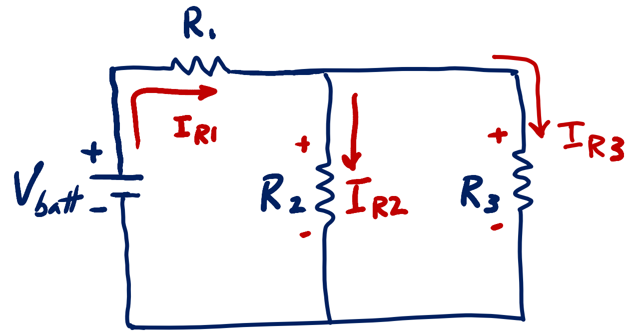
KIRCHOFF'S VOLTAGE LAW: EXAMPLE 2

GIVEN: Consider the circuit shown, where

R_1=12Ω R_2=4Ω R_3=12Ω V_1=12V



REQUIRED:

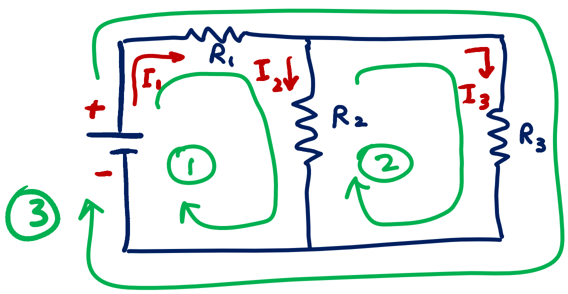
(a) The current through R_1.

(b) The current through R_2.

(c) The current through R_3.

SOLUTION:

(a) First, we identify the loops in the circuit. As shown below, we can choose any two of the three loops.



We use the assumed direction of current to determine whether voltage increases or decreases across a resistor.

Kirchoff's Voltage Law: *The sum of the voltage drops around any closed loop is zero.*

Applying KVL to Loop 1:

$$V_{batt} - V_{R1} - V_{R2} = 0$$

$$V_{batt} = V_{R1} + V_{R2} = I_{R1} \cdot R_1 + I_{R2} \cdot R_2$$

Unfortunately, we have one equation with two unknowns (I_{R1} and I_{R2}). Note that $I_{R1} \neq I_{R2}$ because I_{R1} splits between two resistors (R_2 and R_3). Students often do not recognize that the current SPLITS in this way.

Find R_{eq} for the circuit so that we can find I_{R1}:

$$R_{eq} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = 12\Omega + \frac{1}{\frac{1}{4\Omega} + \frac{1}{12\Omega}} = 12\Omega + \frac{12\Omega}{4} = 15\Omega$$

Apply Ohm's Law:

$$V_{batt} = I_{R1} \cdot R_{eq} \longrightarrow I_{R1} = \frac{V_{batt}}{R_{eq}} = \frac{12V}{15\Omega} = \underline{\underline{0.8A}}$$

(b) Now that we know the current through R_1, we can use KVL around Loop 1 (as shown earlier):

$$\left.\begin{aligned} &V_{batt} - V_{R1} - V_{R2} = 0 \\ &\cancelto{12V}{V_{batt}} = V_{R1} + V_{R2} = \cancelto{0.8A}{I_{R1}} \cdot \cancelto{12\Omega}{R_1} + I_{R2} \cdot \cancelto{4\Omega}{R_2} \end{aligned}\right\} \quad \underline{\underline{I_{R2} = 0.6A}}$$

(c) Apply KVL to Loop 3:

$$\cancelto{12V}{V_{batt}} - \cancelto{I_{R1}\cdot R_1}{V_{R1}} - \cancelto{I_{R3}\cdot R_3}{V_{R3}} = 0$$

$$I_{R3}\cdot \cancelto{12\Omega}{R_3} = 12V - \cancelto{0.8A}{I_{R1}}\cdot \cancelto{12\Omega}{R_1}$$

$$I_{R3} = \frac{1}{12\Omega}\left[12V - 0.8A\cdot 12\Omega\right] = \frac{2.4V}{12\Omega} = \underline{\underline{0.2\,A}}$$

DISCUSSION:

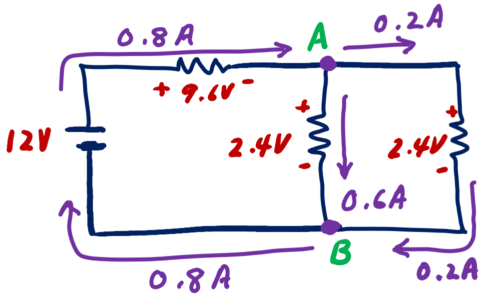
- We can apply KVL to Loop 2 to check our work:

$$V_{R2} - V_{R3} = 0$$

$$\cancelto{0.6A}{I_{R2}}\cdot \cancelto{4\Omega}{R_2} - \cancelto{0.2A}{I_{R3}}\cdot \cancelto{12\Omega}{R_3} = 0$$

$$2.4V - 2.4V = 0 \leftarrow \text{Loop 2 checks out!}$$

- Look at a voltage and current balance:



Notice that the voltages around all three loops sum to zero.

LOOP 1: $12V - 9.6V - 2.4V = 0$ ✓

LOOP 2: $2.4V - 2.4V = 0$ ✓

LOOP 3: $12V - 9.6V - 2.4V = 0$ ✓

Notice that the currents entering and leaving nodes A and B balance

NODE A $\begin{cases} I_{in} = I_{out} \\ 0.8A = 0.2A + 0.6A \end{cases}$ ✓

- Notice that when the current splits at node A, more current goes through R_2 (4Ω) than through R_3 (12Ω). Electricity takes the path of least resistance.