Computer Graphics

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http://www.cs.pdx.edu/~fliu/courses/cs447/

10/13/2021

Last Time

- Dithering
- Signal Processing

Today

- Filering
- Resampling
- Project 1 due 10/29/2021
  - Before you submit, run the grading script on your side
  - Make sure that all the algorithms you implement will be executed by this script

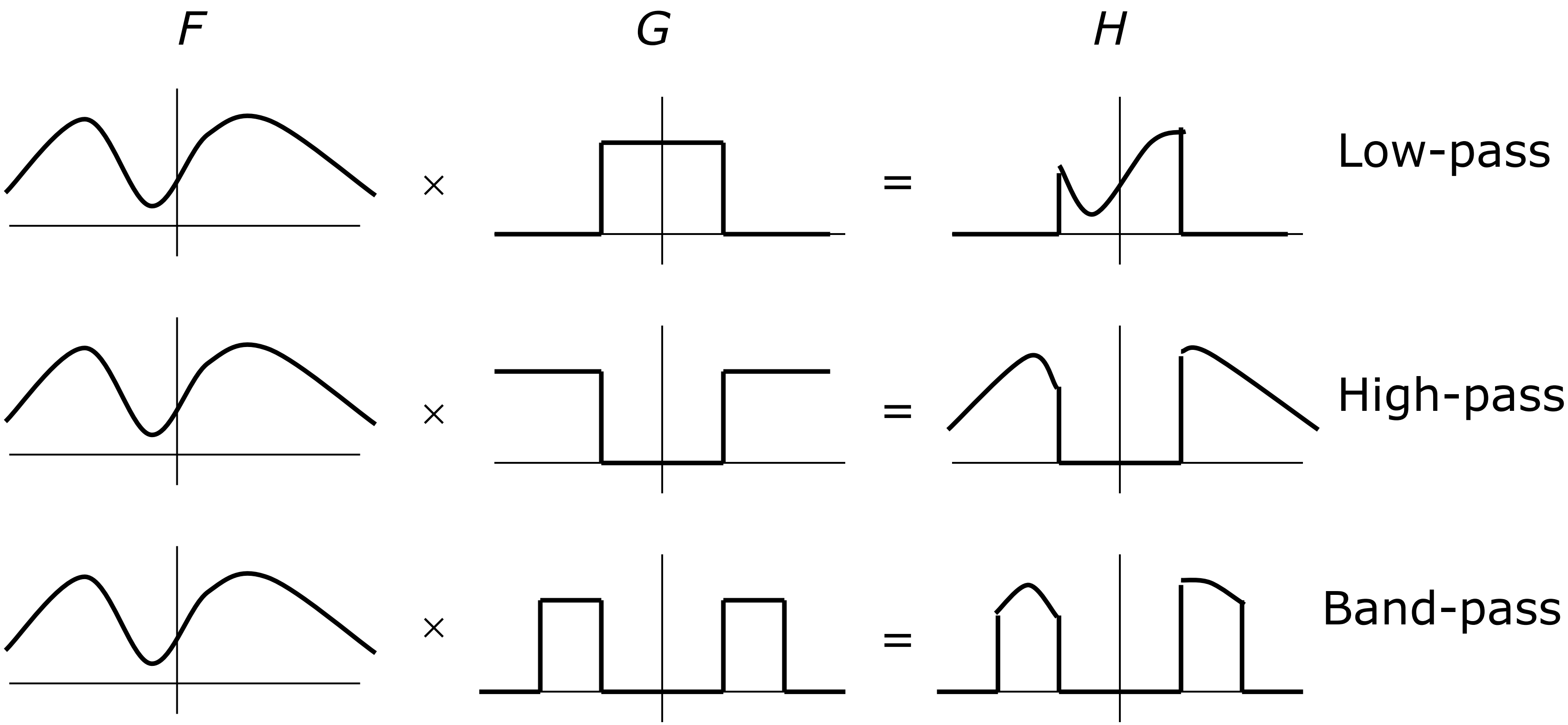
Filters

- A *filter* is something that attenuates or enhances particular frequencies
- Easiest to visualize in the frequency domain, where filtering is defined as multiplication:

$$H(\omega) = F(\omega) \times G(\omega)$$

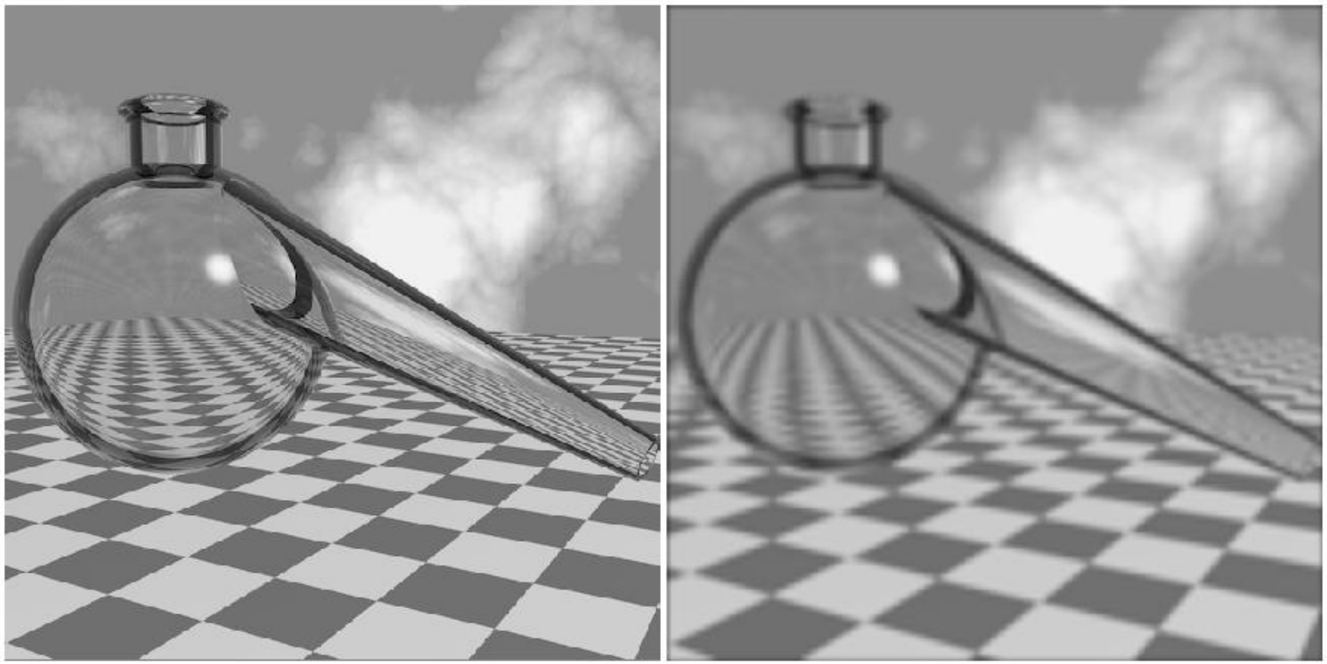
- Here, *F* is the spectrum of the function, *G* is the spectrum of the filter, and *H* is the filtered function. Multiplication is point-wise

Qualitative Filters

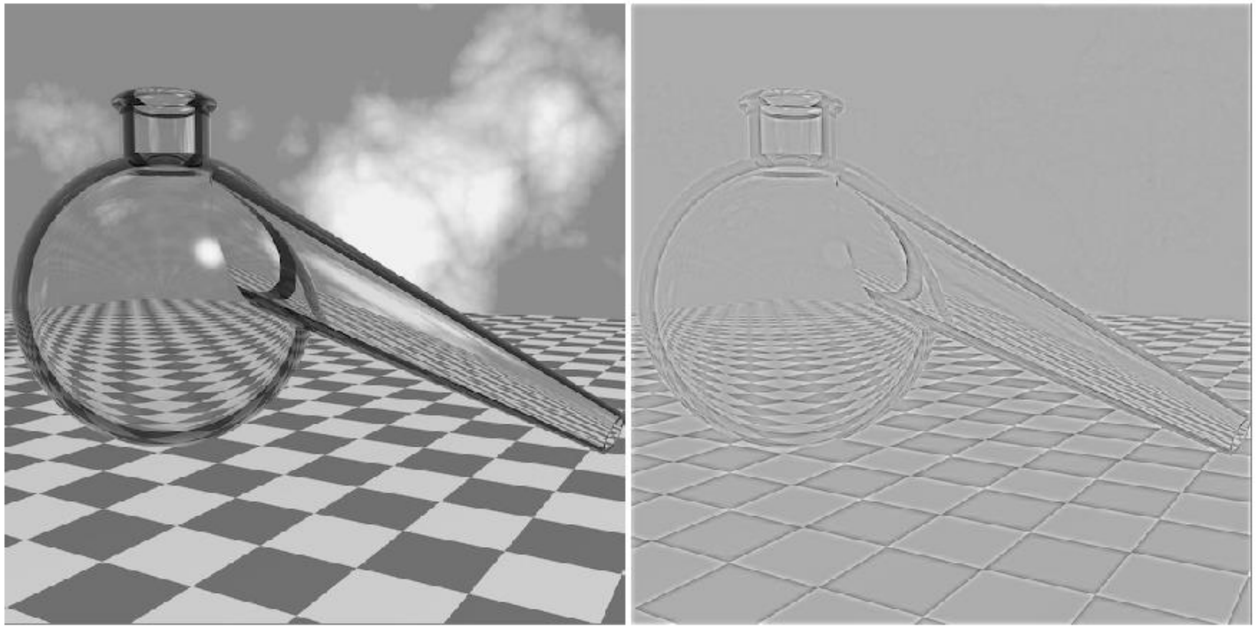


Input spectrum Filter spectrum Filtering result

Low-Pass Filtered Image



High-Pass Filtered Image



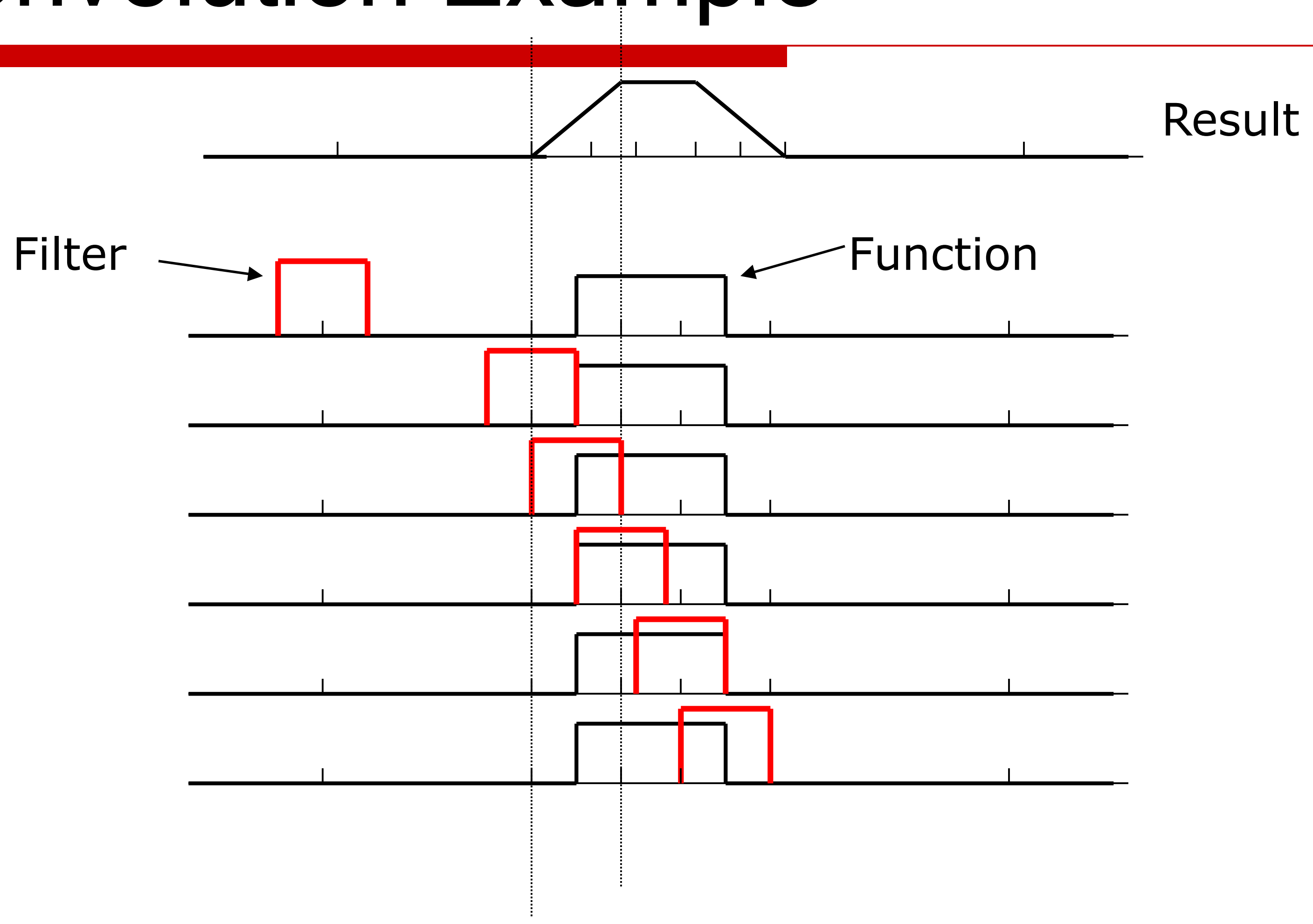
Filtering in the Spatial Domain

- Filtering the spatial domain is achieved by *convolution*

$$h(x) = f \otimes g = \int_{-\infty}^{\infty} f(u)g(x-u)du$$

- Qualitatively: Slide the filter to each position, *x*, then sum up the function multiplied by the filter at that position

Convolution Example



Convolution Theorem

- Convolution in the spatial domain is the same as multiplication in the frequency domain
  - Take a function, f, and compute its Fourier transform, F
  - Take a filter, g, and compute its Fourier transform, G
  - Compute $H=F\times G$
  - Take the inverse Fourier transform of H, to get h
  - Then $h=f\otimes g$
- Multiplication in the spatial domain is the same as convolution in the frequency domain

Filtering Images

$$I_{output}[x][y] = \sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I_{input}[x+i][y+j] M[i+k/2][j+k/2]$$

- Work in the discrete spatial domain
- Convert the filter into a matrix, the *filter mask or kernel*
- Move the matrix over each point in the image, multiply the entries by the pixels below, then sum

Handling Boundaries

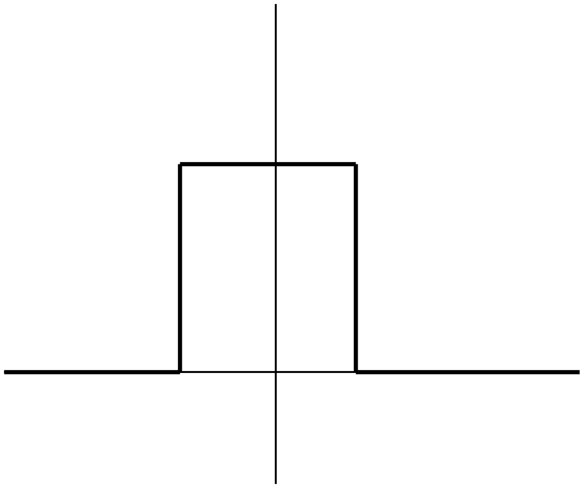
$$I_{output}[x][y] = \sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I_{input}[x+i][y+j] M[i+k/2][j+k/2]$$

- At (0,0) for instance, you might need pixel data for (-1,-1), which doesn't exist
- Option 1: Make the output image smaller – don't evaluate pixels you don't have all the input for
- Option 2: Replicate the edge pixels
  - Equivalent to: posn = x + i; if (posn < 0) posn = 0; and so on for other indices
- Option 3: Reflect image about edge
  - Equivalent to: posn = x + i; if (posn < 0) posn = -posn; and similar for others

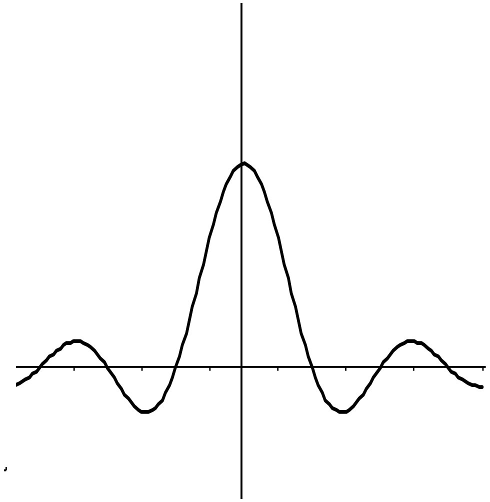
Box Filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Box filters smooth by averaging neighbors
- In frequency domain, keeps low frequencies and attenuates (reduces) high frequencies, so clearly a low-pass filter



Spatial: Box

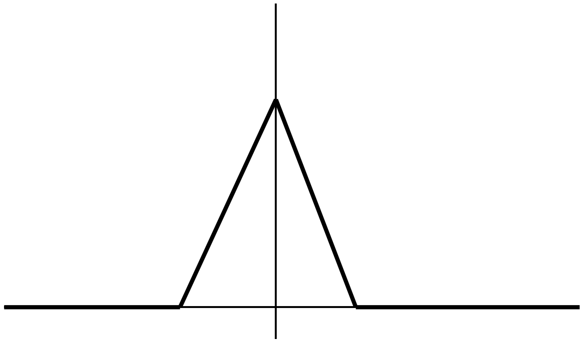


Frequency: sinc

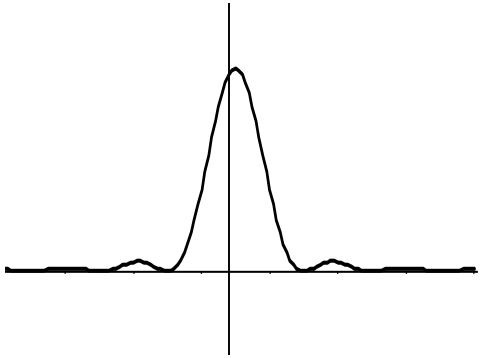
Bartlett Filter

$$\frac{1}{81}\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$

- Triangle shaped filter in spatial domain
- In frequency domain, product of two box filters, so attenuates high frequencies more than a box



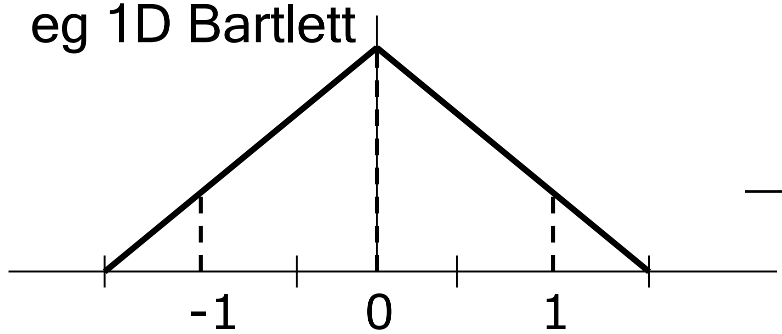
Spatial: Triangle (Box$\otimes$Box)



Frequency: sinc^2

Constructing Masks: 1D

- Sample the filter, then normalize
- eg 1D Bartlett



$$\longrightarrow \frac{1}{5}\begin{array}{|c|c|c|}\hline 1 & 3 & 1 \\ \hline\end{array}$$

- Can go to edge of pixel or middle of next: results are slightly different

Constructing Masks: 2D

- Multiply 2 1D masks together using *outer product*

$$M = mm^T, \text{or } M[i][j] = m[i]m[j]$$

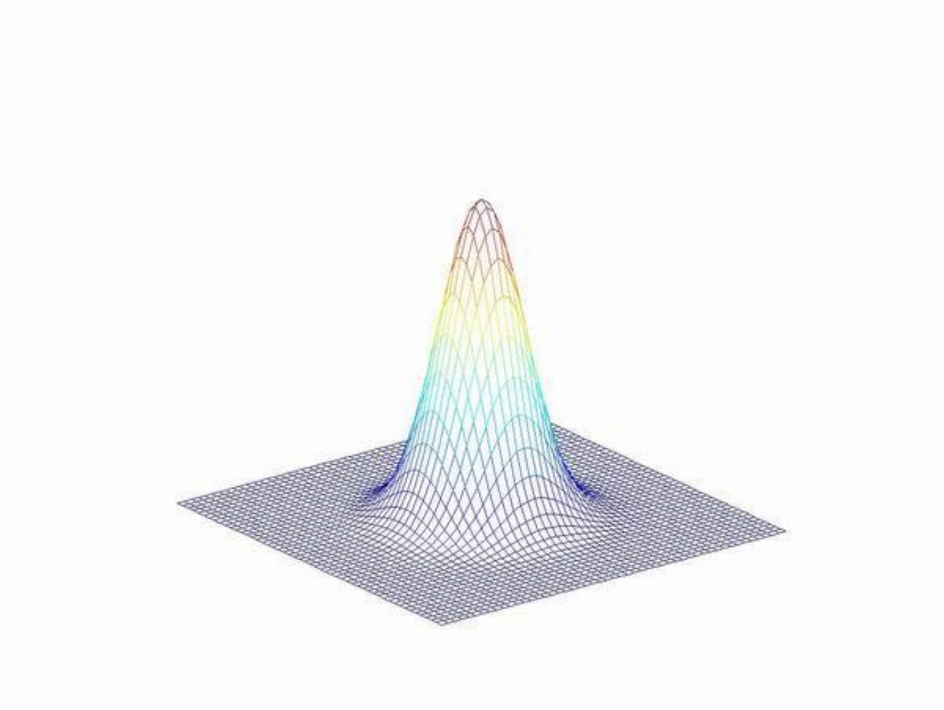
- *M* is 2D mask, *m* is 1D mask

	0.2	0.6	0.2
0.2	0.04	0.12	0.04
0.6	0.12	0.36	0.12
0.2	0.04	0.12	0.04

$$\frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

Gaussian Filter

- Attenuates high frequencies even further
- In 2d, rotationally symmetric, so fewer artifacts



Constructing Gaussian Mask

- Use the binomial coefficients
  - Central Limit Theorem (probability) says that with more samples, binomial converges to Gaussian

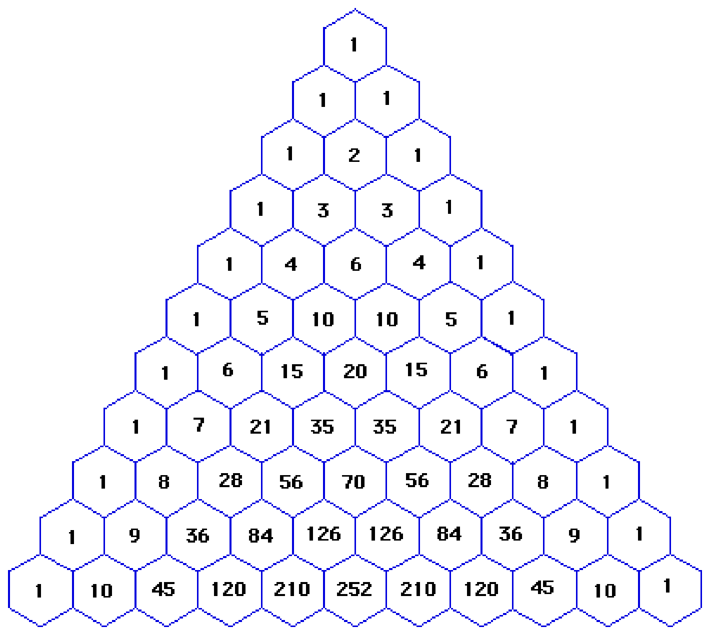
$$\frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline\end{array}$$

$$\frac{1}{16}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline\end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|}\hline 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ \hline\end{array}$$

Constructing Gaussian Mask

- Use the binomial coefficients
  - Central Limit Theorem (probability) says that with more samples, binomial converges to Gaussian



High-Pass Filters

- A high-pass filter can be obtained from a low-pass filter
  - If we subtract the smoothed image from the original, we must be subtracting out the low frequencies
  - What remains must contain only the high frequencies
- High-pass masks come from matrix subtraction:
- eg: 3x3 Bartlett

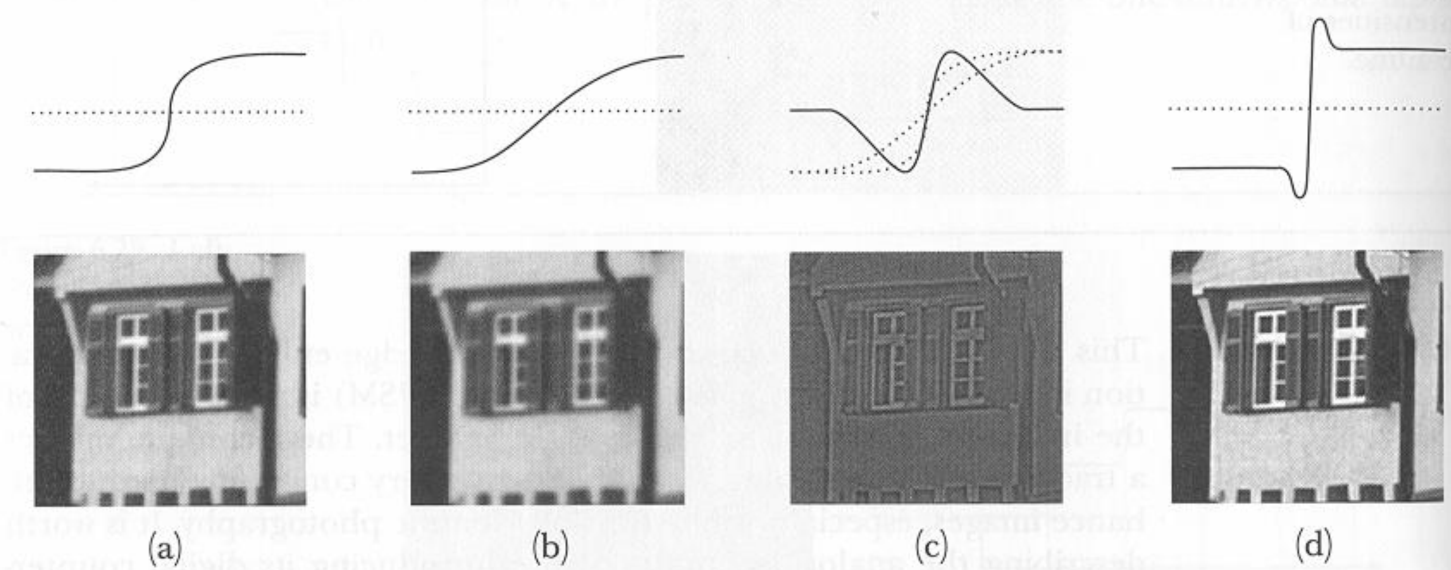
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

Edge Enhancement

- High-pass filters give high values at edges, low values in smooth regions
- Adding high frequencies back into the image enhances edges
- One approach:
  - $\text{Image} = \text{Image} + [\underbrace{\text{Image} - \underbrace{\text{smooth(Image)}}_{\text{Low-pass}}}_{\text{High-pass}}]$

Edge Enhancement

Figure 6.32. (a) Original image. (b) Blurred image. (c) Difference between first two. (d) Enhanced image.



Fixing Negative Values

- The negative values in high-pass filters can lead to negative image values
  - Most image formats don't support this
- Solutions:
  - Truncate: Chop off values below min or above max
  - Offset: Add a constant to move the min value to 0
  - Re-scale: Rescale the image values to fill the range (0,max)

Filtering and Color

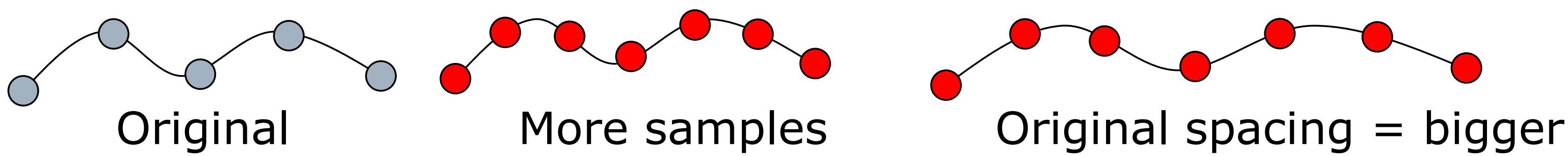
- To filter a color image, simply filter each of R,G and B separately
- Re-scaling and truncating are more difficult to implement:
  - Adjusting each channel separately may change color significantly

Today

- Filtering
- Resampling
- Project 1 due 10/29/2021

Resampling

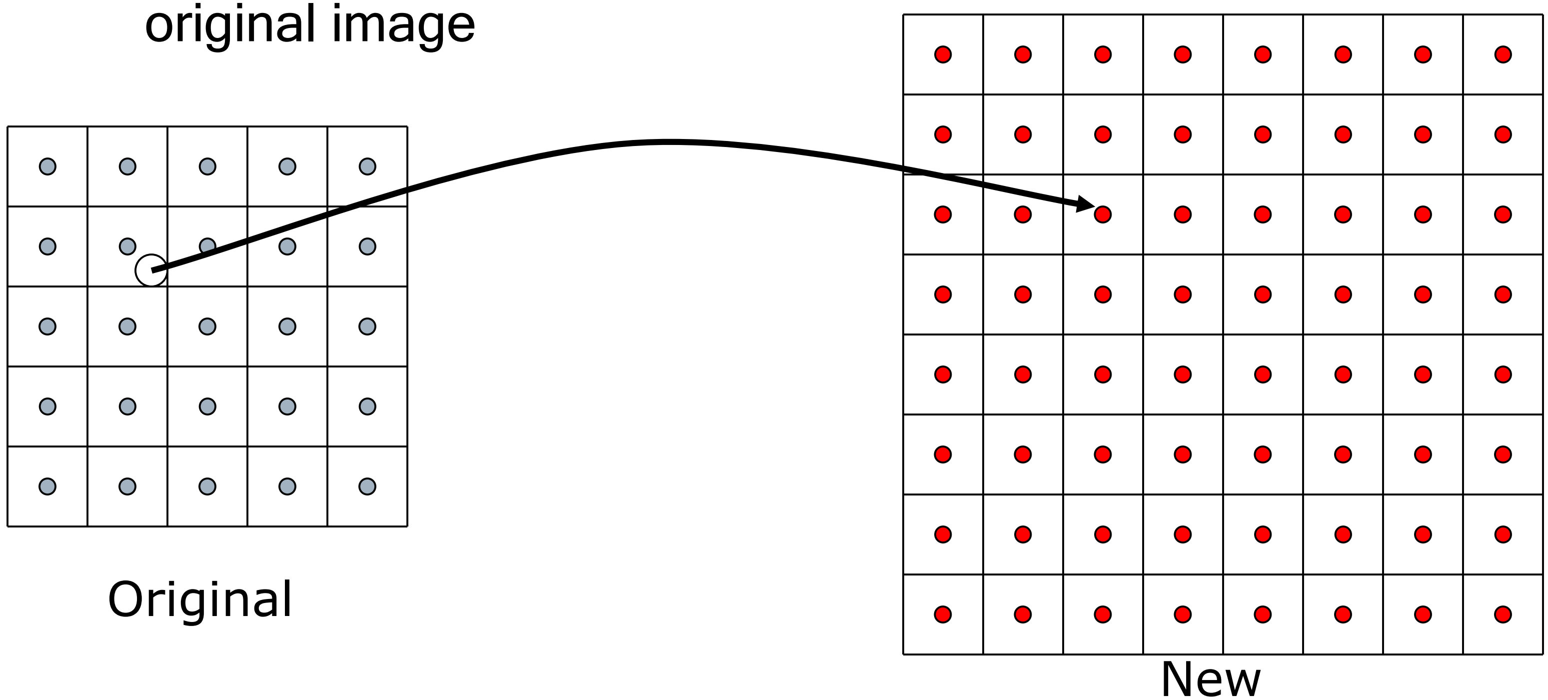
- Making an image larger is like sampling the original function at higher density
  - You need more pixels to represent the same thing, so a higher pixel density



- Reducing an image in size is like sampling at lower density
- Generating new samples of the “same” function is called *resampling*
  - In theory, 2 steps: Reconstruction and sampling – but not in practice
- Many other image manipulation tasks require resampling

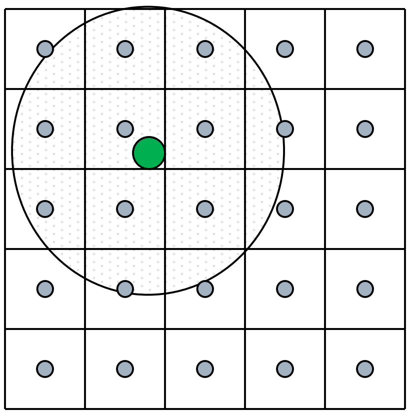
General Scenario

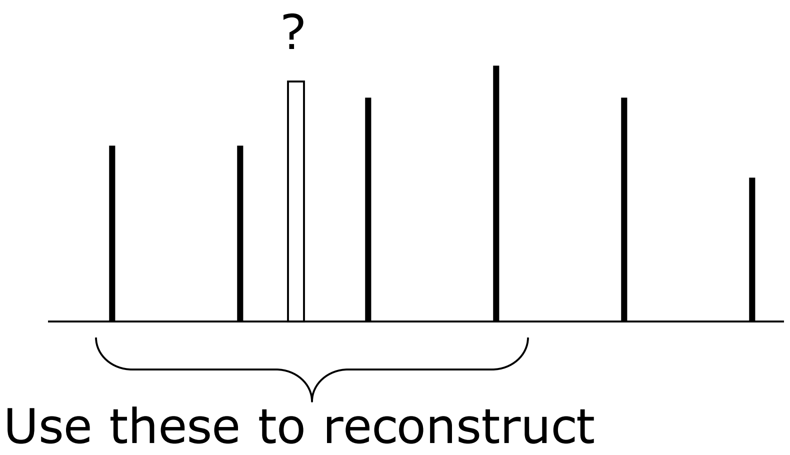
- You are trying to create a new image of some form, and you need data from a particular place in the existing image
  - Always: Figure out where the new sample comes from in the original image



Resampling at a Point

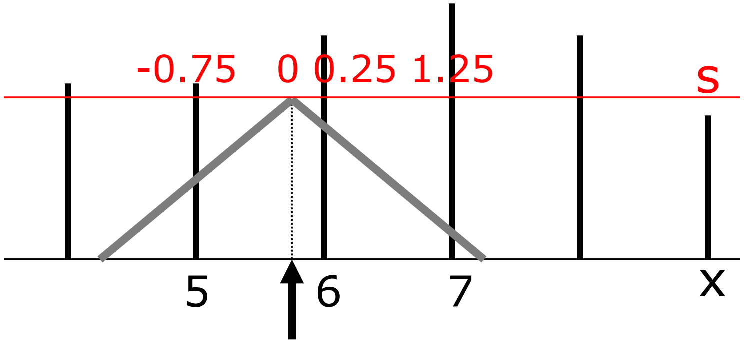
- We want to reconstruct the original "function" at the required point
- We will use information from around the point to do this
- We do it using a filter
- Which filter?
  - We'll look at Bartlett (triangular)
  - Other filters also work





Resampling at a Point

- Place a Bartlett filter at the required point
- Multiply the value of the neighbor by the filter at that point, and add them
  - Convolution with discrete samples
- The filter size is a parameter

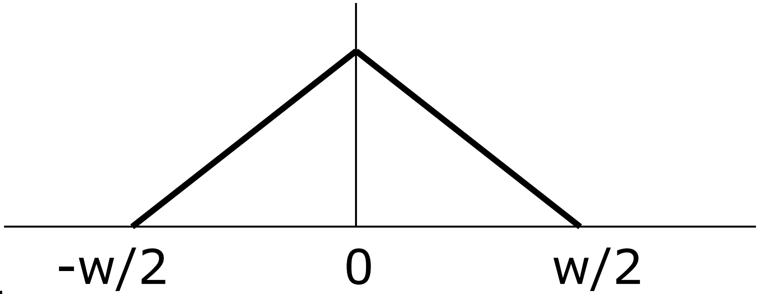


- Say the filter is size 3, and you need the value at x=5.75
  - You need the image samples, I(x), from x=5, x=6 and x=7
  - You need the filter value, H(s), at s=-0.75, s=0.25 and s=1.25
  - Compute: I(5)*H(-0.75)+I(6)*H(0.25)+I(7)*H(1.25)

Functional Form for Filters

- Consider the Bartlett in 1D:

$$H_w(s) = \frac{2}{w}\left(1 - \frac{2|s|}{w}\right)$$



- More generally, to apply it at a point x_c and find the contribution from point x where the image has value I(x)

$$f(x) = \frac{2}{w}\left(1 - \frac{2|x - x_c|}{w}\right) I(x)$$

- Extends naturally to 2D:

$$f(x, y) = \frac{4}{w^2}\left(1 - \frac{2|x - x_c|}{w}\right)\left(1 - \frac{2|y - y_c|}{w}\right) I(x, y)$$

Common Operations

- Image scaling by a factor *k* (e.g. 0.5 = half size):
  - To get x_{orig} given x_{new}, divide by *k*:

$$\left(x_{orig}, y_{orig}\right) = \left(\frac{x_{new}}{k}, \frac{y_{new}}{k}\right)$$

- Image rotation by an angle θ:

$$x_{orig} = \cos(-\theta)x_{new} - \sin(-\theta)y_{new}$$
$$y_{orig} = \sin(-\theta)x_{new} + \cos(-\theta)y_{new}$$

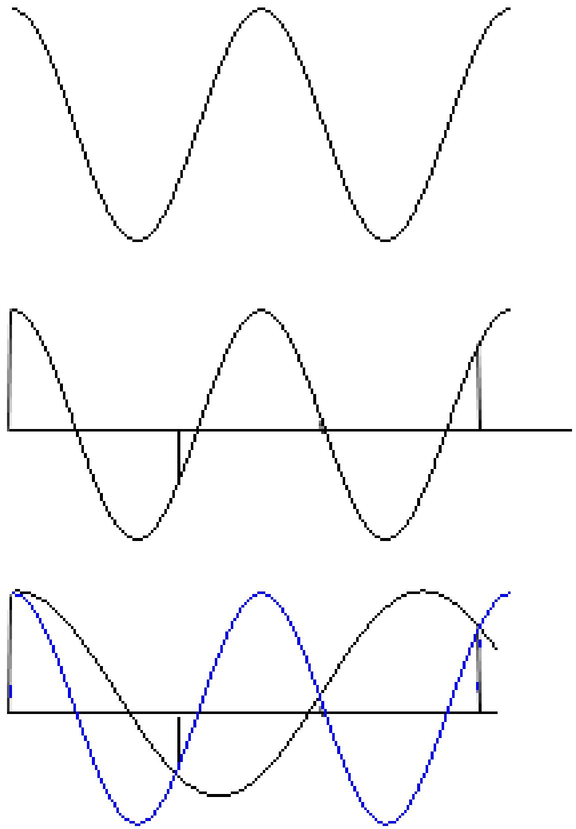
  - This rotates around the bottom left corner. It's up to you to figure out how to rotate about the center
  - Be careful of radians vs. degrees: all C++ standard math functions take radians, but OpenGL functions take degrees

Ideal Image Resize

- To do ideal image resampling, we would reconstruct the original function based on the samples
- A *requirement* for *perfect* enlargement or size reduction
  - Almost never possible in practice, and we'll see why

An Reconstruction Example

- Say you have a sine function of a particular frequency
- And you sample it too sparsely
- You could draw a different sine curve through the samples



Some Intuition

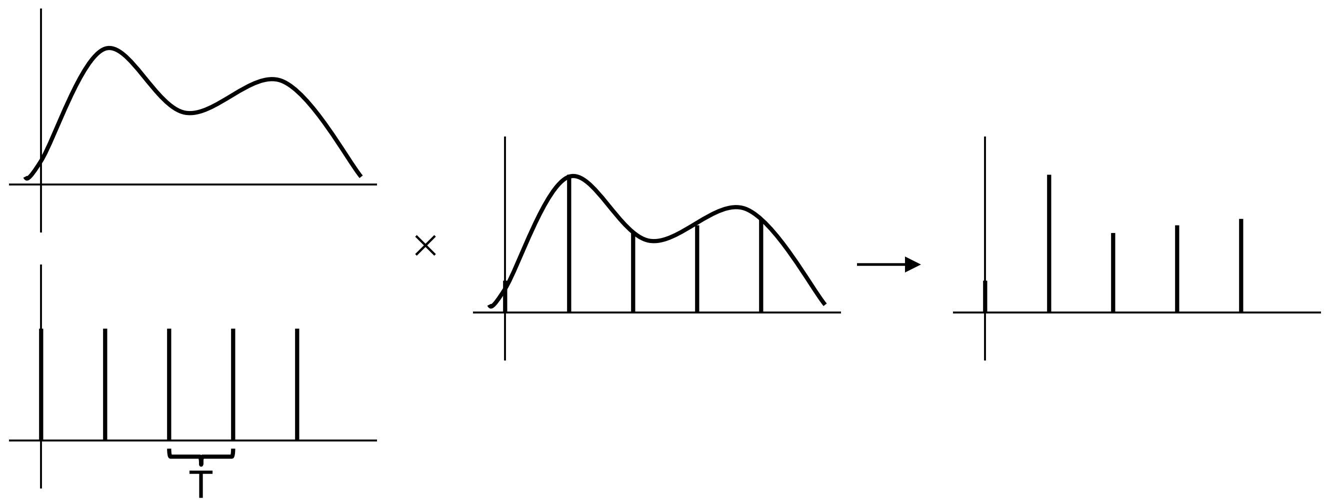
- To reconstruct a function, you need to reconstruct every frequency component that’s in it
  - This is in the frequency domain, but that’s because it’s easy to talk about “components” of the function
- But we’ve just seen that to accurately reconstruct high frequencies, you need more samples
- The effect on the previous slide is called aliasing
  - The correct frequency is *aliased* by the longer wavelength curve

Nyquist Frequency

- Aliasing cannot happen if you sample at a frequency that is twice the original frequency – the *Nyquist* sampling limit
  - You cannot accurately reconstruct a signal that was sampled below its Nyquist frequency – you do not have the information
  - There is no point sampling at higher frequency – you do not gain extra information
- Signals that are not bandlimited cannot be accurately sampled and reconstructed
  - They would require an infinite sampling frequency

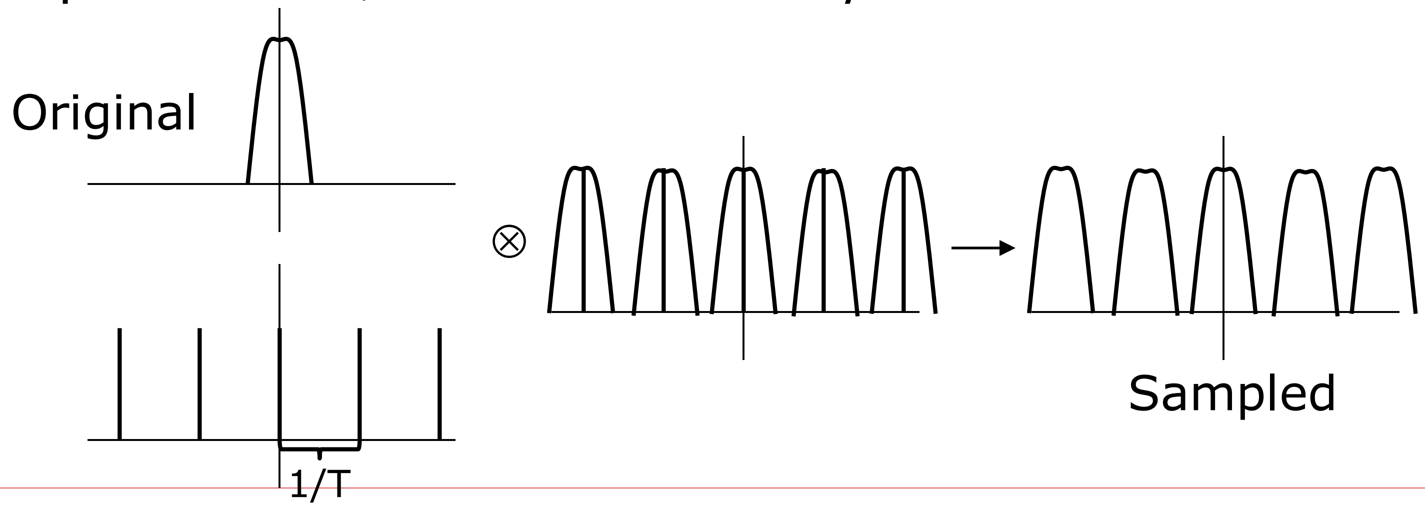
Sampling in Spatial Domain

- Sampling in the spatial domain is like multiplying by a spike function
- You take some ideal function and get data for a regular grid of points



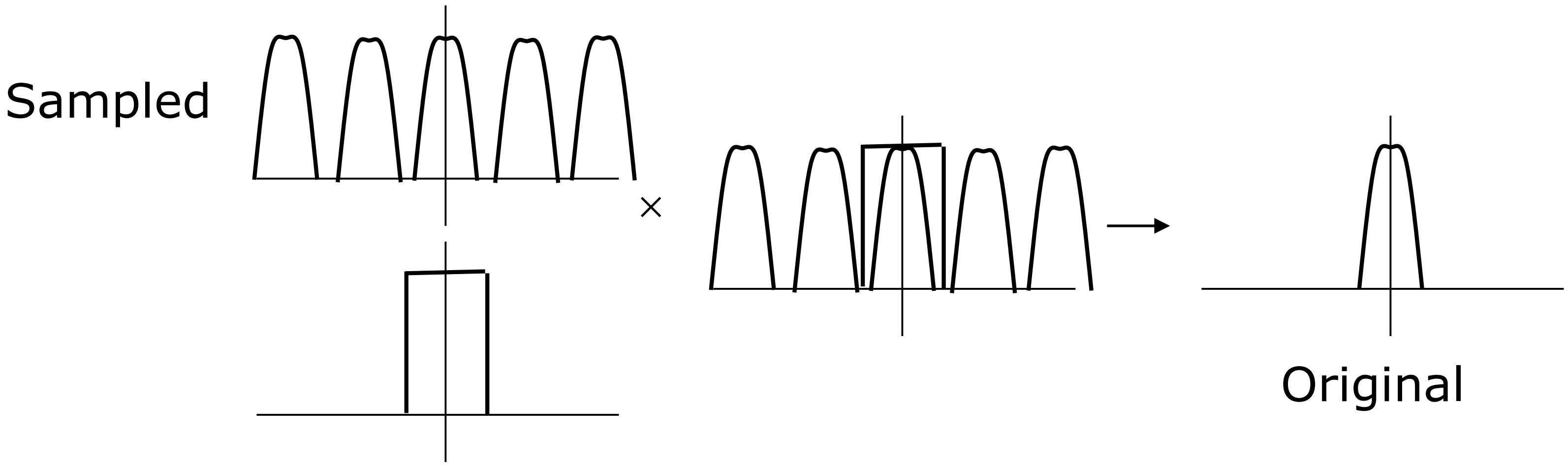
Sampling in Frequency Domain

- Sampling in the frequency domain is like convolving with a spike function
  - Follows from the convolution theory: multiplication in spatial equals convolution in frequency
  - Spatial spike function in the frequency domain is also the spike function, but *with an inverse period*



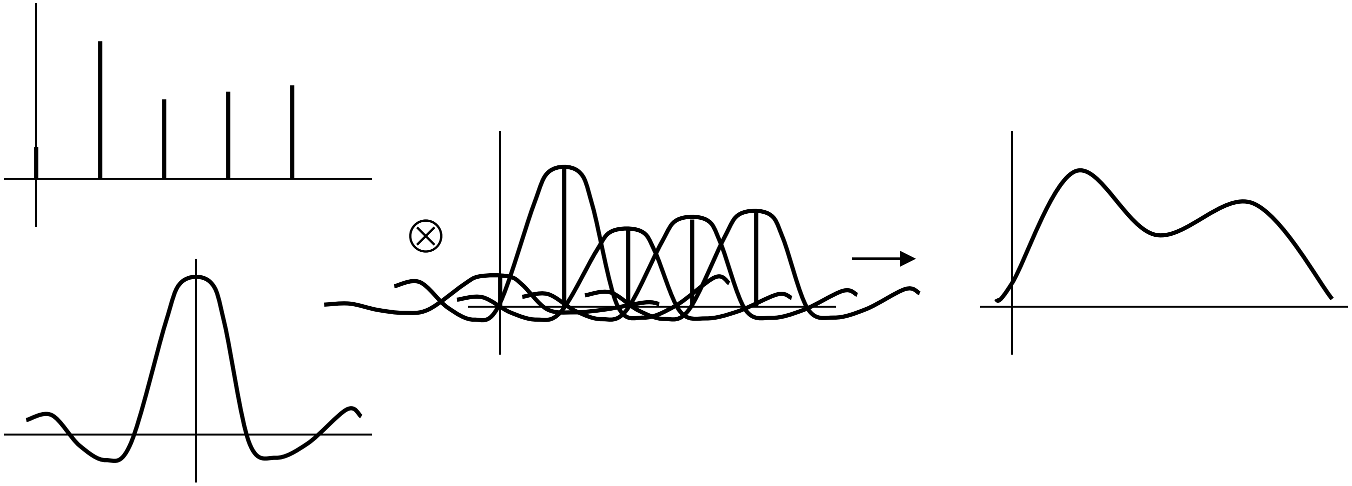
Reconstruction (Frequency Domain)

- To reconstruct, we must restore the original spectrum
- That can be done by multiplying by a square pulse



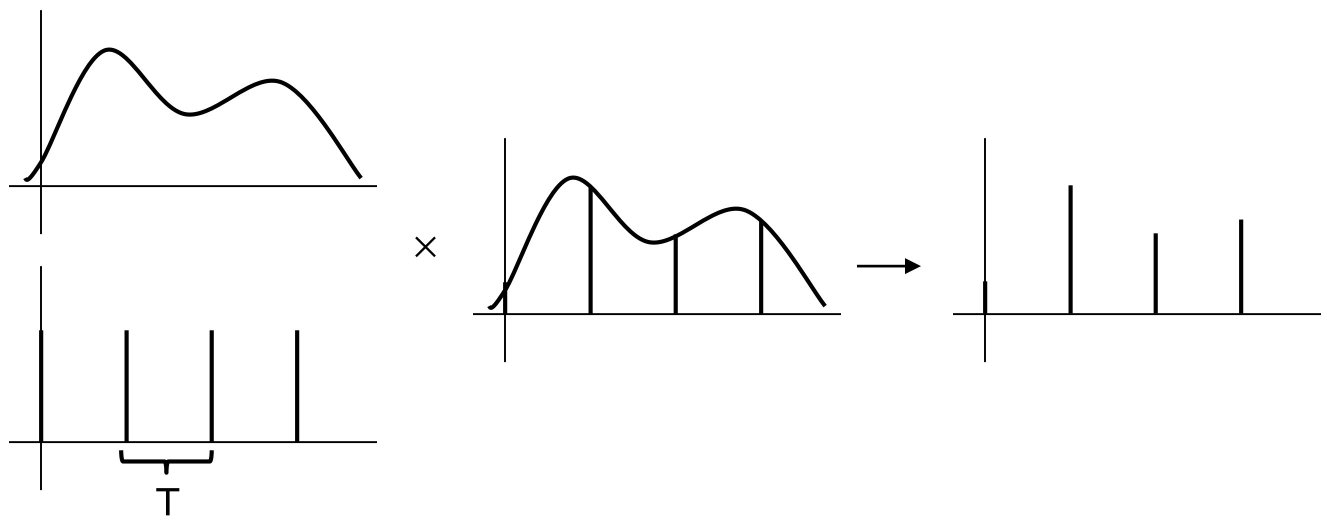
Reconstruction (Spatial Domain)

- Multiplying by a square pulse in the frequency domain is the same as convolving with a sinc function in the spatial domain



Aliasing Due to Under-sampling

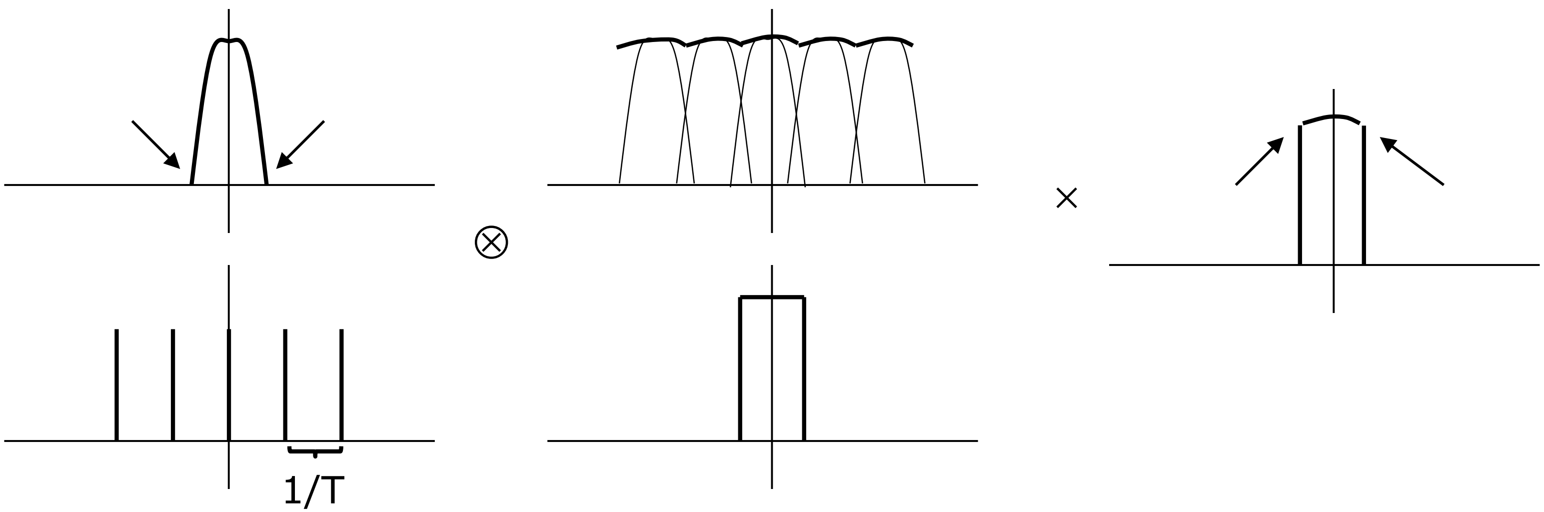
- If the sampling rate is too low, high frequencies get reconstructed as lower frequencies



Sampling in spatial domain

Aliasing Due to Under-sampling

- If the sampling rate is too low, high frequencies get reconstructed as lower frequencies

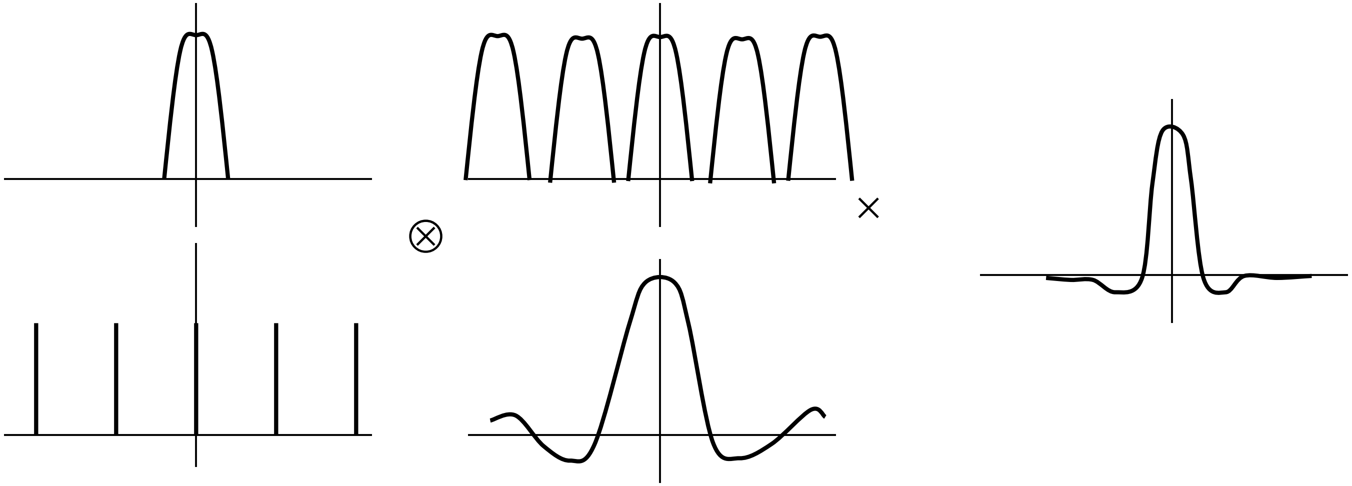


Sampling in frequency domain

- High frequencies from one copy get added to low frequencies from another

More Aliasing

- Poor reconstruction also results in aliasing
- Consider a signal reconstructed with a box filter in the spatial domain (square box pixels, which means using a sinc in the frequency domain):



Aliasing in Practice

- We have two types of aliasing:
  - Aliasing due to insufficient sampling frequency
  - Aliasing due to poor reconstruction
- You have some control over reconstruction
  - If resizing, for instance, use an approximation to the sinc function to reconstruct (instead of Bartlett, as we used last time)
  - Gaussian is closer to sinc than Bartlett
  - But note that sinc function goes on forever (*infinite support*), which is inefficient to evaluate
- You have some control over sampling if creating images using a computer
  - Remove all sharp edges (high frequencies) from the scene before drawing it
  - That is, blur character and line edges *before* drawing

Next Time

- Composition
- NPR
- 3D Graphics Toolkits