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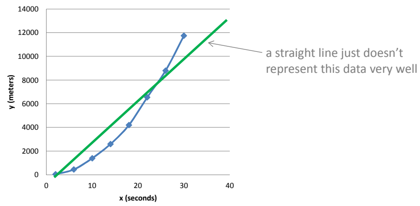
linear regression

exponential and power law relationships



how do we handle cases where x is not linearly related to y?

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Class Problem Enter the data below into a spreadsheet, and complete the following activities:

a. Plot y versus x in Excel, and set the title of the plot to "linear y versus linear x"
b. Copy the plot from (a) to another part of your worksheet. Select the vertical axis on the copied plot, right click and select "format axis." Select logarithmic scale, and change the title of this plot to "log y versus linear x"
c. Copy the plot from (b) to another part of your worksheet. Use a logarithmic axis for the horizontal axis, and change the title of the plot to "log y versus log x"

x (seconds)	y (meters)
2	50
6	450
10	1400
14	2600
18	4200
22	6550
26	8800
30	11750

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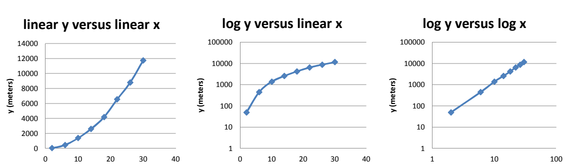
compare the three plots . . .



do you notice anything special about any of these three plots?

plot	form	equation	physical example	
linear versus linear is a straight line	linear	$y = m \cdot x + b$	spring:	$F = k \cdot x$
log versus linear is a straight line	exponential	$y = b \cdot e^{mx}$	radioactive decay:	$A = A_o \cdot e^{-\lambda t}$
log versus log is a straight line	power law	$y = b \cdot x^m$	accelerating body:	$y = \frac{1}{2} a \cdot t^2$

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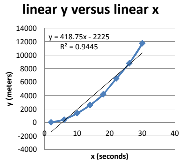
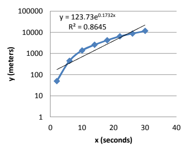
add trendlines & r^2 to the previous plots. . .

linear y versus linear x



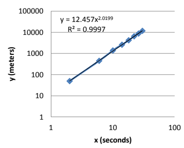
$y = m \cdot x + b$
linear

log y versus linear x



$y = b \cdot e^{mx}$
exponential

log y versus log x



$y = b \cdot x^m$
power law

- which one has the highest r^2 value?
- what does this mean? ⟹ a power law relationship best describes the relationship between x and y

math behind power law relationships

y	$=$	$b \cdot x^m$	logarithm rules
$\log(y)$	$=$	$\log(b \cdot x^m)$	*take log of both sides*
$\log(y)$	$=$	$\log(x^m) + \log(b)$	$\log(p \cdot q) = \log(p) + \log(q)$
$\log(y)$	$=$	$m \cdot \log(x) + \log(b)$	$\log(p^r) = r \cdot \log(p)$

plays part of b

"log(y)" plays the part of "y"

"log(x)" plays the part of "x"

x	y	log x	log y	log x · log y	(log x)²	(log y)²
2	50	0.301	1.699	0.511	0.091	2.886
6	450	0.778	2.653	2.065	0.606	7.040
10	1400	1.000	3.146	3.146	1.000	9.898
14	2600	1.146	3.415	3.914	1.314	11.662
18	4200	1.255	3.623	4.548	1.576	13.128
22	6550	1.342	3.816	5.123	1.802	14.564
26	8800	1.415	3.944	5.581	2.002	15.559
30	11750	1.477	4.070	6.012	2.182	16.565
		8.715	26.367	30.901	10.572	91.302

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{8 \cdot 30.901 - 8.715 \cdot 26.367}{8 \cdot 10.572 - (8.715)^2} = 2.020$$

$$\log b = \frac{\sum y_i - m\sum x_i}{n} = \frac{26.367 - 2.020 \cdot 8.715}{8} = 1.095$$

$$b = 10^{\log b} = 10^{1.095} = 12.457$$

$$y = 12.457 \cdot x^{2.020}$$

rewriting equations for power law relationships

equations for linear fits

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \qquad b = \frac{\sum y_i - m\sum x_i}{n}$$

⇩ replace x with "log x" and y with "log y"

equations for power law fits

$$m = \frac{n\sum(\log x_i \cdot \log y_i) - \sum \log x_i \cdot \sum \log y_i}{n\sum[(\log x_i)^2] - (\sum \log x_i)^2}$$

$$\log b = \frac{\sum \log y_i - m\sum \log x_i}{n}$$

$$b = 10^{\log b}$$

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math behind exponential relationships

			logarithm rules
y	=	$b \cdot e^{mx}$	
$\ln(y)$	=	$\ln(b \cdot e^{mx})$	*take natural log (ln) of both sides*
$\ln(y)$	=	$\ln(e^{mx}) + \ln(b)$	$\ln(p \cdot q) = \ln(p) + \ln(q)$
$\ln(y)$	=	$\mathrm{mx} \cdot \ln(e) + \ln(b)$	$\ln(p^r) = r \cdot \ln(p)$
$\ln(y)$	=	$mx + \ln(b)$	$\ln(e) = 1$

"$\ln(y)$" plays the part of "y" $\ln(b)$plays part of b

equations for exponential fits

$$m = \frac{n\sum(x_i \cdot \ln(y_i)) - \sum x_i \sum \ln(y_i)}{n\sum x_i^2 - (\sum x_i)^2} \qquad \ln(b) = \frac{\sum \ln(y_i) - m\sum x_i}{n}$$

$$b = e^{\ln(b)}$$

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Class Problem The air that we breathe exerts a significant amount of pressure on objects at sea level (101.3 kPa or 14.7 psi). As we go higher into the atmosphere, the pressure decreases and eventually reaches zero in outer space where there is no air. Pressure measurements were recorded as a function of elevation above sea level resulting in the following table (km = kilometers and kPa = kilopascals):.

altitude (km)	pressure (kPa)
0.1	99.5
5	50.7
8	33.8
16	10.1
33	1.0
47	0.1

a) Hand plot pressure versus elevation using linear-linear scales.
b) Hand plot pressure versus elevation using log-linear scales.
c) Hand plot pressure versus elevation using log-log scales.
d) Based on your hand plots, what type of function is most appropriate (linear, exponential, power law)????
e) Create an Excel spreadsheet, and create a plot of the data along with a trendline showing r^2 for linear, exponential and power law functions. Which has the highest r^2?
f) If you have time, manually compute m and b for this function using Excel.

equations for exponential fits

$$m = \frac{n\sum(x_i \cdot \ln(y_i)) - \sum x_i \sum \ln(y_i)}{n\sum x_i^2 - (\sum x_i)^2} \qquad \ln(b) = \frac{\sum \ln(y_i) - m\sum x_i}{n}$$

$$b = e^{\ln(b)}$$

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solution

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