This homework must be done individually. Submission date is October 22, 2015, in class.

**Question 1:** This question concerns human’s intensity perception. Humans are tuned to the ratio of intensities, not their absolute difference. If we want to make a perceptually uniform intensity system with intensities \( l_1 = 1 \), \( l_2 \), \( l_3 \), \( l_4 \), and \( l_5 = 256 \). What are the values of \( l_2 \), \( l_3 \), and \( l_4 \)?

**Question 2:** CIE L*a*b* color space is often considered approximately perceptually uniform. We can convert RGB into L*a*b* in two steps:

1. **Step 1:** Convert RGB to XYZ using the formula in our lecture 2.
2. **Step 2:** Convert XYZ to L*a*b*.

L*a*b* is not a linear color space, so converting XYZ to L*a*b* is more complicated than RGB to XYZ. We will use the following formulas to the conversion.

\[
L^* = \begin{cases} 
116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16 \quad & Y > 0.008856 \\
903.3 \left( \frac{Y}{Y_n} \right) & \text{else}
\end{cases}
\]

\[a^* = 500 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right)\]

\[b^* = 200 \left( f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right)\]

where

\[f(t) = \begin{cases} 
\frac{1}{t^{3}} \quad & t > 0.008856 \\
7.787 t + \frac{16}{116} & \text{else}
\end{cases}\]

Here \( Y_n = 1.0 \) is the luminance, and \( X_n = 0.950455 \), \( Z_n = 1.088753 \).

Suppose we have two colors in RGB color space: \((0.5, 0, 0)\) and \((1, 1, 1)\).

**a.** What are the coordinates for these two colors in L*a*b* color space?

In computer graphics, we often need to perform linear interpolation between two colors. The linear interpolation from \((r_1, g_1, b_1)\) and \((r_2, g_2, b_2)\) can be implemented as follows.

\[r(u) = (1 - u)r_1 + ur_2\]

\[g(u) = (1 - u)g_1 + ug_2\]

\[b(u) = (1 - u)b_1 + ub_2\]

**b.** We want to interpolate from \((0.5, 0, 0)\) to \((1, 1, 1)\) in 5 steps, which can be achieved by using \(u=0\), \(u=0.25\), \(u=0.5\), \(u=0.75\), \(u=1\), respectively. Compute the 5 RGB colors.

**c.** Compute the corresponding coordinates in L*a*b* color space of the above 5 RGB colors.
d. Plot two graphs: one showing $L^*$ as a function of $u$ and the other showing $a^*$ as a function of $u$.
Here we can see that $L^*$ and $a^*$ are not a linear function of $u$.

**Question 3:** Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.

![Graphs of Sensor A, B, and C](image)

What is the response of each of these three sensors to the following spectrum? You need to give actual points.

![Spectrum graph](image)

**Question 4:** The **Sobel operator** is used in image processing for edge detection. The following shows a $3 \times 3$ Sobel filter mask for a horizontal edge detector.

$$
\begin{bmatrix}
1 & 2 & 1 \\
0 & 0 & 0 \\
-1 & -2 & -1
\end{bmatrix}
$$

a. What is the response of this filter to the following $6 \times 6$ image? Ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a $4 \times 4$ image.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

b. What is the response of this filter to the following $6 \times 6$ image? Again, ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a $4 \times 4$ image.
c. Can you design a $3 \times 3$ filter that can detect the vertical edge in the image shown in (b)?

**Question 5:** Gaussian is one of the most popular filters in computer graphics. What is the $9 \times 9$ 2D Gaussian filter mask? (Use the method described in Lecture 4: first construct a 1d filter mask, and then construct the corresponding 2D filter mask. You are not required to fill in the actual number for the 2D filter mask, and a correct formula is good enough.)

$$
\begin{bmatrix}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
$$