

CS 447/547: Computer Graphics

Homework 1

This homework must be done individually.

Question 1: Vectors are very important to computer graphics and they are used to represent both locations in space (points) and directions. Assume you have three points in 2D space, represented by $\mathbf{a} = [a_x, a_y]$, $\mathbf{b} = [b_x, b_y]$, and $\mathbf{c} = [c_x, c_y]$.

- How do you find the direction vector \mathbf{v} that points *from* \mathbf{a} *toward* \mathbf{b} ?
- How is the length, $\|\mathbf{v}\|$, of \mathbf{v} computed?
- A *unit vector*, $\hat{\mathbf{v}}$, in the direction \mathbf{v} is a vector in the same direction as \mathbf{v} but with length 1. How do you compute $\hat{\mathbf{v}}$? Computing $\hat{\mathbf{v}}$ is also referred to as *normalizing* \mathbf{v} .

Question 2: Consider two vectors in 3D, \mathbf{a} and \mathbf{b} .

- How is the dot product $\mathbf{a} \cdot \mathbf{b}$ computed?
- What is the relationship between $\mathbf{a} \cdot \mathbf{b}$ and the angle between \mathbf{a} and \mathbf{b} ?
- How is the cross product vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ computed?
- What is the geometric relationship between \mathbf{a} , \mathbf{b} and \mathbf{c} ?
- What is the geometric relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$?
- What is the relationship between $\mathbf{a} \times \mathbf{b}$ and the angle between \mathbf{a} and \mathbf{b} ?

Question 3: What is the solution to the following quadratic equation?

$$x^2 + 6x + 5 = 0$$

Question 4: What is the distance from a 2D point $\mathbf{p} = [p_x, p_y]$ to a line $ax + by + c = 0$?

Question 5: This question concerns the definition of a 3D parametric line.

- a. What is the minimum number of points needed to define a unique line in 3D that passes through all the points? What other conditions must the points satisfy for the line to be unique?
- b. Given more than the minimum number of points, is it in general possible to find one line that passes through all of them?
- c. A 3D parametric line is usually defined as $\mathbf{p} = \mathbf{o} + t\mathbf{d}$. Label your points $\mathbf{p}_1, \mathbf{p}_2$, etc. Find two vectors \mathbf{o} and \mathbf{d} in terms of the points.

Question 6: What is the result of the following matrix multiplication of a vector?

$$\begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 12 \\ 3 & 1 & 15 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$