1. **[10 points]** Complete the column labeled "*Find*" in the following table. The first column in the table gives the definition of a dimensionless quantity. The second column gives the dimensions of all but one of the quantities in the dimensionless quantity.

   The answer for each row is the correct dimension in the F-L-T-Θ or M-L-T-Θ system of primary dimensions. A sample solution is given below.

   Each row of the table is worth 5 points. **Be sure to show your work**. Answers given without any justification will be given zero points. Even if you see the solution by inspection, provide a simple mathematical justification (or proof that your answer is correct) in the space at the bottom of the page.

   The expression $[V] = \mathrm{L/T}$ is read, "The dimensions of V are length divided by time.

Example:

Definition	Given	Find
$\mathrm{Re} = \frac{V\ell}{\nu}$	$[V] = \mathrm{L/T}$, $[\ell] = L$	$[\nu] = \mathrm{L^2/T}$

Assignment:

Definition	Given	Find
$\mathrm{St} = \frac{\omega\ell}{V}$	$[\ell] = \mathrm{L}$, $[V] = \mathrm{L/T}$,	$[\omega] = \frac{1}{T}$
$\mathrm{Ca} = \frac{\rho V^2}{E}$	$[\rho] = \mathrm{M/L^3}$, $[V] = \mathrm{L/T}$	$[E] = \frac{M}{LT^2}$ or $\frac{F}{L^2}$

$$[St] = 1 \Rightarrow \left[\frac{\omega l}{V}\right] = 1 \Rightarrow [\omega] = \left[\frac{V}{l}\right] = \frac{L/T}{L} = \frac{1}{T}$$

$$[Ca] = 1 \Rightarrow \left[\frac{\rho V^2}{E}\right] = 1 \Rightarrow [E] = [\rho V^2] = \frac{M}{L^3}\cdot\frac{L^2}{T^2} = \frac{M}{LT^2}$$

$$\text{or } \frac{M}{LT^2} = \frac{1}{L^2}\, M\frac{L}{T^2} = \frac{F}{L^2}$$

because $F = \frac{ML}{T^2}$

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   The expression $[V] = \mathrm{L/T}$ is read, "The dimensions of V are length divided by time.

Example:

Definition	Given	Find
$\mathrm{Re} = \frac{V\ell}{\nu}$	$[V] = \mathrm{L/T}$, $[\ell] = L$	$[\nu] = \mathrm{L^2/T}$

Assignment:

Definition	Given	Find
$\mathrm{Ma} = \frac{V}{a}$	$[V] = \mathrm{L/T}$,	$[a] =$ L/T
$\mathrm{Fr} = \frac{U^2}{g\ell}$	$[g] = \mathrm{L/L^2}$, $[\ell] = \mathrm{L}$	$[U] =$ L/T with correct units for g

typo $[g] = L/T^2$

$$[Ma] = 1 \Rightarrow [a] = [V] = \frac{L}{T}$$

$$[Fr] = 1 \Rightarrow [U^2] = [g\ell]$$

with incorrect units as given on the exam:

$$[U^2] = \frac{L}{L^2} \cdot L = 1 \Rightarrow U \text{ is dimensionless}$$

with correct units for g

$$[U^2] = \frac{L}{T^2} L = \frac{L^2}{T^2} \Rightarrow U = \frac{L}{T}$$

1. **[10 points]** Complete the column labeled "*Find*" in the following table. The first column in the table gives the definition of a dimensionless quantity. The second column gives the dimensions of all but one of the quantities in the dimensionless quantity.

   The answer for each row is the correct dimension in the F-L-T-Θ or M-L-T-Θ system of primary dimensions. A sample solution is given below.

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   The expression $[V] = \mathrm{L/T}$ is read, "The dimensions of V are length divided by time.

Example:

Definition	Given	Find
$\mathrm{Re} = \frac{V\ell}{\nu}$	$[V] = \mathrm{L/T}$, $[\ell] = L$	$[\nu] = \mathrm{L^2/T}$

Assignment:

Definition	Given	Find
$\mathrm{Pr} = \frac{\nu}{\alpha}$	$[\nu] = \mathrm{L^2/T}$,	$[\alpha] = L^2/T$
$\mathrm{We} = \frac{\rho V^2 \ell}{\sigma}$	$[\rho] = \mathrm{M/L^3}$, $[V] = \mathrm{L/T}$, $[\ell] = L$	$[\sigma] = \frac{M}{T^2}$ or $\frac{F}{L}$

$[Pr] = 1$, dimensionless $\Rightarrow [\alpha] = [\nu] = L^2/T$

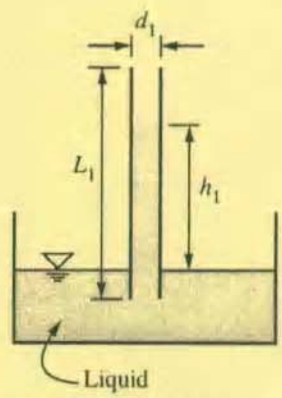
$$[We] = 1 \Rightarrow [\sigma] = [\rho V^2 \ell] = \frac{M}{L^3}\left(\frac{L}{T}\right)^2 L = \frac{M}{L^3}\frac{L^2}{T^2}L = \frac{M}{T^2}$$

$$\text{or } \frac{M}{T^2} = M\frac{L}{T^2}\frac{1}{L} = \frac{F}{L}$$

because $F = M\frac{L}{T^2}$

Choose either problem 2 or problem 3. Clearly indicate which problem you wish to have graded.

2. A tube of diameter d_1 and length L_1 is held vertically so that the lower end is immersed in a pan of liquid. The top surface of the pan and the upper end of the tube are open to the atmosphere.



a. **[3 points]** What is the name of the physical property associated with the movement of liquid up the tube?

Surface tension

b. **[3 points]** Another tube with diameter d_2 and length L_1 is placed parallel to, but not touching or near to the first tube. Both the second tube and the first tube are now held vertically. If $d_2 > d_1$, how does h_2 compare to h_1? $h_2 < h_1$

c. **[9 points]** Justify your answer to part (b).

From the universal cheat sheet $h_1 = \dfrac{2\sigma\cos\theta}{\gamma R_1}$

R_1 = radius of the tube

$$\frac{h_2}{h_1} = \frac{2\sigma\cos\theta/\gamma R_2}{2\sigma\cos\theta/\gamma R_1} = \frac{R_1}{R_2} = \frac{d_1}{d_2}$$

$\therefore$ if $R_2 > R_1$, $h_2 < h_1$

$R_2 < R_1$, $h_2 > h_1$

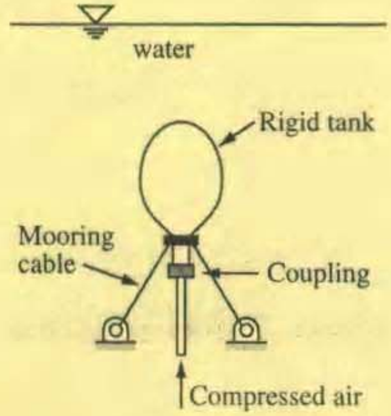
V_t is constant

Choose either problem 2 or problem 3. Clearly indicate which problem you wish to have graded.

3. A perfectly rigid, air-filled tank is held underwater by mooring cables. The tank is connected to a supply of compressed air by a coupling that provides negligible structural support to the tank. As the pressure in tank is increased, its walls do not flex so that the volume of tank is always constant.

    a. **[5 points]** As the pressure in the tank is increased, how does the tension on its mooring cables change?

    b. **[10 points]** Justify your answer to part (a).



a.) Increasing the tank pressure decreases the required tension in the cables

b.) Free body diagram

F_b = buoyancy force = $\gamma_w V_t$

V_t = volume of tank

T = total tension in mooring cables

$T\cos\theta$ = downward vertical force due to mooring cables

W_t = weight of the tank

W_{air} = weight of the air in the tank

$= \rho_{air} V_t g$

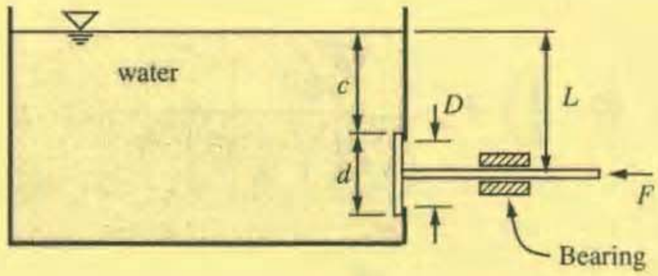
$\Sigma F_y = 0 \Rightarrow F_b = \rho_{air} V_t g + W_t + T\cos\theta \Rightarrow T\cos\theta = F_b - \rho_{air} V_t g - W_t$

Assume the compressed air acts as an ideal gas $\Rightarrow \rho_{air} = \frac{p_{air}}{RT}$

$$T\cos\theta = F_b - \frac{p_{air} V_t}{RT} g - W_t$$

$\therefore$ increasing p_{air} decreases T

4. A stopper of diameter d prevents water from leaking through a hole of diameter D. The rod that actuates the stopper is held in place by a slider bearing.



a. **[8 points]** Neglecting friction in the bearing, what force F is necessary to open the stopper?

b. **[12 points]** The stopper and rod are to be redesigned to reduce the load on the bearings. Where should the axis of the push rod be located so that there is no side load on the bearings? Assume that the attachment point between the push rod and the stopper is to be moved without moving the position of the stopper relative to the tank. In other words, the dimension L is to be changed to eliminate side loads in the bearing. Assume that the stopper is perfectly rigid.

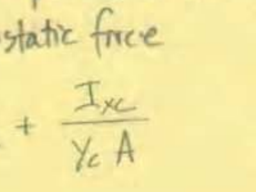
a.) Force need to open the stopper is equal to the net hydrostatic force

$F_w = \gamma h_c A$ $\quad$ h_c = depth to the centroid

A = area of surface exposed to the water $= \frac{\pi}{4} d^2$

$$\therefore \boxed{F_w = \gamma \left(c + \frac{d}{2}\right) \frac{\pi}{4} d^2}$$

b.) The line of action of the hydrostatic force is below the centroid of the stopper. Move the push rod down so that it is colinear with the net hydrostatic force

original: 

modified:

Find y_R, the depth to the line of action of the hydrostatic force

$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

4. (continued)

$$I_{xc} = \frac{\pi r^4}{4} \quad \text{from universal cheat sheet}$$

$$r = \frac{d}{2} \Rightarrow I_{xc} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\frac{d^4}{16} = \frac{\pi d^4}{64}$$

$$\Rightarrow y_R = \left(c+\frac{d}{2}\right) + \frac{\pi d^4/64}{\left(c+\frac{d}{2}\right)\left(\frac{\pi}{4}d^2\right)}$$

$$y_R = c + \frac{d}{2} + \frac{d^2}{16\left(c+\frac{d}{2}\right)}$$

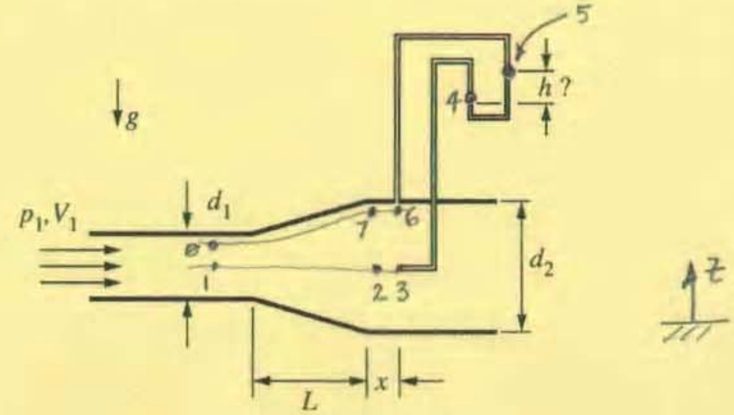
To make F_W and F_R colinear, choose $L = y_R$

$$\therefore \boxed{L = c + \frac{d}{2} + \frac{d^2}{16\left(c+\frac{d}{2}\right)}}$$

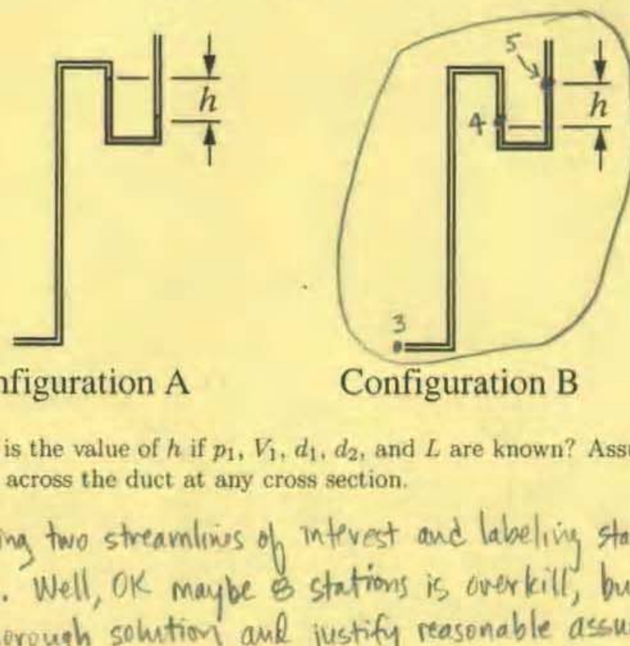
or

$$L_{new} = L + \frac{d^2}{16L} = L + \frac{r^2}{4L} \quad \text{with } L = c + \frac{d}{2}$$

5. A horizontal round pipe of diameter d_1 is connected to a larger pipe of diameter d_2 by a transition of length L. A piezometer is located on the centerline of the duct with its open end in the middle of the transition. The piezometer is connected to a water-filled U-tube manometer.



a. **[6 points]** Which of the following two configurations of the water in the manometer is correct?



Configuration A Configuration B

The stagnation point at 3 has a higher pressure than the static tap at 6. Since $p_3 > p_6$ the free surface at 4 must be lower than the free surface at 5.

b. **[14 points]** What is the value of h if p_1, V_1, d_1, d_2, and L are known? Assume that the velocity is uniform across the duct at any cross section.

Begin by identifying two streamlines of interest and labeling stations ∅ through 7 on the schematic. Well, OK maybe 8 stations is overkill, but it will allow me to give a thorough solution and justify reasonable assumptions.

Simple solution: Assume the flowing fluid is a gas so that elevation effects in the flowing fluid can be neglected.

Assume:
- steady
- incompressible
- no viscous effects

Apply Bernoulli equation along the stagnation streamline from 2 to 3

$$p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 = p_3 + \frac{1}{2}V_3^2 + \gamma z_3$$

$z_2 = z_3$ and $V_3 = 0$ (stagnation point) so $\boxed{p_3 - p_2 = \frac{1}{2}\rho V_2^2}$ (a)

OVER →

Use manometer equations to relate p_3 and p_2 to h.

Neglecting elevation effects in the flowing fluid gives $p_6 = p_5$ and $p_3 = p_4$

For the manometer: $p_4 = p_5 + \gamma_m h \Rightarrow p_4 - p_5 = \gamma_m h$

γ_m = specific weight of liquid in the manometer

$$\boxed{p_3 - p_6 = \gamma_m h} \quad (b)$$

Apply momentum equation <u>normal</u> to streamlines between stations 2 and 7

$$\frac{p_2}{\gamma} + \rho\int \underbrace{\frac{V^2}{R}}_{0} dn + z_2 = \frac{p_7}{\gamma} + \rho\int \underbrace{\frac{V^2}{R}}_{0} dn + z_7$$

The streamlines are parallel so the curvature terms are zero.

$$\Rightarrow p_2 - p_7 = \gamma(z_7 - z_2)$$

But since we are neglecting elevation effects in the flowing fluid, we get $p_2 = p_7$

Furthermore, since there are no area changes, and hence no velocity changes between 7 and 6, $p_6 = p_7$

$\therefore p_2 = p_6$ and Equation (b) becomes

$$\boxed{p_3 - p_2 = \gamma_m h} \quad (c)$$

Combine (a) and (c) to eliminate $p_3 - p_2$

$$\Rightarrow \gamma_m h = \frac{1}{2}\rho V_2^2 \Rightarrow \boxed{h = \frac{1}{2}\frac{\rho V_2^2}{\gamma_m}} \quad (d)$$

The problem statement asked for the value of h in terms of flow conditions in duct at ①. Use mass conservation to relate V_1 to V_2

$$A_1 V_1 = A_2 V_2 \Rightarrow V_2 = V_1 \frac{A_1}{A_2} \quad \therefore V_2 = V_1 \frac{d_1^2}{d_2^2} \text{ and } \boxed{V_2^2 = V_1^2 \frac{d_1^4}{d_2^4}} \quad (e)$$

Substitute (e) into (d)

$$\boxed{h = \frac{\rho}{2\gamma_m}\frac{d_1^4}{d_2^4} V_1^2}$$

Comments on the solution to Problem #5

(1) h is unaffected by p_1

In general, for incompressible flow, the absolute pressure level does not affect the flow other than to determine the (uniform) value of the density

(2) The relationship between p_2 and p_6 cannot be determined through application of the Bernoulli equation because station ② and station ⑥ are not on the same streamline.

(3) If the effect of elevation in the flowing fluid is not negligible the solution is

$$h = \frac{\rho}{2(\gamma_m - \gamma)} \frac{d_1^4}{d_2^4} V_1^2$$

This result is derived in the following pages.

Alternative Solution: Don't neglect elevation effects in the flowing fluid

Apply momentum balance normal to the streamline between station 2 and station 7

$$\frac{p}{\gamma} + \rho \int \frac{V^2}{R} dn + z = \text{constant}$$

streamlines between 2 and 7 are parallel so $\rho \int_2^7 \frac{V^2}{R} dn = 0 \quad (R = \infty)$

$$\Rightarrow \frac{p_2}{\gamma} + z_2 = \frac{p_7}{\gamma} + z_7 \Rightarrow p_2 = p_7 + \gamma(z_7 - z_2) \qquad (\text{I})$$

Apply Bernoulli equation along streamline $7 \rightarrow 6$

$$\frac{p_7}{\gamma} + \frac{V_7^2}{2g} + z_7 = \frac{p_6}{\gamma} + \frac{V_6^2}{2g} + z_6$$

But $V_7 = V_6 = \emptyset$ and $z_7 = z_6$ $\therefore p_7 = p_6$

$\therefore$ Equation (I) becomes

$$p_2 = p_6 + \gamma(z_6 - z_2) \qquad (\text{II}a)$$

or $$p_6 = p_2 - \gamma(z_6 - z_2) \qquad (\text{II}b)$$

Apply hydrostatic equation to each branch of the manometer

in static tube: $p_5 = p_6 - \gamma(z_5 - z_6)$

combine with (IIb)

$$\Rightarrow p_5 = p_2 - \gamma(\cancel{z_6} - z_2) - \gamma(z_5 - \cancel{z_6})$$

$$p_5 = p_2 - \gamma(z_5 - z_2) \qquad (\text{III})$$

in stagnation tube: $p_4 = p_3 - \gamma(z_4 - z_3) \qquad (\text{IV})$

subtract (III) from (IV)

$$p_4 - p_5 = p_3 - \gamma(z_4 - \cancel{z_3}) - p_2 + \gamma(z_5 - \cancel{z_2}) \qquad z_2 = z_3$$

$$p_4 - p_5 = p_3 - p_2 + \gamma(z_5 - z_4)$$

$$p_4 - p_5 = p_3 - p_2 + \gamma h \qquad (\text{V})$$

Substitute $p_4 - p_5 = \gamma_m h$ into Equation (IV) (See equation preceding Equation (b))

$$\gamma_m h = p_3 - p_2 + \gamma h$$

$$p_3 - p_2 = \gamma_m h - \gamma h$$

$$\therefore \; p_3 - p_2 = (\gamma_m - \gamma) h \qquad \text{(VI)}$$

Substitute (a) into (VI)

$$\tfrac{1}{2} \rho V_2^2 = (\gamma_m - \gamma) h$$

Solve for h

$$h = \frac{\rho V_2^2}{2(\gamma_m - \gamma)}$$

substitute (e) for V_2^2

$$\boxed{h = \frac{\rho V_1^2}{2(\gamma_m - \gamma)} \left(\frac{d_1}{d_2}\right)^4} \qquad \text{(VII)}$$