Computer Graphics

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http://www.cs.pdx.edu/~fliu/courses/cs447/

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Last time

- Clipping

Today

- Rasterization
- In-class Mid-term
  - November 1
  - Close-book exam
  - Notes on 1 page of A4 or Letter size paper

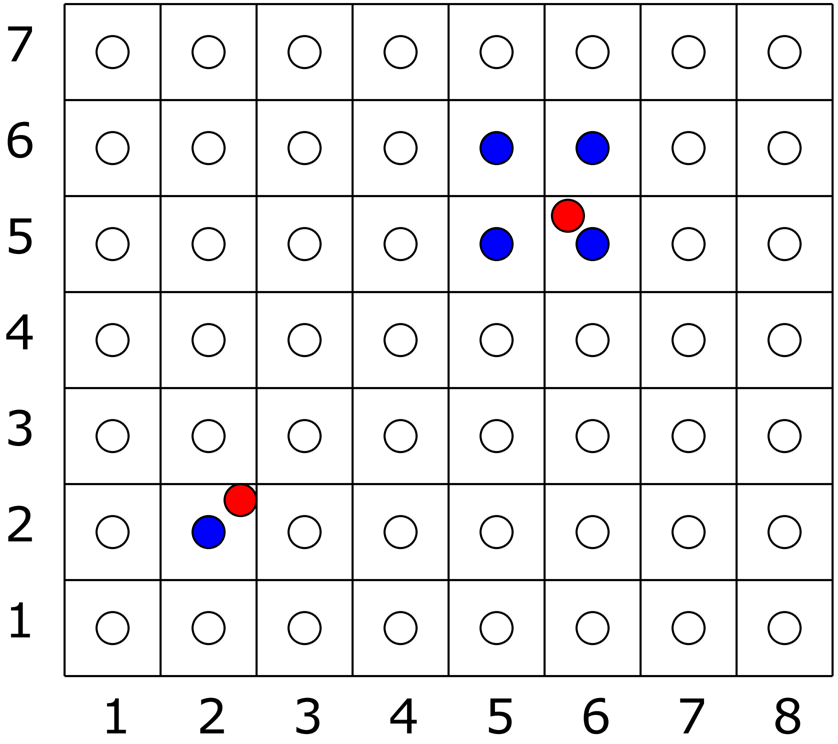
Where We Stand

- At this point we know how to:
  - Convert points from local to screen coordinates
  - Clip polygons and lines to the view volume
- Next thing:
  - Determine which pixels to fill for any given point, line or polygon

Drawing Points

- When points are mapped into window coordinates, they could land anywhere - not just at a pixel center
- Solution is the simple, obvious one
  - Map to window space
  - Fill the closest pixel
  - Can also specify a radius - fill a square of that size, or fill a circle
    - Square is faster

Drawing Points (2)



Drawing Lines

- Task: Decide which pixels to fill (samples to use) to represent a line
- We know that all of the line lies inside the visible region (clipping gave us this!)

Line Drawing Algorithms

- Consider lines of the form $y = m\,x + c$, where $m = \Delta y / \Delta x$, $0 < m < 1$, integer coordinates
  - All others follow by symmetry, modify for real numbers
- Variety of slow algorithms (Why slow?):
  - step x, compute new y at each step by equation, rounding:
  - step x, compute new y at each step by adding m to old y, rounding:

$$x_{i+1} = x_i + 1, \qquad y_{i+1} = round(mx_{i+1} + b)$$

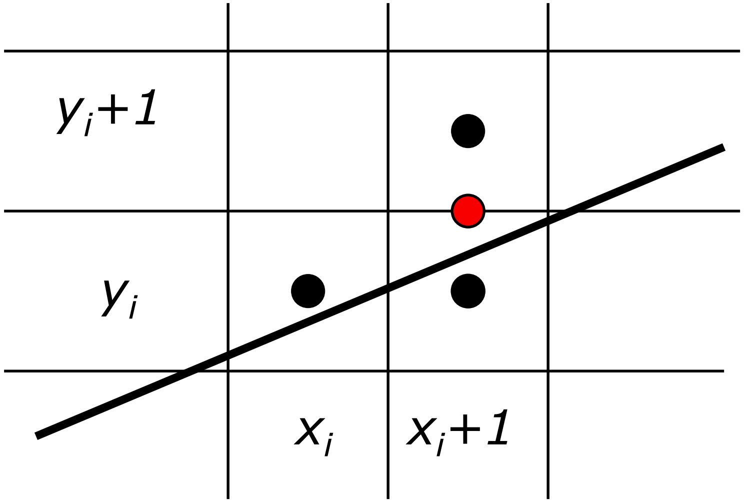
$$x_{i+1} = x_i + 1, \qquad y_{i+1} = round(y_i + m)$$

Bresenham’s Algorithm Overview

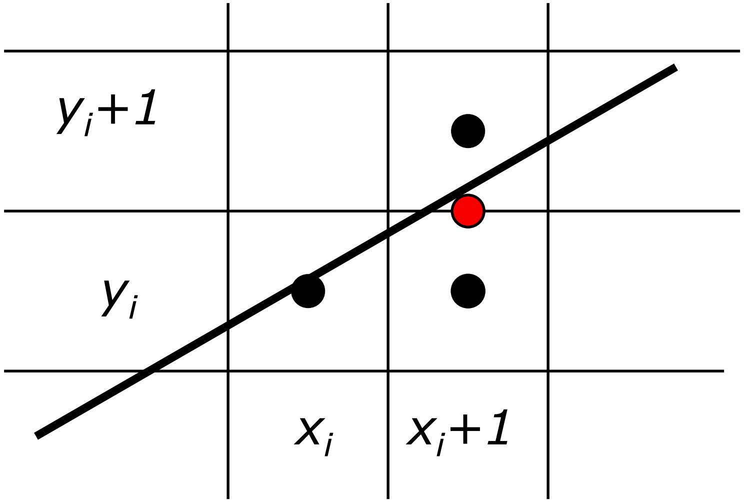
- Aim: For each *x*, plot the pixel whose *y*-value is closest to the line
- Given (x_i,y_i), must choose from either (x_i+1,y_i+1) or (x_i+1,y_i)
- Idea: compute a *decision variable*
  - Value that will determine which pixel to draw
  - Easy to update from one pixel to the next
- Bresenham’s algorithm is the *midpoint algorithm* for lines
  - Other midpoint algorithms for conic sections (circles, ellipses)

Midpoint Methods

- Consider the midpoint between (x_i+1, y_i+1) and (x_i+1, y_i)
- If it's above the line, we choose (x_i+1, y_i), otherwise we choose (x_i+1, y_i+1)



Choose (x_i+1, y_i)



Choose (x_i+1, y_i+1)

Midpoint Decision Variable

- Write the line from (x_1, y_1) to (x_2, y_2) in *implicit form:*
  $$F(x,y) = ax + by + c = \Delta x \cdot y - \Delta y \cdot x + (\Delta y \cdot x_1 - \Delta x \cdot y_1)$$
  - Assume $x_1 <= x_2$
  - $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$
- The value of $F(x,y)$ tells us where points are with respect to the line
  - $F(x,y)=0$: the point is on the line
  - $F(x,y)>0$: The point is above the line
  - $F(x,y)<0$: The point is below the line
- **The decision variable is the value of $d_i = 2F(x_i+1, y_i+0.5)$**
  - The factor of two makes the math easier

What Can We Decide?

$$d_i = 2\Delta x y_i - 2\Delta y(x_i + 1) + 2\left(\Delta y \cdot x_1 - \Delta x \cdot y_1\right) + \Delta x$$

- d_i positive=> next point at (x_i+1, y_i)
- d_i negative => next point at (x_i+1, y_i+1)
- At each point, we compute d_i and decide which pixel to draw
- How do we update it? What is d_{i+1}?

Updating The Decision Variable

- d_{k+1} is the old value, d_k, plus an increment:

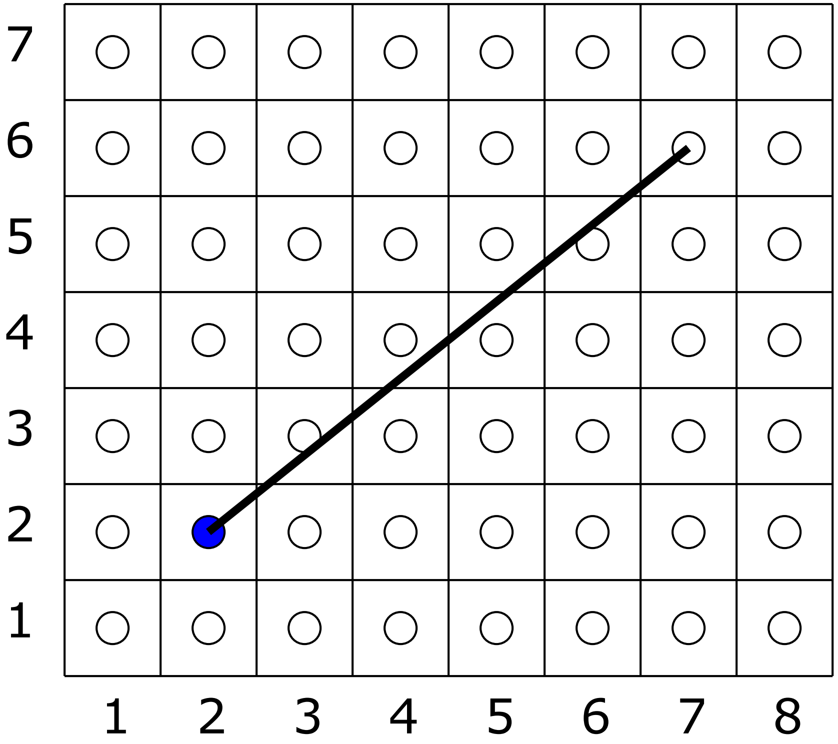
$$d_{k+1} = d_k + (d_{k+1} - d_k)$$

- If we chose $y_{i+1}=y_i+1$: $$d_{k+1} = d_k - 2\Delta y + 2\Delta x$$
- If we chose $y_{i+1}=y_i$: $$d_{k+1} = d_k - 2\Delta y$$
- What is d_1 (assuming integer endpoints)? $$d_1 = \Delta x - 2\Delta y$$
- Notice that we don't need c any more

Bresenham’s Algorithm

- For integers, slope between 0 and 1:
  - $x=x_1$, $y=y_1$, $d= dx -2dy$, draw (x, y)
  - until $x=x_2$
    - $x=x+1$
    - If $d<0$ then { $y=y+1$, draw (x, y), $d=d-2\Delta y + 2\Delta x$ }
    - If $d>0$ then { $y=y$, draw (x, y), $d=d-2\Delta y$ }
- Compute the constants ($2\Delta y-2\Delta x$ and $2\Delta y$) once at the start
  - Inner loop does only adds and comparisons
- For floating point, initialization is harder, Δx and Δy will be floating point, **but still no rounding required**

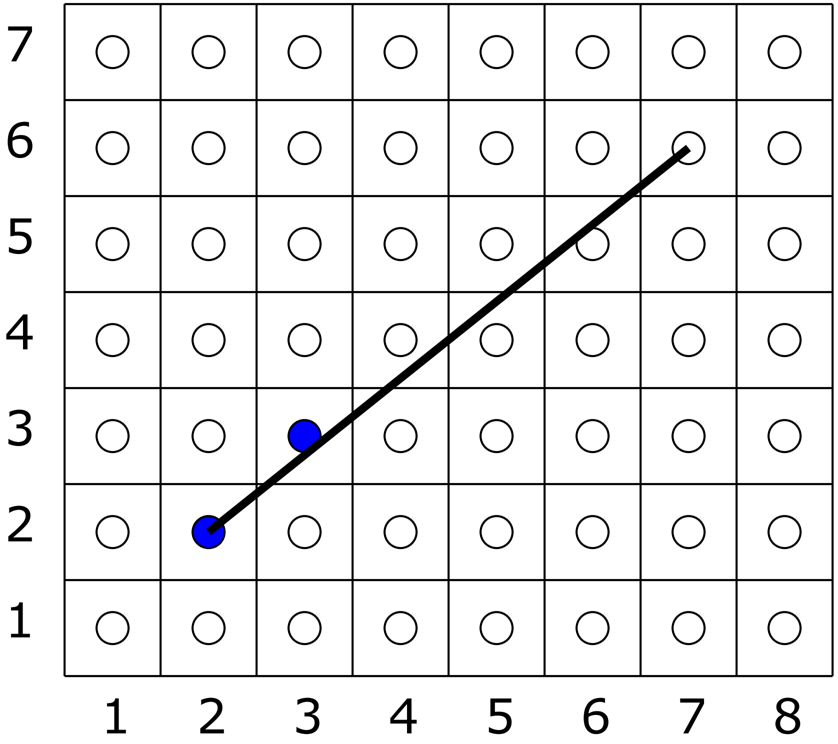
Example: (2,2) to (7,6)



$\Delta x=5$, $\Delta y=4$

x	y	d
2	2	-3

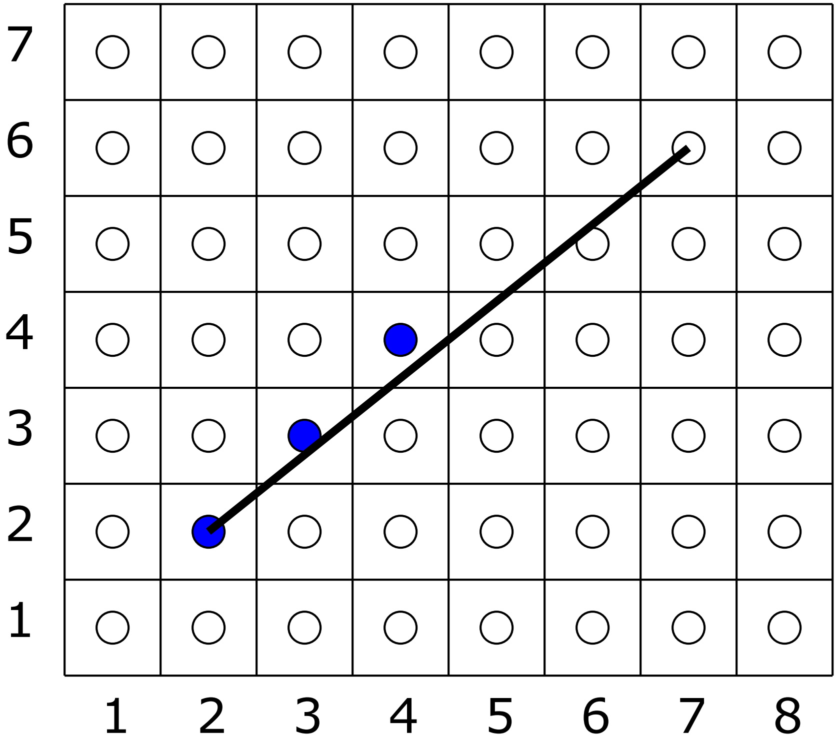
Example: (2,2) to (7,6)



$\Delta x=5$, $\Delta y=4$

x	y	d
2	2	-3
3	3	-1

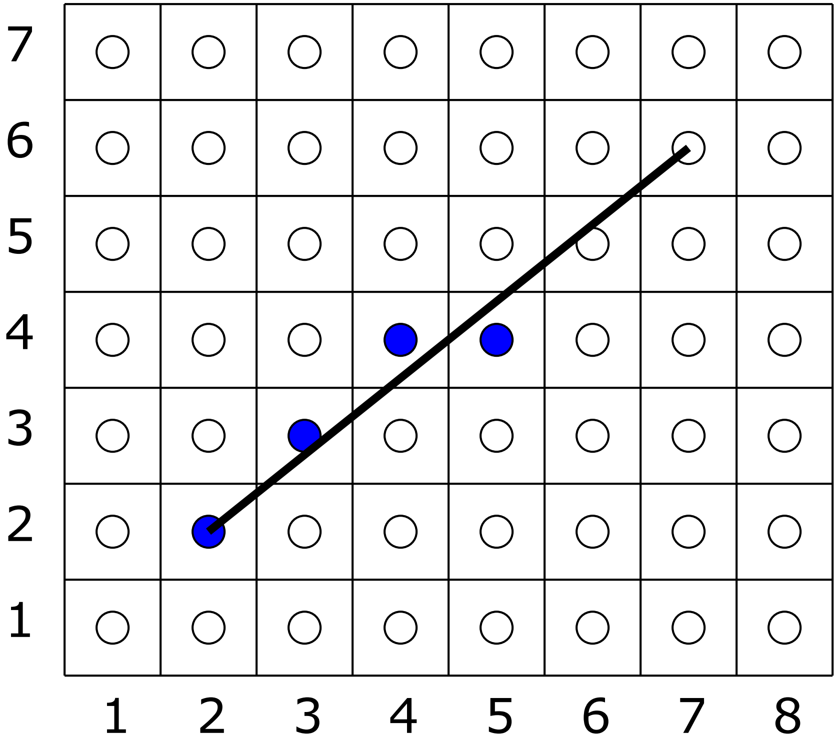
Example: (2,2) to (7,6)



Δx=5, Δy=4

x	y	d
2	2	-3
3	3	-1
4	4	1

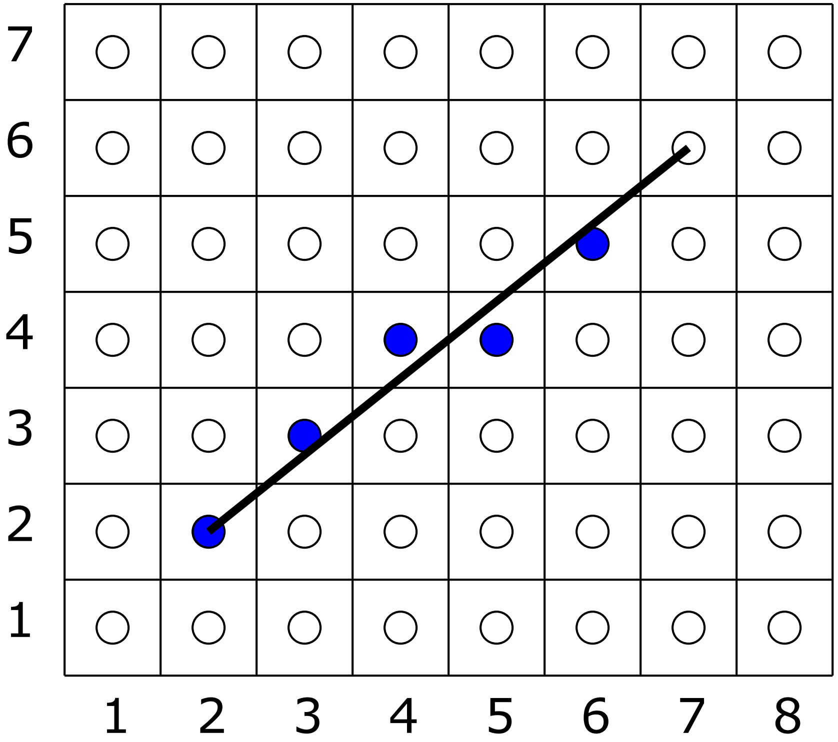
Example: (2,2) to (7,6)



Δx=5, Δy=4

x	y	d
2	2	-3
3	3	-1
4	4	1
5	4	-7

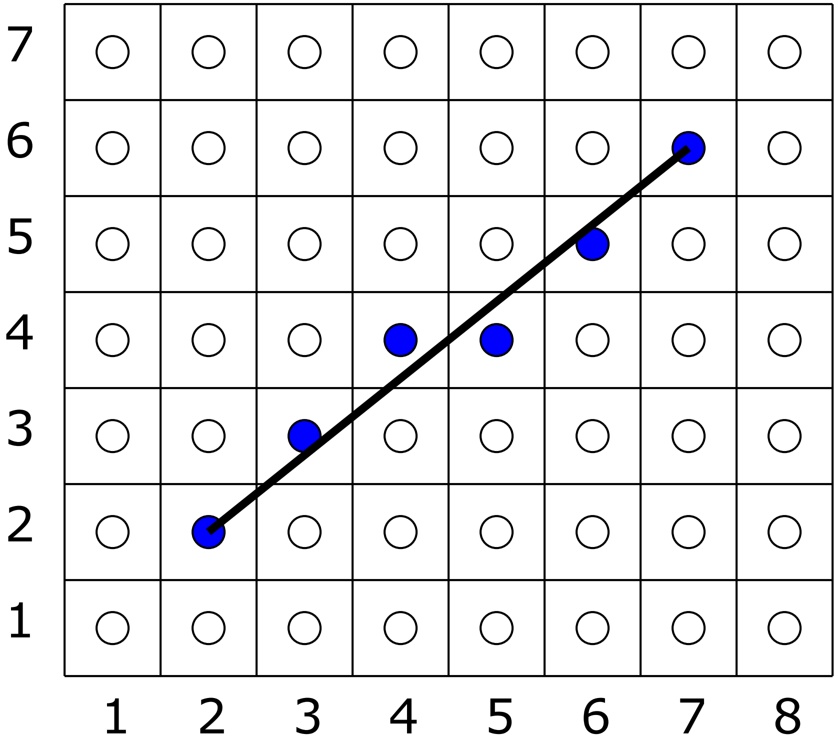
Example: (2,2) to (7,6)



$\Delta x=5$, $\Delta y=4$

x	y	d
2	2	-3
3	3	-1
4	4	1
5	4	-7
6	5	-5

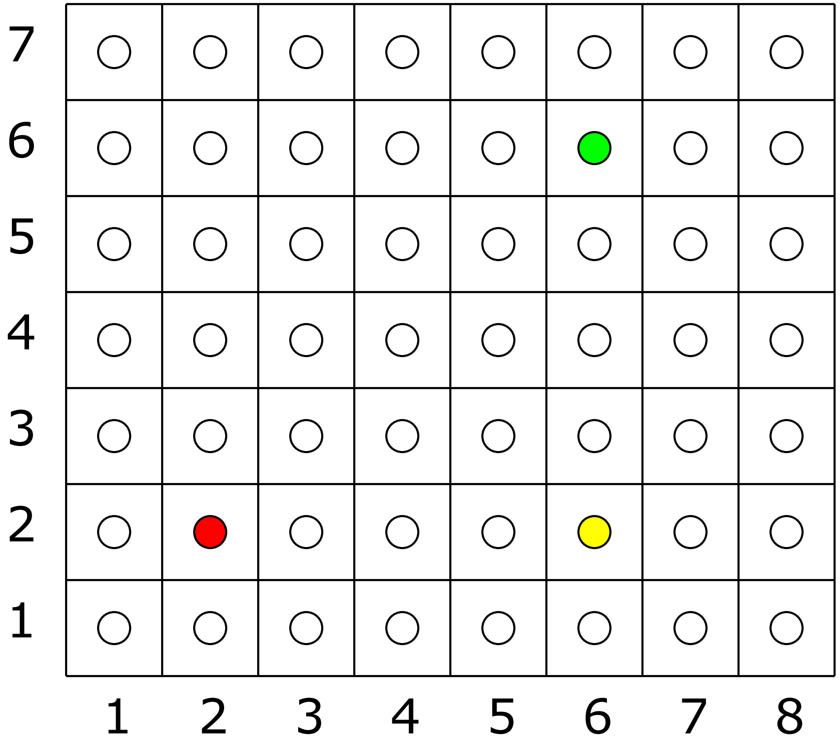
Example: (2,2) to (7,6)



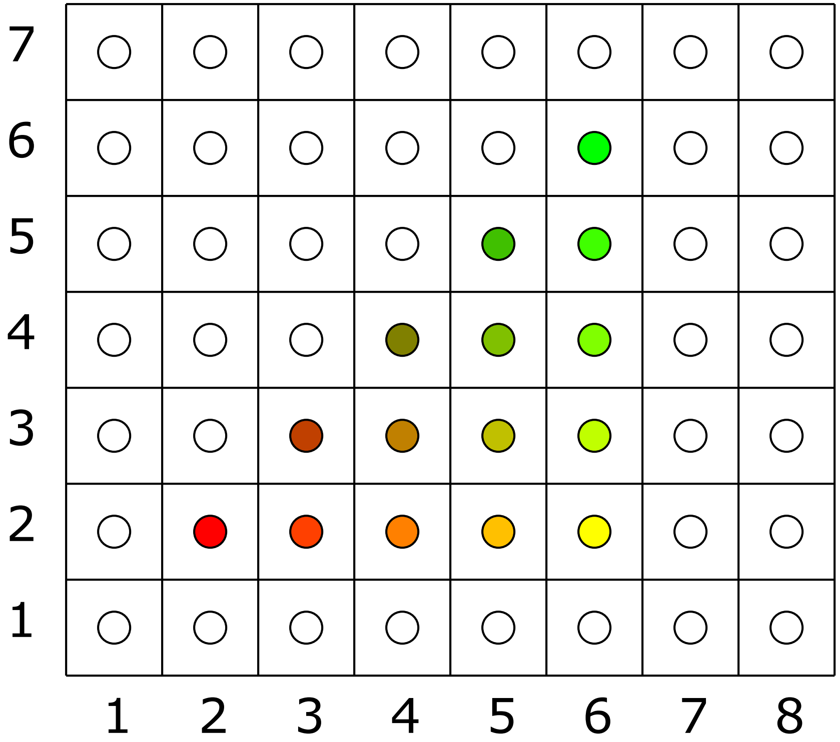
$\Delta x=5$, $\Delta y=4$

x	y	d
2	2	-3
3	3	-1
4	4	1
5	4	-7
6	5	-5
7	6	-3

Filling Triangles



Filling Triangles



Algorithm

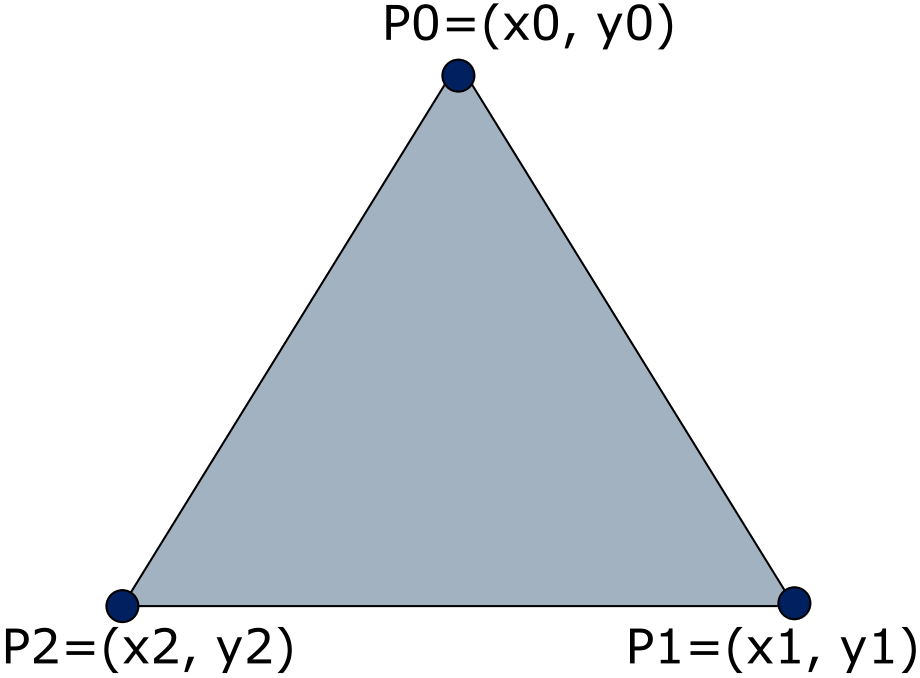
- Decide which pixels to fill (samples to use) to represent a triangle?
- Calculate the color for each pixel?

Barycentric coordinates

$$P = \alpha P_0 + \beta P_1 + \lambda P_2$$

$$\alpha + \beta + \lambda = 1$$

$$0 \le \alpha, \beta, \lambda \le 1$$



Barycentric coordinates

$$\alpha = f_{12}(x, y) / f_{12}(x_0, y_0)$$
$$\beta = f_{20}(x, y) / f_{20}(x_1, y_1)$$
$$\lambda = f_{01}(x, y) / f_{01}(x_2, y_2)$$

$$f_{12}(x, y) = (y_1 - y_2)x + (x_2 - x_1)y + x_1 y_2 - x_2 y_1$$
$$f_{20}(x, y) = (y_2 - y_0)x + (x_0 - x_2)y + x_2 y_0 - x_0 y_2$$
$$f_{01}(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$$

Rasterizing Triangle

- $y_{min}=\min(y_0, y_1, y_2)$, $y_{max}=\max(y_0, y_1, y_2)$
- $x_{min}=\min(x_0, x_1, x_2)$, $x_{max}=\max(x_0, x_1, x_2)$
- for $y=y_{min}$ to y_{max}
  - for $x=x_{min}$ to x_{max}
    - calculate $\alpha, \beta,$ and λ
    - if $0 \le \alpha, \beta,$ and $\lambda \le 1$

      $c = \alpha c_0 + \beta c_1 + \lambda c_2$

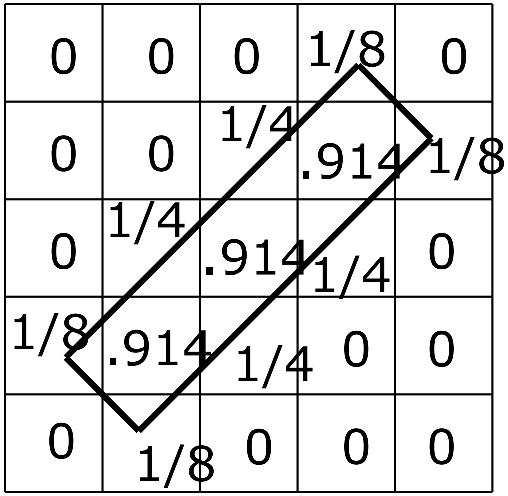
      draw (x, y) with color c

Anti-Aliasing

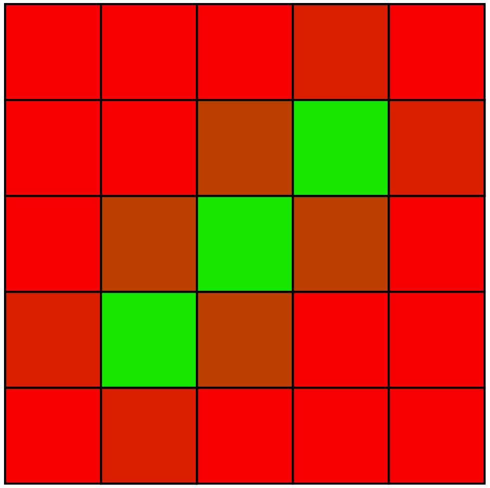
- Recall: We can’t sample and then accurately reconstruct an image that is not band-limited
  - Infinite Nyquist frequency
  - Attempting to sample sharp edges gives “jaggies”, or stair-step lines
- Solution: Band-limit by filtering (pre-filtering)
  - What sort of filter will give a band-limited result?
- In practice, difficult to do for graphics rendering

Alpha-based Anti-Aliasing

- Set the α of a pixel to simulate a thick line
  - The pixel gets the line color, but with α<=1
- This supports the correct drawing of primitives one on top of the other
  - Draw back to front, and **composite** each primitive *over* the existing image
  - Only some hidden surface removal algorithms support it

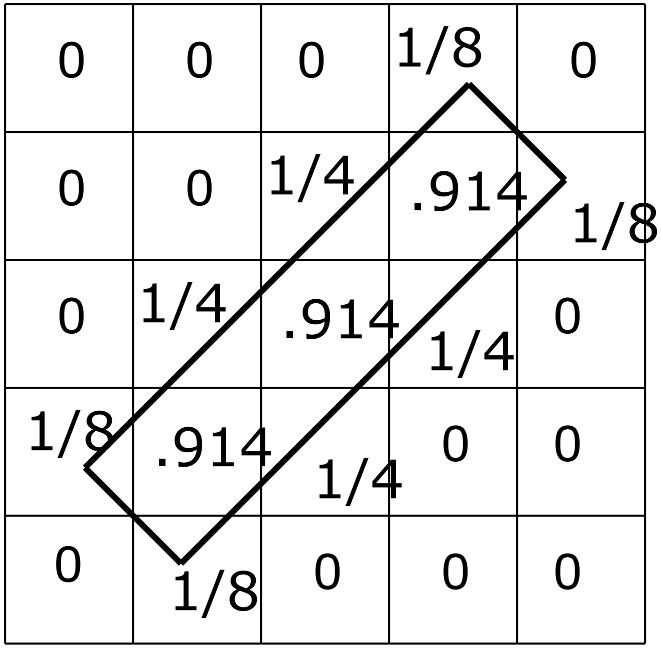


0	0	0	1/8	0
0	0	1/4	.914	1/8
0	1/4	.914	1/4	0
1/8	.914	1/4	0	0
0	1/8	0	0	0



Calculating α

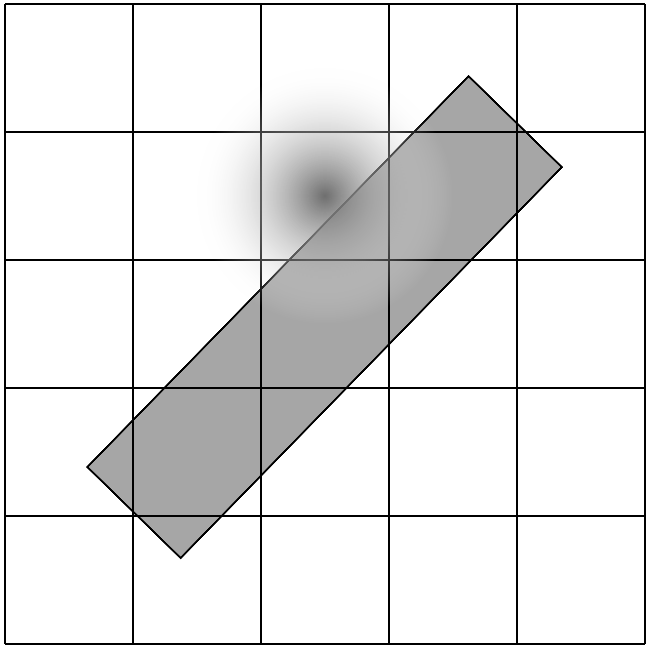
- Consider a line as having thickness (all good drawing programs do this)
- Consider pixels as little squares
- Set α according to the proportion of the square covered by the line
- The sub-pixel coverage interpretation of α



0	0	0	1/8	0
0	0	1/4	.914	1/8
0	1/4	.914	1/4	0
1/8	.914	1/4	0	0
0	1/8	0	0	0

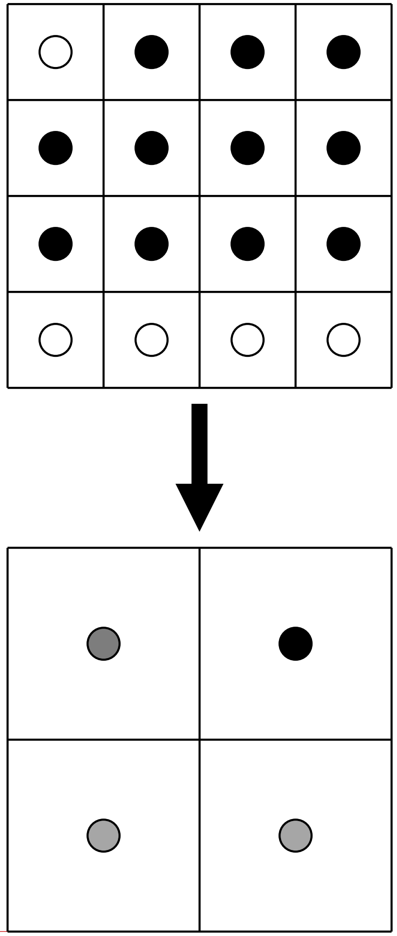
Weighted Sampling

- Instead of using the proportion of the area covered by the line, use convolution to do the sampling
  - Equivalent to filtering the line then point sampling the result
- Place the “filter” at each pixel, and integrate product of pixel and line
- Common filters are cones (like Bartlett) or Gaussians



Post-Filtering (Supersampling)

- Sample at a higher resolution than required for display, and filter image down
  - Easy to implement in hardware
  - Typical is 2x2 sampling per pixel, with simple averaging to get final
    - What kind of filter?
- More advanced methods generate different samples (eg. not on regular grid) and filter properly
  - Issues of which samples to take, and how to filter them



Next Time

- Hidden Surface Removal
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