

# CS 447/547: Computer Graphics

## Homework 1

This homework must be done individually. Submission date is Oct. 06, 2021. Email your homework to [abhijay@pdx.edu](mailto:abhijay@pdx.edu).

**Question 1:** Vectors are very important to computer graphics and they are used to represent both locations in space (points) and directions. Assume you have three points in 2D space, represented by  $\mathbf{a} = [a_x, a_y]$ ,  $\mathbf{b} = [b_x, b_y]$ , and  $\mathbf{c} = [c_x, c_y]$ .

- How do you find the direction vector  $\mathbf{v}$  that points *from*  $\mathbf{a}$  *toward*  $\mathbf{b}$ ?
- How is the length,  $\|\mathbf{v}\|$ , of  $\mathbf{v}$  computed?
- A *unit vector*,  $\hat{\mathbf{v}}$ , in the direction  $\mathbf{v}$  is a vector in the same direction as  $\mathbf{v}$  but with length 1. How do you compute  $\hat{\mathbf{v}}$ ? Computing  $\hat{\mathbf{v}}$  is also referred to as *normalizing*  $\mathbf{v}$ .

**Question 2:** Consider two vectors in 3D,  $\mathbf{a}$  and  $\mathbf{b}$ .

- How is the dot product  $\mathbf{a} \cdot \mathbf{b}$  computed?
- What is the relationship between  $\mathbf{a} \cdot \mathbf{b}$  and the angle between  $\mathbf{a}$  and  $\mathbf{b}$ ?
- How is the cross product vector  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$  computed?
- What is the geometric relationship between  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ ?
- What is the geometric relationship between  $\mathbf{a} \times \mathbf{b}$  and  $\mathbf{b} \times \mathbf{a}$ ?
- What is the relationship between  $\mathbf{a} \times \mathbf{b}$  and the angle between  $\mathbf{a}$  and  $\mathbf{b}$ ?

**Question 3:** What is the solution to the following quadratic equation?

$$x^2 + 6x + 5 = 0$$

**Question 4:** What is the distance from a 2D point  $\mathbf{p} = [p_x, p_y]$  to a line  $ax + by + c = 0$ ?

**Question 5:** This question concerns the definition of a 3D parametric line.

- What is the minimum number of points needed to define a unique line in 3D that passes through all the points? What other conditions must the points satisfy for the line to be unique?
- Given more than the minimum number of points, is it in general possible to find one line that passes through all of them?
- A 3D parametric line is usually defined as  $\mathbf{p} = \mathbf{o} + t\mathbf{d}$ . Label your points  $\mathbf{p}_1$ ,  $\mathbf{p}_2$ , etc. Find two vectors  $\mathbf{o}$  and  $\mathbf{d}$  in terms of the points.

**Question 6:** What is the result of the following matrix multiplication of a vector?

$$\begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 12 \\ 3 & 1 & 15 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$