A Breathing LED Indicator EAS 199A Notes

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EAS 199A: Breathing LED equations

Motivation

- 1. A reverse engineering exercise: emulate the breathing style of LED pulsing on a Macintosh laptop
- 2. Controlling LED brightness requires Pulse-width modulation (PWM), which can also used to control the speed of DC motors.
- 3. This is also an opportunity to practice algebra and Excel plotting

US Patent # 6,658,577 B2

(12) United States Patent Huppi et al.			US006658577B2 (10) Patent No.: US 6,658,577 B2 (45) Date of Patent: Dec. 2, 2003	
(54)	BREATHING STATUS LED INDICATOR		(52) U.S. Cl	
(75)	Inventors:	Brian Q. Huppi, Cupertino, CA (JS); Christopher J. Stringer, Pacifica, CA (US); Jory Bell, San Francisco, CA (US); Christopher L. Capener, Cupertino, CA (US)	 (56) References Cited U.S. PATENT DOCUMENTS 5.608.225 A. * 3/1997 Kamimura et al 250/458 	
(73)	Assignee:	Apple Computer, Inc., Cupertino, CA (US)	5,659,465 A * 8/1997 Flack et al	
(*)	Notice:	Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.		
(21)	Appl. No.:	: 10/197,542	A new and improved status LED indicator provides	
(22)	Filed:	Jul. 15, 2002	pleasing visual appeal. An embodiment of the present investion includes a sleep-mode indicator for laptop computer	
(65)	Prior Publication Data		The LED indicator is energized by pulse-width modulated electrical pulses. The effect of these pulses on the indicator varies in intensity and mimics a rhythm typical of breathing. It is another aspect of the invention to provide an electrical	
	US 2002/0178388 A1 Nov. 28, 2002			
	Rel	ated U.S. Application Data	apparatus that generates a sleep-mode indicator blinking pattern based on a sinusoidal function using PWM (pulse	
(63)	Continuation of application No. 09/332,242, filed on Jun. 14, 1999.		width modulation) designs.	
(51)	Int. Cl. ⁷		9 Claims, 3 Drawing Sheets	

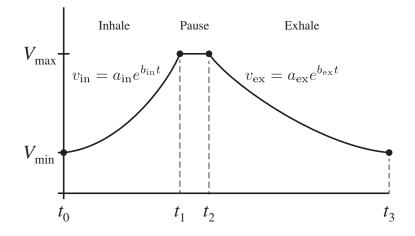
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US Patent # 6,658,577 B2

	US006658577B2		
12) United States Patent Huppi et al.	(10) Patent No.: US 6,658,577 B2 (45) Date of Patent: Dec. 2, 2003		
54) BREATHING STATUS LED INDICATOR 75) (57) ABST	(52) U.S. Cl		
A new and improved status LED indicator provides a pleasing visual appeal. An embodiment of the present invention includes a sleep-mode indicator for laptop computers. The LED indicator is energized by pulse-width modulated electrical pulses. The effect of these pulses on the indicator varies in intensity and mimics a rhythm typical of breathing. It is another aspect of the invention to provide an electrical paratus that generates a sleep-mode indicator blinking pattern based on a sinusoidal function using PWM (pulse width modulation) designs.			
 (63) Continuation of application No. 09/332,242, filed on J 14, 1999. (64) Int. Cl.⁷	, e		

The breathing pattern has three phases: inhale, pause, and exhale

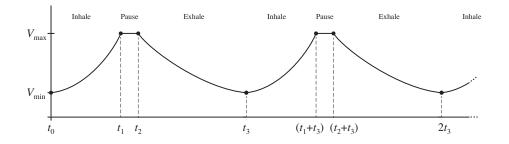


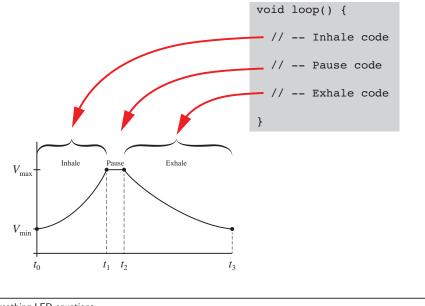
Note: This not the pattern claimed on US patent # 6658577.

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The breathing pattern repeats indefinitely





The repeated pattern is the body of the loop function

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Inhale and exhale Functions have the same form

Inhale and exhale functions are of the form

$$v = a e^{bt} \tag{1}$$

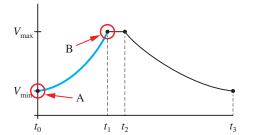
Require that this function pass through two points (t_A, v_A) and (t_B, v_B) .

Substituting these data pairs into Equation (1) gives

$$v_A = a e^{bt_A} \tag{2}$$

$$v_B = a e^{bt_B} \tag{3}$$

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Linearize the Equations to Simply the Algebra

Recall that if

then

$$\ln(z) = \ln(x) + \ln(y).$$

z = xy

In words: the logarithm of a product is the sum of the logarithms of the terms being multiplied.

Also recall that if

then

$$\ln(r) = st.$$

 $r = e^{st}$

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Linearize the Equations to Simply the Algebra

Take the logarithm of Equation (1) and apply the rules for manipulating logarithms of products:

$$\ln(v) = \ln\left[ae^{bt}\right] \longrightarrow \ln(v) = \ln(a) + \ln\left[e^{bt}\right] \longrightarrow \ln(v) = \ln(a) + bt$$

Apply the transformation to Equations (2) and (3) to get

$$\ln(v_A) = \ln(a) + bt_A \tag{4}$$

$$\ln(v_B) = \ln(a) + bt_B \tag{5}$$

These are two *linear* equations for the two unknowns, $\ln(a)$ and b. The linear equations can be solved more easily.

Solve for a and b (1)

Subtract Equation (5) from Equation (4) to get

$$\ln(v_A) - \ln(v_B) = b(t_A - t_B)$$
(6)

Since $\mathit{t_{\!A}}, \mathit{v_{\!A}}, \mathit{t_{\!B}}$ and $\mathit{v_{\!B}}$ are known, we can solve for b

$$b = \frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}.$$
(7)

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Solve for a and b (2)

Now that the formula for b is known, we can substitute Equation (7) into either Equation (4) or Equation (5) to solve for a.

$$\ln(v_A) = \ln(a) + \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$
$$\ln(a) = \ln(v_A) - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$
$$= \ln(v_A) \left[\frac{t_A - t_B}{t_A - t_B}\right] - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$
$$= \frac{\ln(v_A) \left[t_A - t_B\right] - \left[\ln(v_A) - \ln(v_B)\right] t_A}{t_A - t_B}$$
$$= \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

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Solve for
$$a$$
 and b (3)

Therefore,

$$\ln(a) = \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

Applying the exponential function to both sides of the preceding equation gives the formula for computing \boldsymbol{a}

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right]$$
(8)

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We have created the formulas that model either inhale or exhale

Given (t_A, v_A) and (t_B, v_B) , we can compute

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right]$$

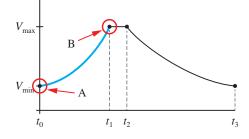
 $\quad \text{and} \quad$

$$b = \frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}$$

which allows us to evaluate

$$v = ae^{bt}$$

for any t in the interval $t_A \leq t \leq t_B$.



Applying the Equations to find a and b (1)

- 1. Choose appropriate values of V_{\min} , V_{\max} , t_1 , t_2 and t_3 . These are somewhat arbitrary *design* choices that you make to achieve a desired look to your inhale and exhale functions.
- 2. Use Equations (8) and (7) to compute $a_{\rm in}$ and $b_{\rm in}$.
- 3. Use Equations (8) and (7) (again) to compute $a_{\rm ex}$ and $b_{\rm ex}$.

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Applying the Equations to find a and b (2)

Once you have obtained values for $a_{
m in}$, $b_{
m in}$, $a_{
m ex}$ and $b_{
m ex}$, it is a good idea to add this step

4. Plot the $v_{in}(t)$ and $v_{ex}(t)$ functions (say, with Excel or MATLAB) to make sure you do not have an error in your algebra.

Recapitulation

So far:

- 1. Both the inhale and exhale phases can be modeled with $v = ae^{bt}$
- 2. We choose the end points to give a desired shape.
- 3. When the endpoints are known, we can solve for a and b.
- 4. With known values of a and b for each phase, we can write code to control the brightness of the LED.

Next: translate the $v = ae^{bt}$ function into Arduino code.

But first: Let's make a plot of our $v = ae^{bt}$ functions to make sure we understand the math.

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