Salt Water Mixtures

We wish to create a system that can control the temperature and salinity of a small volume of water. We will mix laboratory grade salt (non-iodized sodium chloride or NaCl) with deionized water (DI water) to produce four different concentrations:

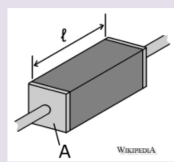
Pure DI water	0.05% salt (by weight)	0.10% salt	0.15% salt

For comparison, the average salt concentration of the world's oceans is about 3.5% (35 grams of salt in every 1 kg of water).

Standard tap water contains a variety of dissolved ions including sodium, calcium, iron, copper, chloride, and bromide. These ions have been removed from DI water. The electrical resistivity of DI water is around 18 MΩ-cm whereas the resistivity of standard tap water is around 15 kΩ-cm. We are using DI water because we want our conductivity sensors to respond predictably.

Did you notice the strange units on resistivity (Ω-cm)? The electrical resistance R of a material (measured in Ω) is related it its electrical resistivity ρ (measured in Ω-cm) as

$$R = \rho\ell/A$$



where A is the cross-sectional area of the specimen and ℓ is the length of specimen. For a wire, A would be the cross sectional area of the wire and ℓ would be the length of the wire over which the resistance is measured. We will discuss this more fully later in the quarter when we design our temperature sensor.

Calculating the Percent Weight of a Salt Water Mixture

Assume we have 19 grams of water mixed with 1 gram of NaCl. What is the weight percent of NaCl?

$$\text{weight \% NaCl} = \frac{\text{weight of NaCl}}{\text{weight of mixture}} \times 100\%$$

$$= \frac{W_{NaCl}}{W_{NaCl} + W_{H_2O}} \times 100\%$$

$$= \frac{M_{NaCl} \cdot \text{gravity}}{M_{NaCl} \cdot \text{gravity} + M_{H_2O} \cdot \text{gravity}} \times 100\%$$

gravity = 9.81 m/s^2

$$= \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\%$$

$$= \frac{1\,g}{1\,g + 19\,g} \times 100\% = \frac{1}{20} \times 100\%$$

$$\boxed{\text{weight \% NaCl} = 5\%}$$

The equation for computing the percent weight of salt is . . .

$$\text{weight \% NaCl} = \frac{W_{NaCl}}{W_{NaCl} + W_{H_2O}} \times 100\% = \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\%$$

Useful conversion factors:
Density of water = ρ_{H_2O} = 1000 kg/m^3 = 1 g/cm^3 = 1 kg/L at 4^{o}C (maximum density) 1 cm^3 = 1 cc = 1 ml 1 L = 0.001 m^3 1 gallon = 3.7853 L

Example: If you add 9.5 grams of NaCl to 5 gallons of DI water, what weight percent of salt will the mixture contain?

$$\text{wt \% NaCl} = \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\%$$

M_{NaCl} – 9.5g

M_{NaCl} – 9.5g

M_{H_2O} – what is the mass of 5 gallons of water?

$$M_{H_2O} = 5\,\text{gal} \left[\frac{3.7853\,\text{L}}{\text{gal}}\right] \frac{1\,\text{kg}}{\text{L}} = 18.93\,\text{kg}$$

Thus,

$$\text{wt \% NaCl} = \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\% = \frac{9.5}{9.5 + 18\,930} \times 100\%$$

$$\boxed{\text{wt \% NaCl} = 0.05\ \text{wt \%}}$$

Class Problem: How much salt would you need to add to 2 L of water to have a concentration of 3.5 weight percent NaCl?

$$\text{wt \% NaCl} = \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\%$$

wt % NaCl: 3.5 %

M_{NaCl} – unknown

M_{H_2O}: Mass of 2L of water

$$M_{H_2O} = 2\,L \cdot \frac{1\,kg}{L} = 2\,kg = 2000\,g$$

Solving for M_{NaCl} . . .

$$3.5\% = \frac{M_{NaCl}}{M_{NaCl} + 2} \times 100\%$$

$$.035\,(M_{NaCl} + 2) = M_{NaCl}$$

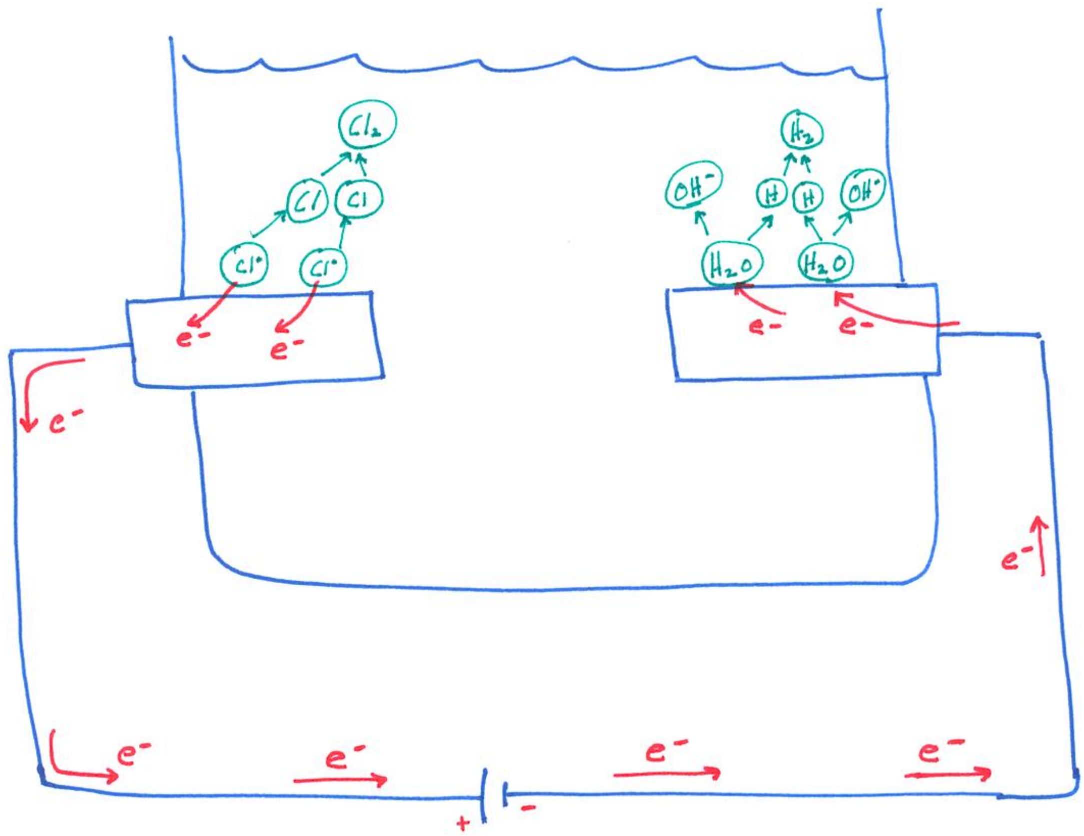
$$.035\,M_{NaCl} + 0.07 = M_{NaCl}$$

$$(1 - .035)\,M_{NaCl} = 0.07$$

$$\boxed{M_{NaCl} = 0.0725\,kg = 72.5\,g}$$

Basic Electrochemistry for Salt Water Solutions

Recall that the NaCl is hydrated in water, resulting in Na^+ and Cl^- ions in the solution.



The oxidation reaction at the anode is . . .

$$2Cl^-_{(aq)} \longrightarrow Cl_{2(g)} + 2e^-$$

oxidation since Cl^- LOSES electrons

and the reduction reaction at the cathode is . . .

$$2H_2O_{(l)} + 2e^- \longrightarrow H_{2(g)} + 2OH^-_{(aq)}$$

reduction since H_2O GAINS electrons

Useful information:

Atomic Weight of Na = 22.99 $\frac{g}{mol}$
Atomic Weight of Cl = 35.45 $\frac{g}{mol}$
Atomic Weight of NaCl = 58.44 $\frac{g}{mol}$
Avogadro's Number = 6.022 x 10^{23} mol^{-1}
1 Coulomb = 6.28 x 10^{18} electrons

Caricature of Amedeo Avogadro.

Avogadro's number was named for him due to his early work on molarity and molecular weight.

Class Problem: Assume you have a 5 gallons of water to which you add salt to create a mixture with 0.2 weight percent NaCl. Determine:

(a) The mass of the water?
(b) The mass of the salt?
(c) The number of moles of NaCl
(d) The number of Cl^- ions.

$$\text{wt\% NaCl} = \frac{M_{NaCl}}{M_{NaCl} + M_{H_2O}} \times 100\%$$

wt% NaCl: 0.2%

M_{H_2O}: mass of 5 gallons of water

(a) $M_{H_2O} = 5\,gal \left(\frac{3.7853\,L}{gal}\right) \frac{kg}{L} = 18.93\,kg$

(b) $.2 = \frac{M_{NaCl}}{M_{NaCl} + 18930\,g} \times 100\%$

$.002(M_{NaCl} + 18930\,g) = M_{NaCl}$

$(1 - .002)M_{NaCl} = .002(18930\,g) \longrightarrow \boxed{M_{NaCl} = 37.94\,g}$

(c) $mol_{NaCl} = 37.94\,g\,NaCl \left(\frac{mol}{58.44\,g}\right) = 0.649\,mol$

(d) There are $6.022(10)^{23}$ Cl^- ions in each mol of hydrated NaCl.

$\#Cl^-\,ions = 0.649\,mol\,NaCl \;\frac{6.022(10)^{23}\,Cl^-}{mol\,NaCl}$

$\boxed{\#Cl^-\,ions = 3.91(10)^{23}\,Cl^-\,ions}$

Class Problem: If a constant current of 0.1 mA passes through the probes of a conductivity sensor, how many H_2 gas molecules would be formed over a 1 minute time period?
Hint: Use the definition of an amp and a Coulomb along with the chemical reaction at the cathode (see previous page of notes).

$$2H_2O + 2e^- \longrightarrow H_2 + 2OH^-$$

2e⁻ for each H_2 molecule

$$1\ \text{Amp} = 1\ \frac{\text{coulomb}}{s} \quad \text{and} \quad 1\ \text{coulomb} = 6.28(10)^{18}\ e^-$$

$$\frac{e^-}{s} = 0.1\ mA\left(\frac{A}{1000\ mA}\right)\frac{C/s}{A}\ \frac{6.28(10)^{18}e^-}{C} = 6.28(10)^{14}\ \frac{e^-}{s}$$

Now, find the number of H_2 molecules over 1 minute:

$$\#\ H_2\ \text{Molecules} = 6.28(10)^{14}\ \frac{e^-}{s}\left(\frac{60\ s}{1\ min}\right)\frac{H_2\ \text{molecule}}{2e^-}\cdot 1\ min$$

$$\boxed{\#\ H_2\ \text{molecules} = 1.88(10)^{16}\ H_2\ \text{Molecules}}$$