

# CS 447/547: Computer Graphics

## Homework 5

This homework must be done individually. Submission date is November 29, 2018.

**Question 1:** A student is planning a polygon mesh data structure in which vertices are stored in a vertex array, and then the triangular faces in the mesh each store the indices of the vertices and the triangle's face plane normal vector. The face data structure is given below.

```
class Triangle {  
int vertices[3]; // The vertex indices.  
float nx, ny, nz; // The face-plane normal.  
};
```

- a. Is this a convenient way to represent a mesh if used with flat shading? Explain your reasoning. (2 points)

**Answer:** This is a convenient way to represent a mesh if used with flat shading, because during flat shading, shading is computed at a representative point and apply to the whole face. For example, OpenGL uses one of the vertices.

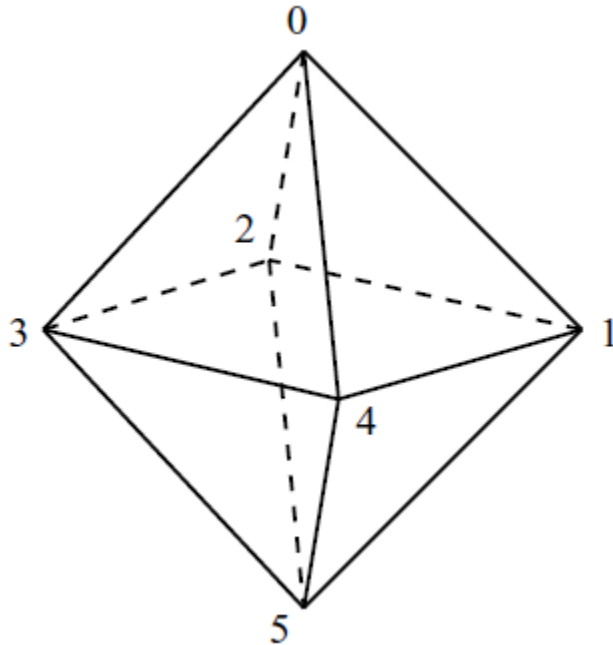
- b. Suggest an object for which this is a good mesh format when used with Gouraud shading. Explain. (2 points)

**Answer:** This mesh format is good to construct a *cube* when shading with Gouraud shading algorithm. Because for a cube, which consists of 6 faces, even a common normal is used for all the 3 vertices in a triangle, it will still produce realistic enough result, smooth in each face and sharp between adjacent faces.

- c. Suggest an object for which this is a bad mesh format when used with Gouraud shading? Explain. (2 points)

**Answer:** This mesh format is bad to construct a *sphere* when shading with Gouraud shading algorithm. Because for a sphere, if a common normal is used for all the 3 vertices, the result will not be smooth.

**Question 2:** The following figure on the left shows the octahedron used as the starting shape for sphere subdivision. On the right are the vertex locations.



- 0: (0,0,1)
- 1: (1,0,0)
- 2: (0,1,0)
- 3: (-1,0,0)
- 4: (0,-1,0)
- 5: (0,0,-1)

Perform sphere subdivision of the face 3-4-5 (the one using vertices 3, 4 and 5). Give the location of the new vertices by splitting edge 3-4, 4-5, and 3-5. (3 points)

**Answer:**

The new vertex by splitting 3-4 is  $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$ .

The new vertex by splitting 4-5 is  $(0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ .

The new vertex by splitting 3-5 is  $(-\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$ .

**Question 3:** A Bezier curve will be used to represent a straight line of length 1. The first control point,  $x_0$ , is at (0,0,0).

- a. The line is to point along the y-axis. Where should the final control point,  $x_3$ , be located? (1 point)

**Answer:** (0,1,0)

- b. Say we want the magnitude of the parametric derivative of the curve to equal 1 at both the start and end of the curve. Where should we place the other two control points,  $x_1$  and  $x_2$ ? (2 points)

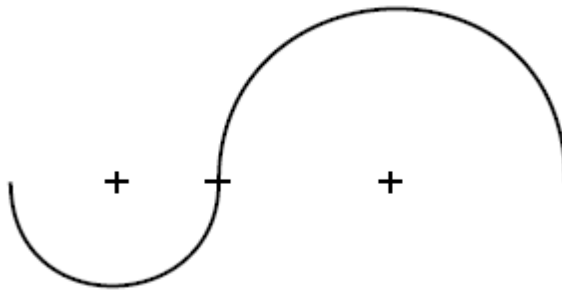
**Answer:**  $x_1=(0,1/3,0)$ ,  $x_2=(0,2/3,0)$

- c. Show that the magnitude of the parametric derivative is always 1 for the curve you have created. (2 points)

**Answer:**

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{x}_0(-3 + 6t - 3t^2) + \mathbf{x}_1 3(1 - 4t + 3t^2) + \mathbf{x}_2 3(2t - 3t^2) + \mathbf{x}_3 3t^2 \\ &= \begin{bmatrix} 0 \\ (1 - 4t + 3t^2) + 2(2t - 3t^2) + 3t^2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{aligned}$$

**Question 4:** The figure below shows two pieces of circular arcs, joined at the point indicated by the small horizontal bar. The center of each circle is also marked, and the centers lie on a straight line through the join point.



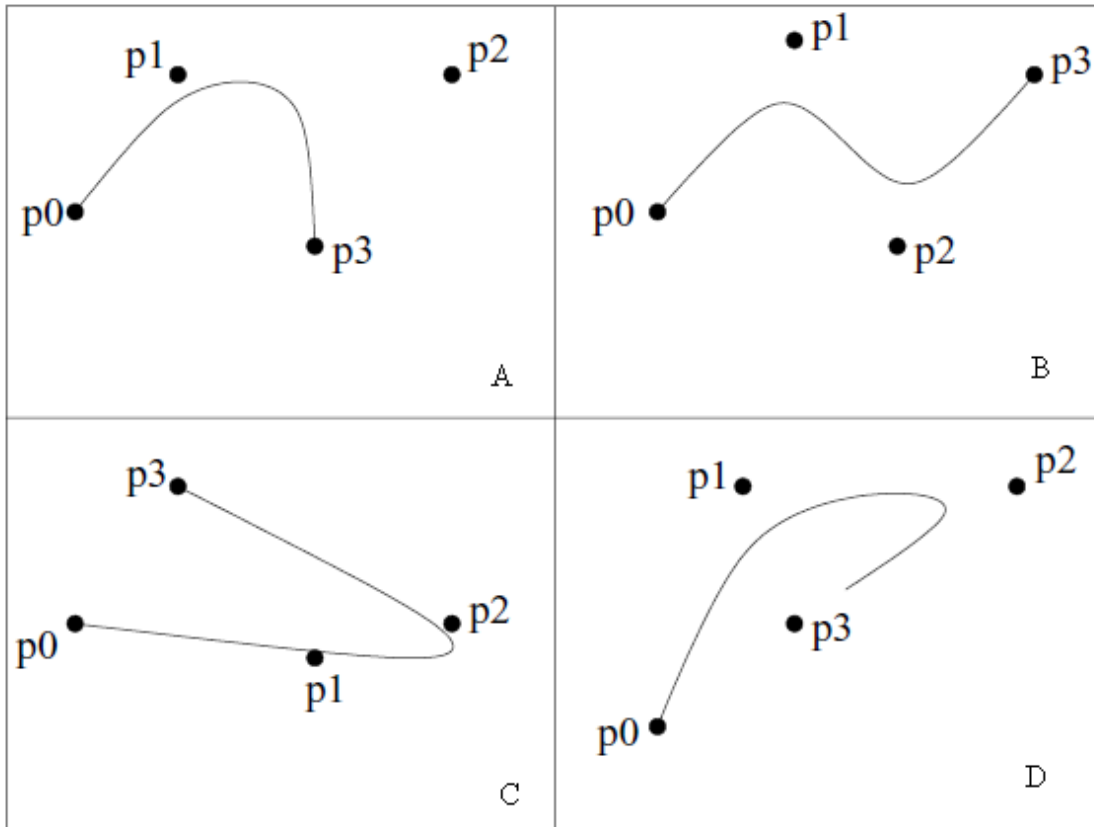
- a. Each arc is uniformly parameterized such that  $t = 0$  is at the left-most point on the arc,  $t = 0.5$  is at the midpoint of each arc, and  $t = 1$  is at the right-most point. Do the arcs join with  $C^1$  continuity? (1 point)

Answer: No. While the parametric derivatives of these two curves are of the same direction, but their magnitudes are different.

- b. Do the arcs join with  $G^1$  continuity? (1 point)

Answer: Yes.

**Question 5:** Which of the following must **not** be cubic Bezier curves, and why not? (4 points)



**Answer:** A is not, because the tangent at  $p_3$  is not along line  $p_2p_3$ . C is not, because the curve is not totally inside the convex hull formed by all the control points. D is not, because the curve does not interpolate  $p_3$ .