CS 447/547: Computer Graphics

Homework 2

This homework must be done individually. Submission date is October 18, 2018, in class.

Question 1: This question concerns human's intensity perception. Humans are tuned to the *ratio* of intensities, not their absolute difference. If we want to make a perceptually uniform intensity system with intensities $l_1 = 1,\ l_2,\ l_3, l_4, l_{5,} l_6 = 32$. What are the values of $l_2,\ l_3, l_4,$ and l_5 ?

Answer:

$$\begin{cases} \frac{l_2}{l_1} = \frac{l_3}{l_2} = \frac{l_4}{l_3} = \frac{l_5}{l_4} = \frac{l_6}{l_5} \\ l_6 = 32 \\ l_1 = 1 \end{cases} \Rightarrow l_6 = l_2^5 = 32 \Rightarrow l_2 = 2,\ l_3 = 4, l_4 = 8\ and\ l_5 = 16.$$

Question 2: CIE L*a*b* color space is often considered approximately perceptually uniform. We can convert RGB into L*a*b* in two steps:
Step 1: Convert RGB to XYZ using the formula in our lecture 2.
Step 2: Convert XYZ to L*a*b*.

L*a*b* is not a linear color space, so converting XYZ to L*a*b* is more complicated than RGB to XYZ. We will use the following formulas to the conversion.

$$L^* = \begin{cases} 116 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 & \frac{Y}{Y_n} > 0.008856 \\ 903.3 * \left(\frac{Y}{Y_n}\right) & else \end{cases}$$

$$a^* = 500 * \left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right)$$

$$b^* = 200 * \left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right)$$

where

$$f(t) = \begin{cases} t^{\frac{1}{3}} & t > 0.008856 \\ 7.787 * \mathrm{t} + \frac{16}{116} & else \end{cases}$$

Here Y_n = 1.0 is the luminance, and X_n = 0.950455, Z_n = 1.088753.

Suppose we have two colors in RGB color space: (0.5, 0, 0) and (1, 1, 1).

a. What are the coordinates for these two colors in L*a*b* color space? (2 points)

Answer:

(38.9502, 63.5844,53.3516) and (99.9923,0.0412,-0.0285)

In computer graphics, we often need to perform linear interpolation between two colors. The linear interpolation from (r_1, g_1, b_1) and (r_2, g_2, b_2) can be implemented as follows.

$$r(u) = (1-u)r_1 + ur_2$$
$$g(u) = (1-u)g_1 + ug_2$$
$$b(u) = (1-u)b_1 + ub_2$$

b. We want to interpolate from (0.5, 0, 0) to (1, 1, 1) in 5 steps, which can be achieved by using *u*=0, *u*=0.25, *u*=0.5, *u*=0.75, *u*=1, respectively. Compute the 5 RGB colors. (5 points)

Answer:

Let $\mathbf{C}_i$ $(1 \le i \le 5)$ be the resulting color by interpolating from $\mathbf{R}_1(0.5,0,0)$ to $\mathbf{R}_2(1,1,1)$,

$\mathbf{C}_i = u\mathbf{R}_2 + (1-u)\mathbf{R}_1$

$\mathbf{C}_1 = (0.5,0,0)$; $\mathbf{C}_2 = (0.625,0.25,0.25)$; $\mathbf{C}_3 = (0.75,0.5,0.5)$; $\mathbf{C}_4 = (0.875,0.75,0.75)$; $\mathbf{C}_5 = (1,1,1)$

c. Compute the corresponding coordinates in L*a*b* color space of the above 5 RGB colors. (5 points)

Answer:

Let L_i $(1 \le i \le 5)$ be $(\mathrm{L}^*{}_i, a^*{}_i, b^*{}_i)$ corresponding to the RGB colors in (c),

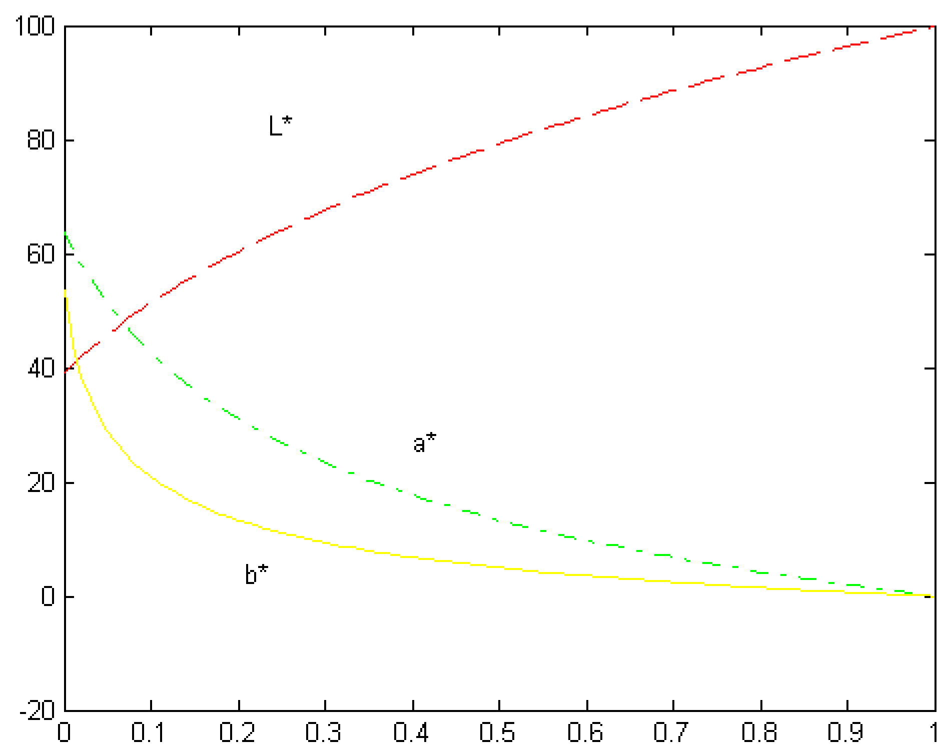
$\mathrm{L}_1 = (38.9502, 63.5844, 53.3516)$; $\mathrm{L}_2 = (64.1346, 26.8609, 11.0547)$

$\mathrm{L}_3 = (79.2166, 13.2815, 4.9469)$; $\mathrm{L}_4 = (90.6165, 5.4298, 1.9170)$

$\mathrm{L}_5 = (99.9923, 0.0412, -0.0285)$

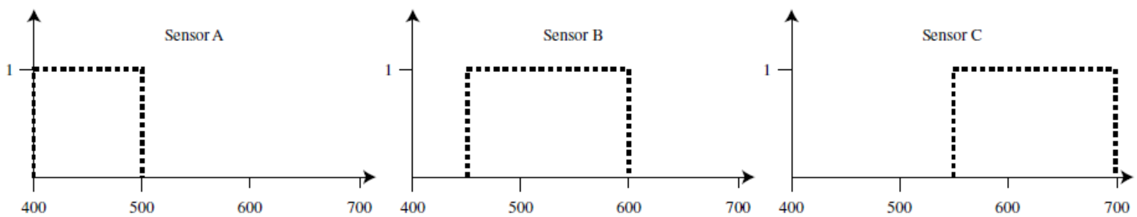
d. Plot two graphs: one showing $L\rightharpoonup$ as a function of u and the other showing a*. (2 points)

Answer:

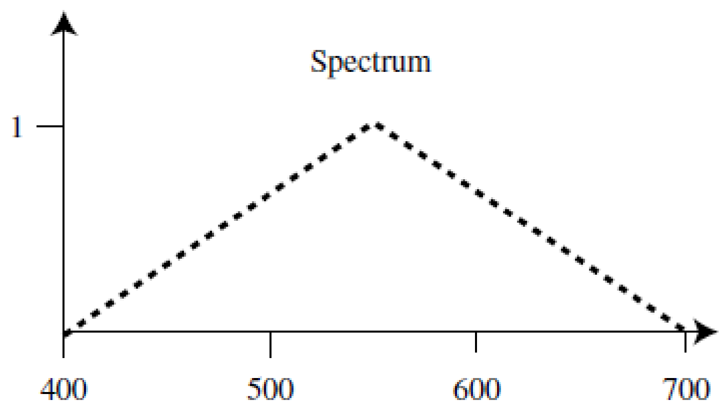


Here we can see that L* and a* are not a linear function of *u*.

Question 3: Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.



What is the response of each of these three sensors to the following spectrum? You need to give actual points.



Answer:
r_A=100/3=33.3; r_B=325/3=108.3; r_C=75

Here is how we can get answers. Take sensor A as an example. Denote the sensitivity curve of sensor A as $\rho(\lambda)$ and *the spectrum as* $E(\lambda)$, the response r can be computed using the following equation (in Lecture 2).

$$r = \int \rho(\lambda)E(\lambda)d\lambda$$

$$where\ \rho(\lambda) = \begin{cases} 1, 400 \le \lambda \le 500 \\ 0, otherwise \end{cases} \quad and\ E(\lambda) = \begin{cases} \frac{\lambda-400}{150}, 400 \le \lambda \le 550 \\ 1 - \frac{\lambda-550}{150},\ 550 \le \lambda \le 700 \\ 0, otherwise \end{cases}$$

Denote $f(\lambda) = \rho(\lambda)E(\lambda)$. We can get $f(\lambda) = \begin{cases} \frac{\lambda-400}{150}, 400 \le \lambda \le 500 \\ 0, otherwise \end{cases}$. Therefore,

$$r = \int \rho(\lambda)E(\lambda)d\lambda = \int f(\lambda)d\lambda = 33.3$$

Question 4: The **Sobel operator** is used in image processing for edge detection. The following shows a 3×3 Sobel filter mask for a vertical edge detector.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

a. What is the response of this filter to the following 6×6 image? Ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b. What is the response of this filter to the following 6×6 image? Again, ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 0 & -4 & -4 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & -4 & -4 & 0 \end{bmatrix}$$

c. Can you design a 3×3 filter that can detect the horizontal edge in the image shown in (a)?
Answer:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Question 5: Gaussian is one of the most popular filters in computer graphics. What is the 9×9 2D guassian filter mask? (Use the method described in Lecture 4: first construct a 1d filter mask, and then construct the corresponding 2D filter mask. You are not required to fill in the actual number for the 2D filter mask, and a correct formula is good enough.).

Answer: Our lecture gave a 1×7 1D Gaussian filter mask. We can then first construct the following 1×9 1D Gaussian filter mask.

G$_9$= $\frac{1}{256}[1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1]^T$ (2 points)

We can use obtain the corresponding 9×9 2D Gaussian filter mask by vector outer product. (1 point)

$$\mathbf{G}_{9\times 9} = \mathbf{G}_9 \mathbf{G}_9^T$$