}\right)^b\left(\frac{\mathrm{M}}{\mathrm{L}^2\mathrm{T}^2}\right)^c = \mathrm{M}^0\mathrm{L}^0\mathrm{T}^0$$

The powers of M, L, and T in the preceding equation must be the same on both sides.

$$\mathrm{M}: \quad b + c = 0$$
$$\mathrm{L}: \quad 1 + a - b - 2c = 0$$
$$\mathrm{T}: \quad -1 - b - 2c = 0$$

The M equation gives $b = -c$. Substitute $b = -c$ into the T equation and solve for c:

$$-1 + c - 2c = 0 \Longrightarrow -1 - c = 0 \Longrightarrow c = -1 \quad \text{and} \quad b = +1.$$

Substitute $c = -1$ and $b = +1$ into the L equation to solve for a:

$$1 + a - 1 + 2 = 0 \quad \Longrightarrow \quad a = -2$$

Therefore

$$\Pi = V d^{-2}\mu^1\gamma^{-1} \quad \text{or} \quad \boxed{\Pi = \frac{V\mu}{\gamma d^2}}$$

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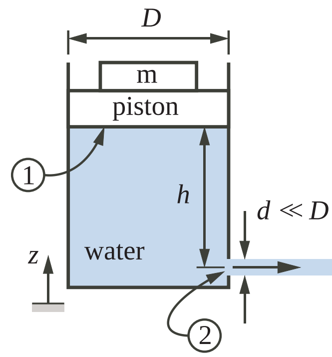
9. A cylindrical tank of water is fitted with a frictionless piston. A mass m is placed on the top of the piston. There is a hole of diameter d in the side of the tank. For $t < 0$ a plug is placed in the hole in the side of the tank and the mass and piston are held in place by a mechanism not shown in the sketch. At $t = 0$ the plug is removed and the mass and piston are released. Derive a formula for the velocity of the water jet as a function of h.



Solution: Identify station 1 as the underside of the piston and station 2 as the exit hole through which the water leaves the tank. Neglect losses on the streamline between stations 1 and 2, and apply the Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

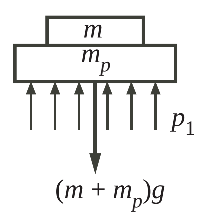
Since $d \ll D$ the downward velocity of the piston is negligible, i.e. $V_1 \approx 0$. Station 2 is the beginning of the free jet so $p_2 = 0$. With these simplifications the Bernoulli equation can be rearranged as

$$\frac{V_2^2}{2g} = \frac{p_1}{\gamma} + z_1 - z_2 = \frac{p_1}{\gamma} + h.$$

Solve for V_2:

$$V_2^2 = \frac{2p_1}{\rho} + 2gh \quad \Longrightarrow \quad V_2 = \sqrt{2\left(\frac{p_1}{\rho} + gh\right)}.$$

The pressure on the underside of the piston is determined by a force balance on the piston:



$$p_1 \frac{\pi}{4} D^2 = (m + m_p)g \quad \Longrightarrow \quad p_1 = \frac{4(m + m_p)g}{\pi D^2}$$

where m_p is the mass of the piston.

Substitute this expression for p_1 into the equation for V_2 to get

$$\boxed{V_2 = \sqrt{2\left(\frac{4(m + m_p)g}{\rho \pi D^2} + gh\right)}}$$

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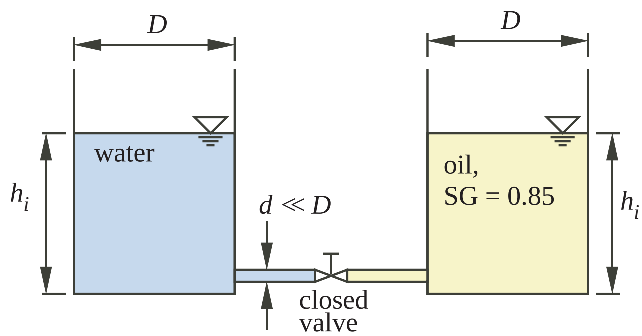
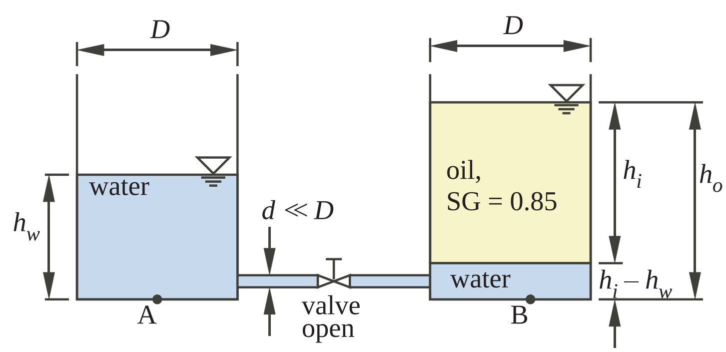
10. Two identical cylindrical tanks are connected by a pipe with a shut-off valve. One tank is filled to a depth h_i with water, and the other tank is filled to a depth h_i with oil having SG = 0.85. Both tanks are open to atmospheric pressure at the top. The diameter of the connecting pipe is much smaller than the diameter of the tanks.

    The valve is opened and the two fluids reach a new static equilibrium. Let h_w be the height of the free surface for the tank that originally contained only water. Let h_o be the height of the free surface for the tank that originally contained only oil. What are the equilibrium values of h_w and h_o when the valve is open? Neglect the volume of fluid in the connecting pipe. Assume h_i is known. The solutions are two simple formulas for h_w and h_o in terms of h_i and SG.



Solution: Water has a greater density than the oil (with SG = 0.85). When the valve is opened, water will flow into the bottom of the tank that originally had oil in it. Both oil and water can be assumed to be incompressible, therefore the total volume of fluid in the two tanks must be equal before and after the valve is opened. The fluid configuration after the valve is opened is depicted in the following sketch.



Neglecting the volume of the connecting pipe means that the water side is lower by $h_i - h_w$ and the oil side is higher by $h_i - h_w$.

Since the water is continuous at the bottom of the tank, Pascal's law requires that $p_A = p_B$, where A and B are two points on the bottoms of the two tanks.

$$p_A = p_B \Longrightarrow \gamma_w h_w = \gamma_o h_i + \gamma_w (h_i - h_w)$$

Solve for h_w:

$$2\gamma_w h_w = \gamma_o h_i + \gamma_w h_i = h_i(\gamma_o + \gamma_w) \quad \Longrightarrow \quad h_w = h_i \frac{\gamma_o + \gamma_w}{2\gamma_w} \quad \Longrightarrow \quad \boxed{h_w = h_i \frac{\text{SG} + 1}{2}}$$

Note that SG < 1 so the formula for h_w gives $h_w < h_i$, as expected.

From the sketch of the configuration with the valve open,

$$h_o = h_i + (h_i - h_w) = 2h_i - h_w$$

Substitute $h_w = h_i(\mathrm{SG}+1)/2$ and solve for h_o:

$$h_o = 2h_i - h_i\frac{\mathrm{SG}+1}{2} = \left(2 - \frac{\mathrm{SG}+1}{2}\right)h_i = \left(\frac{4-\mathrm{SG}-1}{2}\right)h_i = \left(\frac{3-\mathrm{SG}}{2}\right)h_i$$

Therefore

$$\boxed{h_o = \frac{3-\mathrm{SG}}{2}h_i}$$

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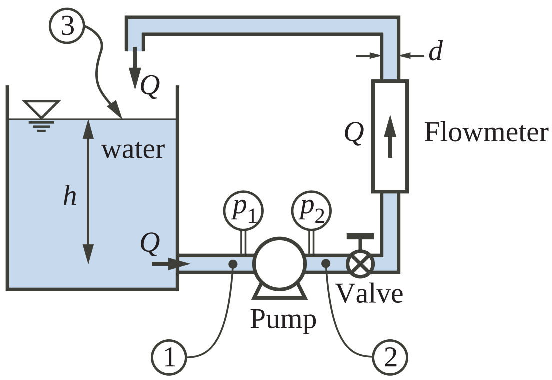
11. The schematic depicts a pump test stand that is used to measure the ability of a pump to deliver a volumetric flow rate Q when working against a flow resistance. The table to the right shows the type of data measured during a pump test. The $\sim\!\sim$ symbol represents a numerical value recorded during the test. All piping in the test stand has the same diameter d.

Valve	Q	p_1	p_2	$\dot{W}_{\text{in}}$
closed	0	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$
1/4 open	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$
1/2 open	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$
3/4 open	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$	$\sim\!\sim$

a. Assume that measured data for Q, p_1 and p_2 are available for each setting of the control valve. Derive an expression for the head loss between the downstream side of the pump and the free surface of the tank. Consider only the head loss downstream of the pump.

b. The $\dot{W}_{\text{in}}$ column is the measured electrical power supplied to the motor that drives the pump. Derive an expression for efficiency $\eta = \dot{W}_{\text{shaft}}/\dot{W}_{\text{in}}$ in terms of the measured data where $\dot{W}_{\text{shaft}}$ is the shaft power actually delivered to the water. Neglect head loss in the pump. (Head loss in the pump reduces the head gain that the pump can produce, so internal head loss is already accounted for in the data.)



Solution (a): Apply the steady flow energy equation between station 2 and station 3

$$\frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_s - h_L$$

Station 3 is the free surface at the top of the (large) tank, so $p_3 = 0$ and $V_3 = 0$. There is no pump between station 2 and station 3 so $h_s = 0$. Rearranging the energy equation to solve for h_L gives

$$h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - z_3$$

Use $z_3 - z_2 = h$ and $V_2 = Q/A_2$

$$\boxed{h_L = \frac{p_2}{\gamma} + \frac{Q^2}{2gA_2^2} - h}$$

Solution (b):

$$\eta = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{in}}} = \frac{\dot{m}gh_s}{\dot{W}_{\text{in}}} = \frac{\rho Q g h_s}{\dot{W}_{\text{in}}} = \frac{\gamma Q h_s}{\dot{W}_{\text{in}}}$$

Apply the steady flow energy equation between stations 1 and 2

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L.$$

Neglect the head loss inside the pump. The pipe diameter is the same on both sides of the pump, so under steady flow conditions, $V_1 = V_2$. Stations 1 and 2 are also at the same elevation so $z_1 = z_2$. With these simplifications the steady flow energy equation becomes

$$h_s = \frac{p_2 - p_1}{\gamma}.$$

Substitute this expression for h_s into the formula for η to get

$$\boxed{\eta = \frac{(p_2 - p_1)Q}{\dot{W}_{\text{in}}}}.$$

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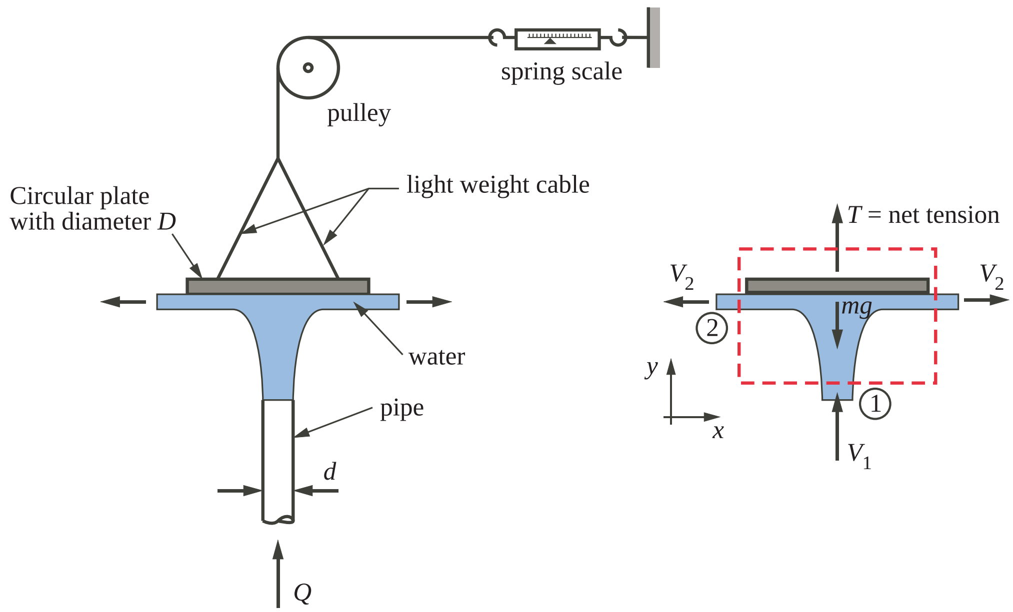
12. An upward jet of water from a pipe of diameter d impinges on a circular plate of diameter D. The plate has a mass m, and it is suspended by thin cable attached to a spring scale. The other end of the spring scale is attached to a wall. What flow rate Q is needed so that the spring scale reads zero? Neglect friction in the pulley and the mass of the cable and mass of the spring scale.



Solution: Draw the control volume, coordinate axes, and label the inlet (1) and outlet (2). Station 2 is the circular jet of water spraying outward from the center of the plate. Students in EAS 361 will recognize this as being nearly identical to the jet momentum experiment in the lab.

Apply the linear momentum equation in the y direction.

$$\sum F_y = \frac{\partial}{\partial t}\int_{CV} V_y\,\rho\,d\mathcal{V} + \int_{CS} V_y \rho(\mathbf{V}\cdot\hat{\boldsymbol{n}})\,dA$$

The flow is steady so the time derivative is zero. Assume the velocity profiles at the inlet and outlet are uniform. Under these restrictions the right hand side of the momentum equation is simplified to

$$\frac{\partial}{\partial t}\int_{CV} V_y\,\rho\,d\mathcal{V} + \int_{CS} V_y \rho(\mathbf{V}\cdot\hat{\boldsymbol{n}})\,dA = \rho V_1(-V_1)A_1 = -\rho V_1 Q = -\rho\frac{Q^2}{A_1} \qquad (\star)$$

The left hand side of the momentum equation is

$$\sum F_y = T - mg \qquad (\star\star)$$

where T is the net tension measured by the spring scale, and mg is the weight of the plate. Put the pieces of the momentum equation back together, i.e. set the right hand side of Equation ($\star$) equal to the right hand side of Equation ($\star\star$).

$$-\rho\frac{Q^2}{A_1} = T - mg$$

Solve for Q

$$\rho\frac{Q^2}{A_1} = mg - T \quad\Longrightarrow\quad Q = \sqrt{\frac{(mg-T)A_1}{\rho}}$$

Therefore, the flow rate necessary to give $T = 0$ is

$$\boxed{Q(T=0) = \sqrt{\frac{mgA_1}{\rho}}}$$

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