CS 447/547: Computer Graphics

Homework 2

This homework must be done individually.

Question 1: This question concerns human's intensity perception. Humans are tuned to the *ratio* of intensities, not their absolute difference. If we want to make a perceptually uniform intensity system with intensities $l_1 = 1,\ l_2,\ l_3,\ and\ l_4 = 64$. What are the values of $l_2,$ and l_3?

Question 2: CIE L*a*b* color space is often considered approximately perceptually uniform. We can convert RGB into L*a*b* in two steps:
Step 1: Convert RGB to XYZ using the formula in our lecture 3.
Step 2: Convert XYZ to L*a*b*.

L*a*b* is not a linear color space, so converting XYZ to L*a*b* is more complicated than RGB to XYZ. We will use the following formulas to the conversion.

$$L^* = \begin{cases} 116 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 & \frac{Y}{Y_n} > 0.008856 \\ 903.3 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} & else \end{cases}$$

$$a^* = 500 * \left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right)$$

$$b^* = 200 * \left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right)$$

where

$$f(t) = \begin{cases} t^{\frac{1}{3}} - 16 & t > 0.008856 \\ 7.787 * \mathrm{t} + \frac{16}{116} & else \end{cases}$$

Here $Y_n = 1.0$ is the luminance, and $X_n = 0.950455$, $Z_n = 1.088753$.

Suppose we have two colors in RGB color space: (0.5, 0, 0) and (1, 1, 1).

a. What are the coordinates for these two colors in L*a*b* color space?

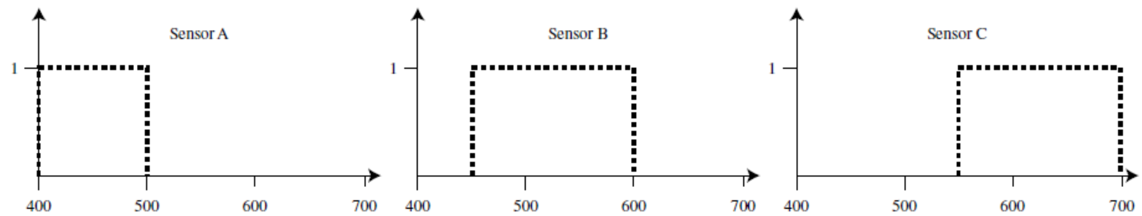
In computer graphics, we often need to perform linear interpolation between two colors. The linear interpolation from (r_1, g_1, b_1) and (r_2, g_2, b_2) can be implemented as follows.

$$r(u) = (1-u)r_1 + ur_2$$
$$g(u) = (1-u)g_1 + ug_2$$
$$b(u) = (1-u)b_1 + ub_2$$

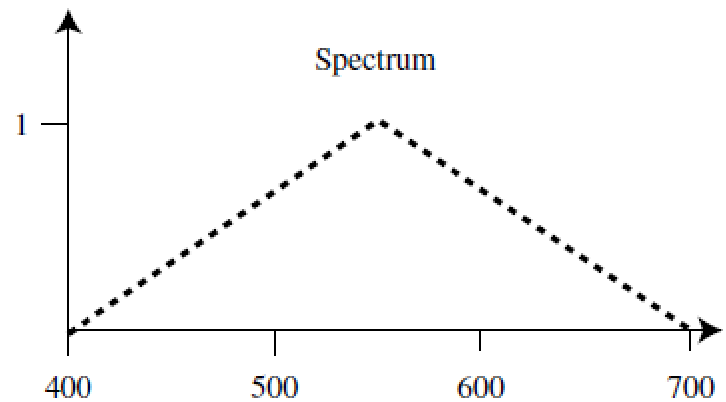
b. We want to interpolate from (0.5, 0, 0) to (1, 1, 1) in 5 steps, which can be achieved by using u=0, u=0.25, u=0.5, u=0.75, u=1, respectively. Compute the 5 RGB colors.

c. Compute the corresponding coordinates in L*a*b* color space of the above 5 RGB colors.
d. Plot two graphs: one showing L* as a function of *u* and the other showing a* as a function of *u*. Here we can see that L* and a* are not a linear function of *u*.

Question 3: Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.



What is the response of each of these three sensors to the following spectrum?



Question 4: The **Sobel operator** is used in image processing for edge detection. The following shows a 3×3 Sobel filter mask for a horizontal edge detector.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

a. What is the response of this filter to the following 6×6 image? Ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

b. What is the response of this filter to the following 6×6 image? Again, ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

c. Can you design a 3×3 filter that can detect the vertical edge in the image shown in (b)?

Question 5: Gaussian is one of the most popular filters in computer graphics. What is the 7×7 2D guassian filter mask? (Use the method described in Lecture 4: first construct a 1d filter mask, and then construct the corresponding 2D filter mask. You are not required to fill in the actual number for the 2D filter mask, and a correct formula is good enough.).