CLASS PROBLEM 1: Polystyrene is a plastic that is used in many applications. Some familiar applications include computer covers, drinking cups, which can be considered hard plastic. Other applications include products made from the brand name Styrofoam, a soft form of the same substance.

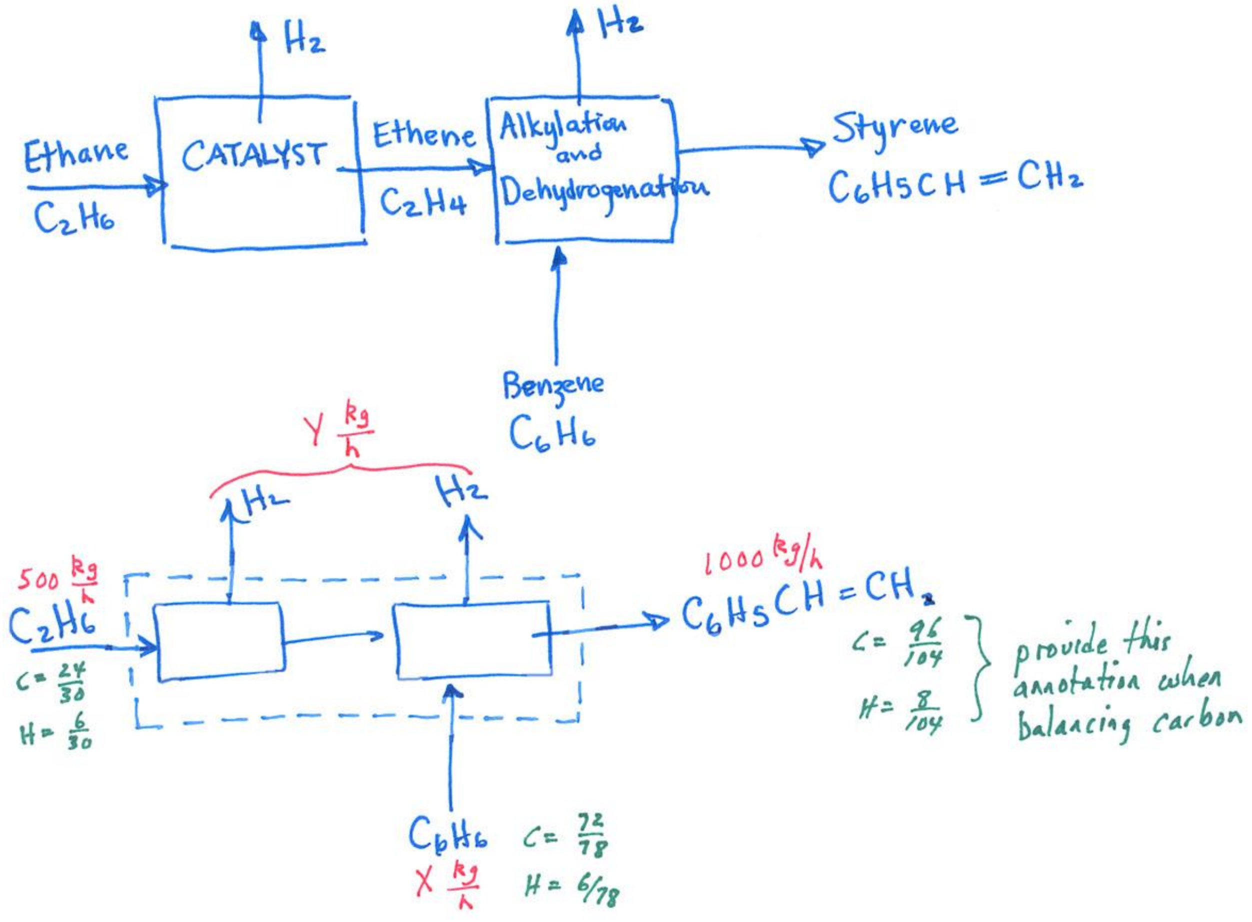
One process for the manufacture of polystyrene is to generate styrene, which is then polymerized into polystyrene. To produce styrene, the raw material *ethane* (C_2H_6) is converted to *ethene* (C_2H_4) by the removal of hydrogen using a nickel catalyst. The resulting *ethene* is mixed with *benzene* (C_6H_6) in a process called alkylation, then dehydrogenated, resulting in the styrene monomer ($C_6H_5CH=CH_2$).

Suppose I want to produce 1000 kg/hour of styrene for use in the manufacture of polystyrene. There is an available supply of 500 kg/hour of ethane (C_2H_6). What is the rate of benzene (C_6H_6) that will be required? What is the rate of hydrogen gas (H_2) extraction for the entire process?

Assume a steady state process.

Reference: http://www.shellchemicals.com/styrene_monomer/1,1098,1513,00.html

Steps 1 and 2:



provide this annotation when balancing carbon

Step 3

Cons of Mass
Rate

$$M_{IN} + M_G - M_{OUT} - M_C = \Delta M_{sys}$$

its steady so $\Delta M_{sys} = 0$

there is no M_G nor M_C

$$\underline{\underline{M_{IN} = M_{OUT}}}$$

Steps 4 (step 5 which is solving for unknowns is combined here)

$$\underline{\underline{M_{IN} = M_{OUT}}}$$

now
overall: $\dot{M}_{Ethane} + \dot{M}_{Benzene} - \dot{M}_{\substack{Hydrogen\\Gas}} - \dot{M}_{styrene} = 0$ dots for rate

$$500\frac{kg}{hour} + X\frac{kg}{hour} - Y\frac{kg}{hour} - 1000\frac{kg}{hour} = 0$$

$$X - Y = 500\frac{kg}{hour}$$

Carbon: $\frac{24}{30}\left(500\frac{kg}{hour}\right) + \frac{72}{78}(X) - 0Y - \frac{96}{104}\left(1000\frac{kg}{hr}\right) = 0$

recall that
% by weight
$= \frac{M_{carbon}}{M_{compound}}$ ← C_2H_6 ex

$$\longrightarrow \frac{M_C}{M_{C_2H_6}} = \frac{2(12)}{2(12)+6(1)} = \frac{24}{30}$$

$$\frac{M_C}{M_{C_6H_6}} = \frac{6(12)}{6(12)+6(1)} = \frac{72}{78}$$

$$\frac{M_C}{M_{C_6H_5CH=CH_2}} = \frac{8(12)}{8(12)+8(1)} = \frac{96}{104}$$

$$X = \frac{\left(\frac{96}{104}\right)\left(1000\frac{kg}{hr}\right) - \frac{24}{30}\left(500\frac{kg}{hr}\right)}{\frac{72}{78}}$$

$\boxed{X = 566\,^2/_3\ \frac{kg}{hr}}$ Benzene required

$$Y = X - 500 = 566\tfrac{2}{3} - 500$$

$\boxed{Y = 66\tfrac{2}{3}\ \frac{kg}{hr}}$ Hydrogen gas produced

Check with
Hydrogen: $\frac{6}{30}(500) + \frac{6}{78}(X) - Y - \frac{8}{104}(1000) = 0$

$$100 + 43.590 - 66\,^2/_3 - 76.923 = 0 = 0\ ✓$$

CLASS PROBLEM 2: The nutritional information for a chocolate chip cookie recipe is shown below:

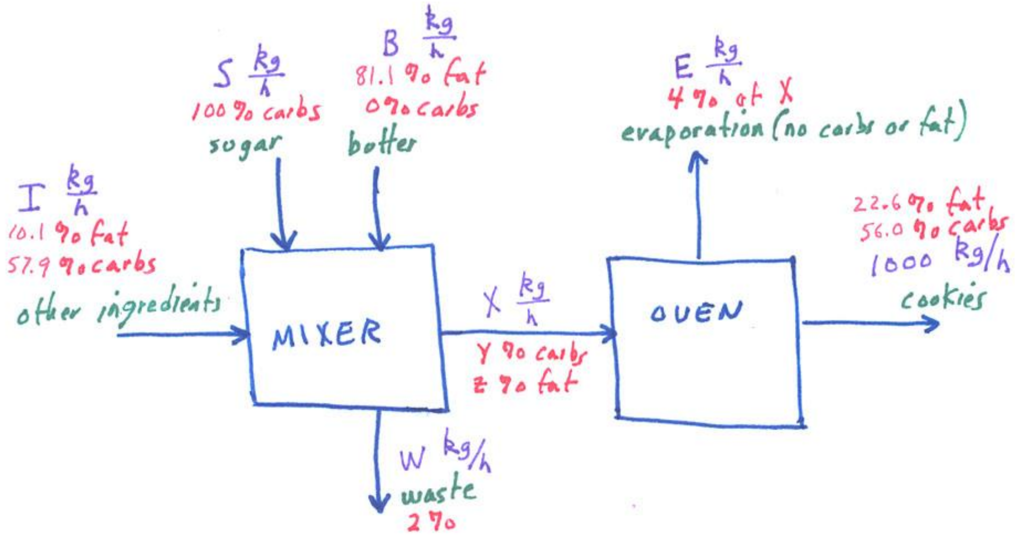
ingredient	amount	calories	total mass (g)	total fat (g)	cholesterol (g)	sodium (g)	carbs (g)	protein (g)	minerals & mass not listed on label (g)
butter	2 sticks	1626	227	184	0.5	1.9	0	2	39
brown sugar	2 cups	720	192	0	0.0	0.0	192	0	0
eggs	2	150	100	10	0.4	0.7	2	12	75
vanilla	1 tablespoon	37	13	0	0.0	0.0	2	0	11
whole wheat flour	2 cups	880	240	4	0.0	0.0	184	24	28
salt	1 teaspoon	0	6	0	0.0	2.4	0	0	4
whole oats	3 cups	900	240	18	0.0	0.0	162	30	30
baking powder	1 tablespoon	6	15	0	0.0	1.1	3	0	11
baking soda	1 tablespoon	0	14	0	0.0	3.8	0	0	10
chocolate chips	1/2 package	840	168	48	0.0	0.0	108	5	7
totals:		**5159**	**1215**	**264**	**0.9**	**10**	**653**	**73**	**214**

A cookie machine produces 1000 kg/h of chocolate chip cookies approximating the recipe above. Butter, sugar and the other ingredients are combined in a mixer before being sent to the oven, and 2% of the resulting cookie batter is lost to waste in the mixing area. As the cookies are baked in the oven, 4% of the mass of the entering batter is lost to evaporation.

ingredient	fat (% wt)	carbs (% wt)
butter	81.1	0
brown sugar	0	100
additional ingredients	10.1	57.9

Find the flow rate (kg/h) of the three streams of ingredients that enter the mixer if the finished cookies contain 22.6% weight fat and 56.0% weight carbohydrates (no fat or carbs evaporate in the oven). The recipe is provided only for background and is not needed in the solution.

Solution Steps 1 and 2:



STEP 3: This is a rate problem.

STEP 4: ① Analyze oven first.

overall balance

Mass in = Mass out

$$X = \overset{.04\,X}{\cancel{E}} + 1000 \longrightarrow (1-.04)\,X = 1000\ \frac{kg}{h}$$

$$\boxed{X = 1041.7\ \frac{kg}{h}}$$

carb balance

carbs in = carbs out

$$X\left(\frac{Y}{100}\right) = .56\left(1000\ \frac{kg}{hr}\right)$$

$$Y = \frac{100(.56)\,1000\ kg/h}{1041.7\ kg/h} = \boxed{53.8\ \%\ \text{carbs} = Y}$$

fat in = fat out

$$X\left(\frac{Z}{100}\right) = .226\left(1000\ \frac{kg}{h}\right) \longrightarrow \boxed{Z = 21.7\ \%}$$

② Now analyze mixer.

overall: mass in = mass out

$$I + S + B = \overset{.02(I+S+B)}{\cancel{W}} + \overset{1041.7\ \frac{kg}{hr}}{\cancel{X}}$$

$$(1-.02)\, I + S + B = 1041.7$$

$$\boxed{I + S + B = 1063.0\ \frac{kg}{h}} \quad (1) \qquad W = .02\left(1063\ \frac{kg}{h}\right) = 21.3\ \frac{kg}{h}$$

carbs: carbs in = carbs out

$$.579I + S = .538\,(W + X) = .538\left(21.3\ \frac{kg}{h} + 1041.7\ \frac{kg}{h}\right)$$

$$\boxed{.579I + S = 571.9\ \frac{kg}{h}} \quad (2)$$

fat: fat in = fat out

$$.101I + .811B = .217\,(W + X) = .217\left(21.3\ \frac{kg}{h} + 1041.7\ \frac{kg}{h}\right)$$

$$\boxed{.101I + .811B = 230.7\ \frac{kg}{h}} \quad (3)$$

Step 5:

SOLVE FOR UNKNOWNS:

$$\boxed{\begin{array}{rcl} 1I + 1S + 1B &=& 1063 \\ .579I + 1S + 0B &=& 571.9 \\ .101I + 0S + .811B &=& 230.7 \end{array}} \text{ – Solve using Mathcad}$$

keystrokes

$$a := \begin{pmatrix} 1 & 1 & 1 \\ .579 & 1 & 0 \\ .101 & 0 & .811 \end{pmatrix}$$

a : <insert> <matrix> <enter 3 rows and 3 columns>

Manually enter the coefficients - you can tab between entries.

$$b := \begin{pmatrix} 1063 \\ 571.9 \\ 230.7 \end{pmatrix}$$

b : <insert> <matrix> <enter 3 rows and 1 column>

Manually enter the coefficients - you can tab between entries.

$$\mathrm{lsolve}(a, b) = \begin{pmatrix} 697.007 \\ 168.333 \\ 197.66 \end{pmatrix}$$

lsolve(a,b)=

ANSWER:

additional ingredients	**I = 597 kg/h**
sugar	**S = 168 kg/h**
butter	**B = 198 kg/h**

CHECK: Adding up I+S+B equals 1063 kg/hr. Two percent or 21.3 kg/hr is lost to waste in the mixing area leading to 1041.7 kg/hr entering the oven. Evaporation leads to a loss of 41.7 kg/hr of moisture in the oven, and the remaining 1000 kg/hr is output as cookies. The ratios of the computed ingredients also compare favorably with the ratios of ingredients in the recipe.