

CS 447/547: Computer Graphics

Homework 3

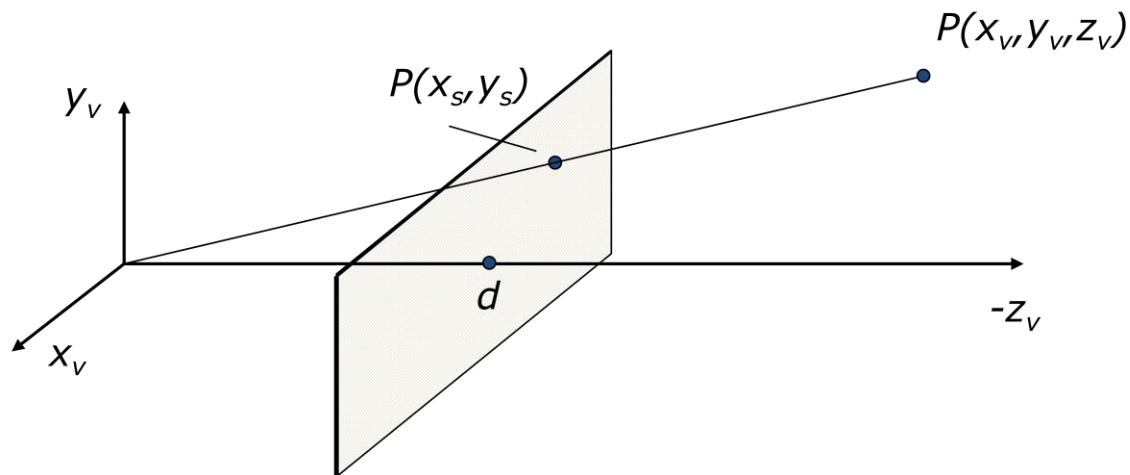
This homework must be done individually. Submission date is October 30, 2018, in class.

Question 1: We have an orthographic view volume that is defined by the following 6 planes: near plane: $n=-2$; far plane $f=-6$; left plane $l=-2$; right plane $r=6$; top plan $t=4$; bottom plan $b=-4$. We need to transform this orthographic view volume to canonical view volume, which is the cube $[-1,1] \times [-1,1] \times [-1,1]$.

Compute the projection matrix from the orthographic view volume to the canonical view volume.

Answer:
$$\begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.5 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2: We assume a basic perspective projection as illustrated in the following figure.



Here a 3D point $P(x_v, y_v, z_v)$ in the view space is projected onto the image plane $P(x_s, y_s)$. We know that the focal distance $d=10$.

- a. We know the coordinate of a point in the view space is (100, 20, 20). What is the coordinate of this point in the image plane? (2 points)

Answer: (50, 10)

- b. We know the coordinate of a point in the image plane is (20, 10). Can we uniquely determine its corresponding 3D point's coordinate in the view space? (1 point)

Answer: No. (Or there are an infinite number of points in the view space).

Question 3: You wish to use compositing operations to perform a stencil operation. You have a foreground image, f , which you wish to place into a background image, b , only at places where a stencil mask, s , has a particular α value. (2 points)

For example, if the foreground image is all white with $\alpha = 1$, the background is all red with $\alpha = 1$ and the stencil has holes for a word, inserting the foreground into the background would result in a white word on a red background. Assume that the α value in the stencil is 1 in the holes and 0 elsewhere. You plan to use two compositing operations to combine the images, with the form $(f \text{ op}_1 s) \text{ op}_2 b$, where brackets indicate precedence. Which compositing operations would you use for op_1 and op_2 ?

Answer: op_1 is **inside**, and op_2 is **over**.

Question 4: It takes 3 pairs of point correspondences to determine a 2D affine transformation shown as follows:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

We know that $[1, 1]$ is transformed to $[\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}-1}{2}]$, $[0, 0]$ is transformed to $[1, -1]$, and $[1, 0]$ is transformed into $[\frac{\sqrt{3}+2}{2}, \frac{-1}{2}]$.

- a. Compute the values for the unknowns a, b, c, d, e , and f . (**Hint:** you can set up two linear equations for each pair of equations, so eventually you get six linear equations. You can verify whether you get the right answers or not by use your answers to form the affine transformation matrix, and transform each (x, y) to its corresponding (u, v) .) (3 points)

Answer: $\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\ 0 & 0 & 1 \end{bmatrix}$

- b. Determine a sequence of rotation, scaling or translations required for the transformation. (Note, this question is not in your original homework. You can consider this as a practice.)

Answer: Rotate counterclockwise 30 degree and translate $[1 -1]$.