A Breathing LED Indicator

EAS 199A Notes

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EAS 199A: Breathing LED equations

Motivation

- 1. A reverse engineering exercise: emulate the breathing style of LED pulsing on a Macintosh laptop
- 2. Controlling LED brightness requires Pulse-width modulation (PWM), which can also used to control the speed of DC motors.
- 3. This is also an opportunity to practice algebra and Excel plotting

US Patent # 6,658,577 B2



(12)	United	States	Patent
	Huppi et a	l.	

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(54) BREATHING STATUS LED INDICA	CATOR
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- (*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.
- (21) Appl. No.: 10/197,542
- (22) Filed: Jul. 15, 2002
- (65) **Prior Publication Data**

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Related U.S. Application Data

- (63) Continuation of application No. 09/332,242, filed on Jun. 14, 1999.
- (51) **Int. Cl.**⁷ **G06F 1/26**; G06F 1/28

(32) 0.0. 01	(52)	U.S. Cl.		713/323;	713/3	20
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(56) References Cited

U.S. PATENT DOCUMENTS

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^{*} cited by examiner

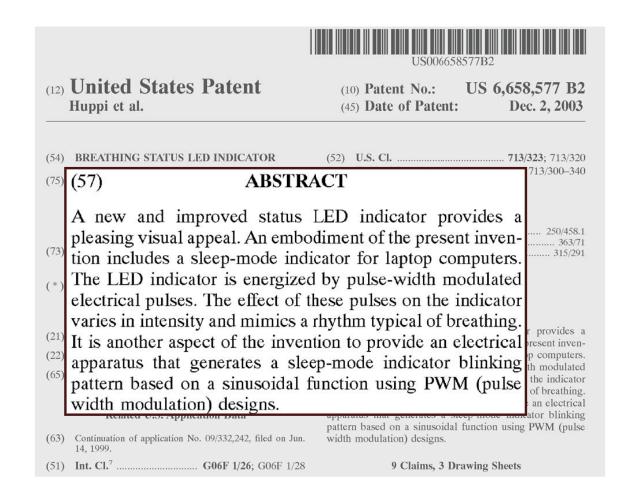
Primary Examiner—Rupal Dharia

(57) ABSTRACT

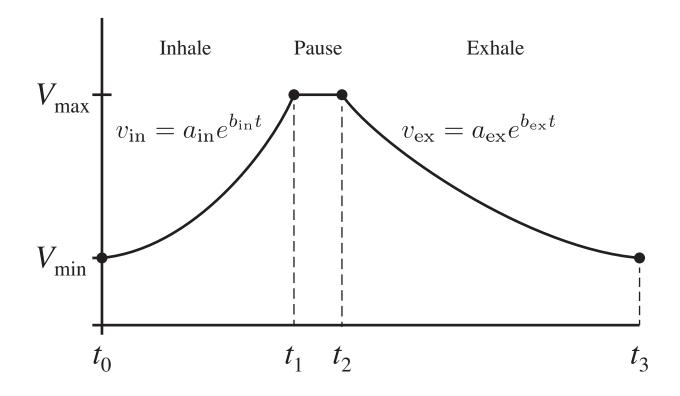
A new and improved status LED indicator provides a pleasing visual appeal. An embodiment of the present invention includes a sleep-mode indicator for laptop computers. The LED indicator is energized by pulse-width modulated electrical pulses. The effect of these pulses on the indicator varies in intensity and mimics a rhythm typical of breathing. It is another aspect of the invention to provide an electrical apparatus that generates a sleep-mode indicator blinking pattern based on a sinusoidal function using PWM (pulse width modulation) designs.

9 Claims, 3 Drawing Sheets

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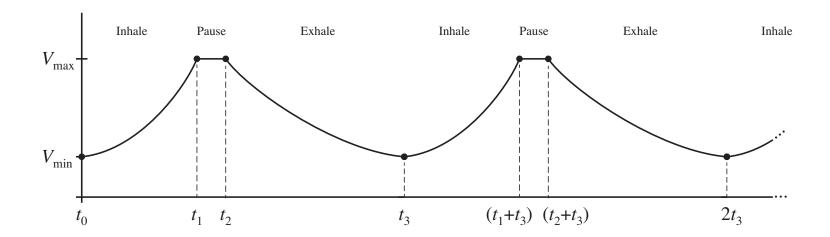


The breathing pattern has three phases: inhale, pause, and exhale

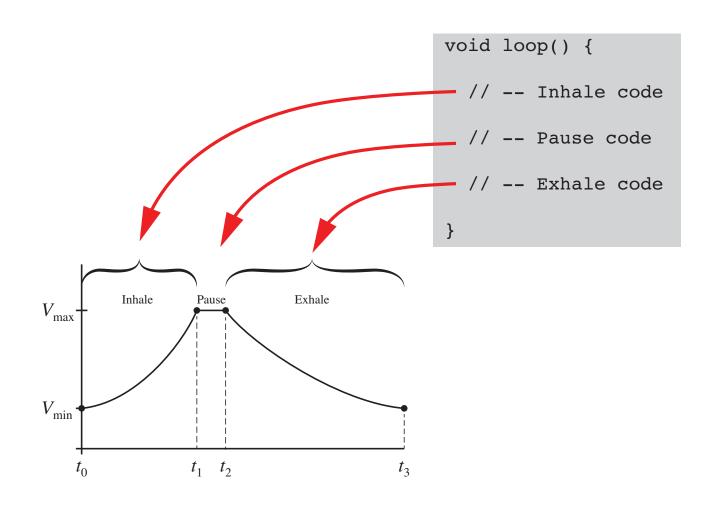


Note: This *not* the pattern claimed on US patent # 6658577.

The breathing pattern repeats indefinitely



The repeated pattern is the body of the loop function

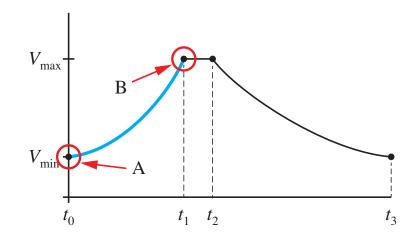


Inhale and exhale Functions have the same form

Inhale and exhale functions are of the form

$$v = ae^{bt} \tag{1}$$

Require that this function pass through two points (t_A, v_A) and (t_B, v_B) .



Substituting these data pairs into Equation (1) gives

$$v_A = ae^{bt_A} (2)$$

$$v_B = ae^{bt_B} (3)$$

Linearize the Equations to Simply the Algebra

Recall that if

$$z = xy$$

then

$$\ln(z) = \ln(x) + \ln(y).$$

In words: the logarithm of a product is the sum of the logarithms of the terms being multiplied.

Also recall that if

$$r = e^{st}$$

then

$$ln(r) = st.$$

Linearize the Equations to Simply the Algebra

Take the logarithm of Equation (1) and apply the rules for manipulating logarithms of products:

$$\ln(v) = \ln\left[ae^{bt}\right] \longrightarrow \ln(v) = \ln(a) + \ln\left[e^{bt}\right] \longrightarrow \ln(v) = \ln(a) + bt$$

Apply the transformation to Equations (2) and (3) to get

$$\ln(v_A) = \ln(a) + bt_A \tag{4}$$

$$ln(v_B) = ln(a) + bt_B$$
(5)

These are two *linear* equations for the two unknowns, ln(a) and b. The linear equations can be solved more easily.

Solve for a and b (1)

Subtract Equation (5) from Equation (4) to get

$$\ln(v_A) - \ln(v_B) = b(t_A - t_B) \tag{6}$$

Since t_A , v_A , t_B and v_B are known, we can solve for b

$$b = \frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}. (7)$$

Solve for a and b (2)

Now that the formula for b is known, we can substitute Equation (7) into either Equation (4) or Equation (5) to solve for a.

$$\ln(v_A) = \ln(a) + \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$\ln(a) = \ln(v_A) - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$= \ln(v_A) \left[\frac{t_A - t_B}{t_A - t_B}\right] - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$= \frac{\ln(v_A) \left[t_A - t_B\right] - \left[\ln(v_A) - \ln(v_B)\right] t_A}{t_A - t_B}$$

$$= \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

Solve for a and b (3)

Therefore,

$$\ln(a) = \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

Applying the exponential function to both sides of the preceding equation gives the formula for computing \boldsymbol{a}

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right] \tag{8}$$

We have created the formulas that model either inhale or exhale

Given (t_A, v_A) and (t_B, v_B) , we can compute

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right]$$

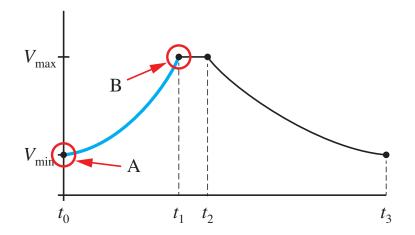
and

$$b=rac{\ln(v_A)-\ln(v_B)}{t_A-t_B}.$$

which allows us to evaluate

$$v = ae^{bt}$$

for any t in the interval $t_A \leq t \leq t_B$.



Applying the Equations to find a and b (1)

- 1. Choose appropriate values of V_{\min} , V_{\max} , t_1 , t_2 and t_3 . These are somewhat arbitrary design choices that you make to achieve a desired look to your inhale and exhale functions.
- 2. Use Equations (8) and (7) to compute $a_{\rm in}$ and $b_{\rm in}$.
- 3. Use Equations (8) and (7) (again) to compute $a_{\rm ex}$ and $b_{\rm ex}$.

Applying the Equations to find a and b (2)

Once you have obtained values for $a_{\rm in}$, $b_{\rm in}$, $a_{\rm ex}$ and $b_{\rm ex}$, it is a good idea to add this step

4. Plot the $v_{\rm in}(t)$ and $v_{\rm ex}(t)$ functions (say, with Excel or MATLAB) to make sure you do not have an error in your algebra.

Recapitulation

So far:

- 1. Both the inhale and exhale phases can be modeled with $v=ae^{bt}$
- 2. We choose the end points to give a desired shape.
- 3. When the endpoints are known, we can solve for a and b.
- 4. With known values of a and b for each phase, we can write code to control the brightness of the LED.

Next: translate the $v = ae^{bt}$ function into Arduino code.

But first: Let's make a plot of our $v=ae^{bt}$ functions to make sure we understand the math.