5.3. SINGLE SIDEBAND (SSB) MODULATION

- Double SideBand (both SC and LC) doubles the bandwidth of original baseband signal, resulting an inefficient usage of the valuable bandwidth.
- From symmetry condition of the Fourier transform of a real signal $F(-\omega) = F^*(\omega)$, this doubling is unnecessary or redundant.
- Using single sideband (either upper or lower) for transmission will be bandwidth efficient.

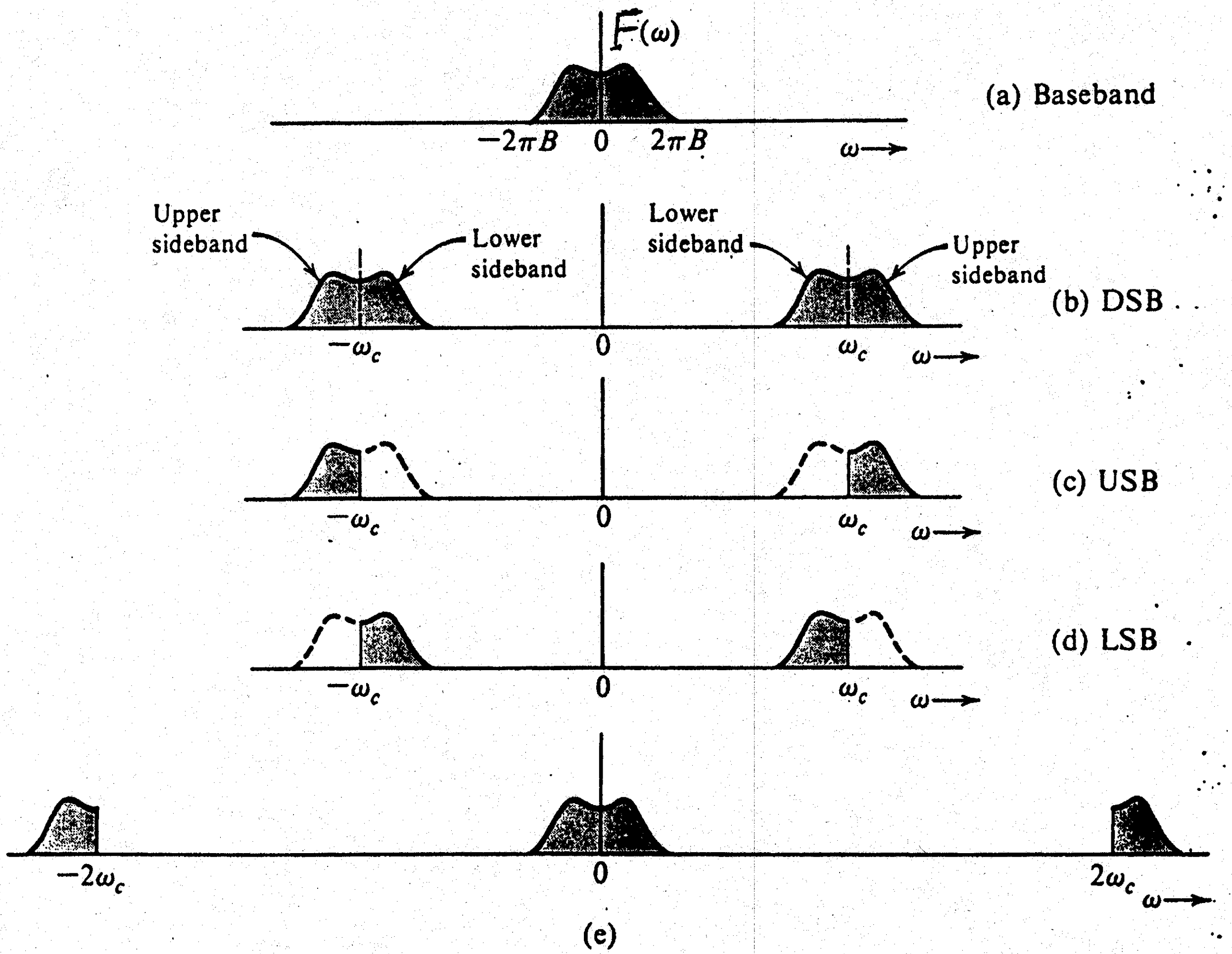


Figure SSB spectra.

- One side spectrum corresponds to complex signal (called *analytical signal* or *pre-envelope*).
- Define an analytical signal as

$$z(t) = f(t) + j\hat{f}(t). \tag{5.19}$$

Its Fourier transform

$$Z(\omega) = F(\omega) + j\hat{F}(\omega) \tag{5.20}$$

is one-sided (say upper sided)

$$Z(\omega) = 0 \text{ for } \omega < 0$$

which requires

$$\hat{F}(\omega) = jF(\omega) \text{ for } \omega < 0.$$

To further maintain an odd symmetry of the phase:

$$\hat{F}(\omega) = -jF(\omega) \text{ for } \omega > 0.$$

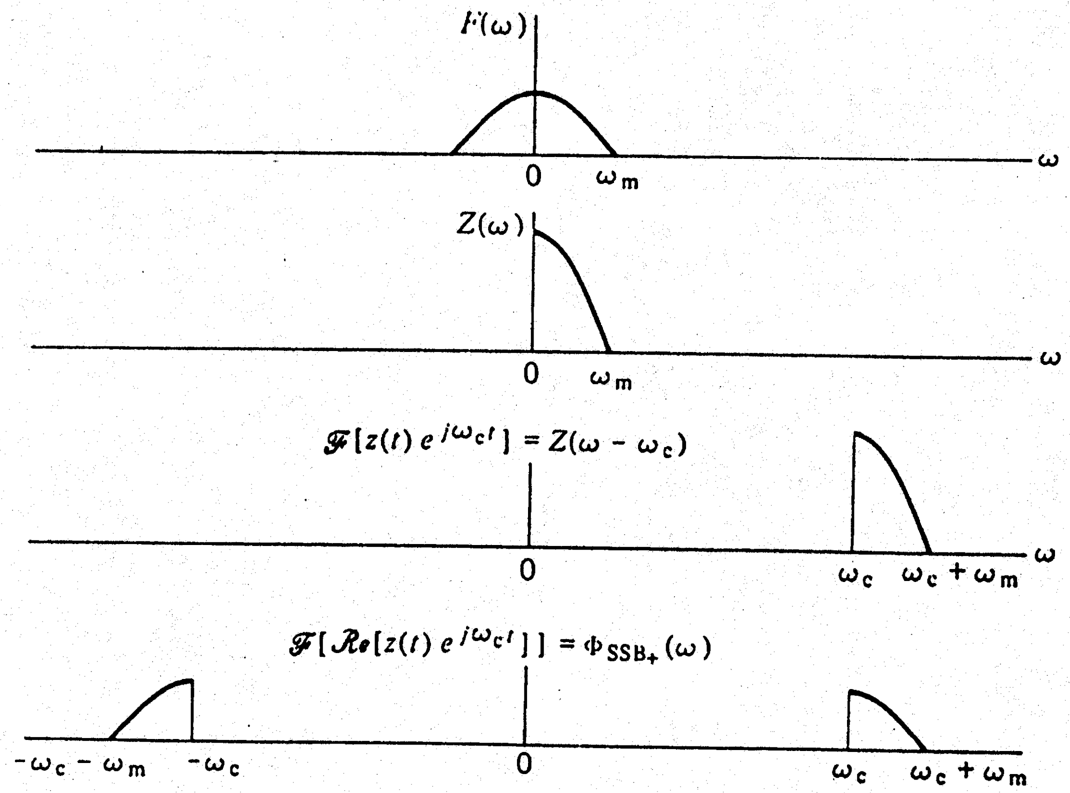


Figure Generation of SSB using analytic signals.

Combine

$$\hat{F}(\omega) = \begin{Bmatrix} jF(\omega) & \text{for } \omega < 0 \\ -jF(\omega) & \text{for } \omega > 0 \end{Bmatrix} = -j\ sgn(\omega)F(\omega). \quad (5.21)$$

The spectrum of the analytical signal is one sided

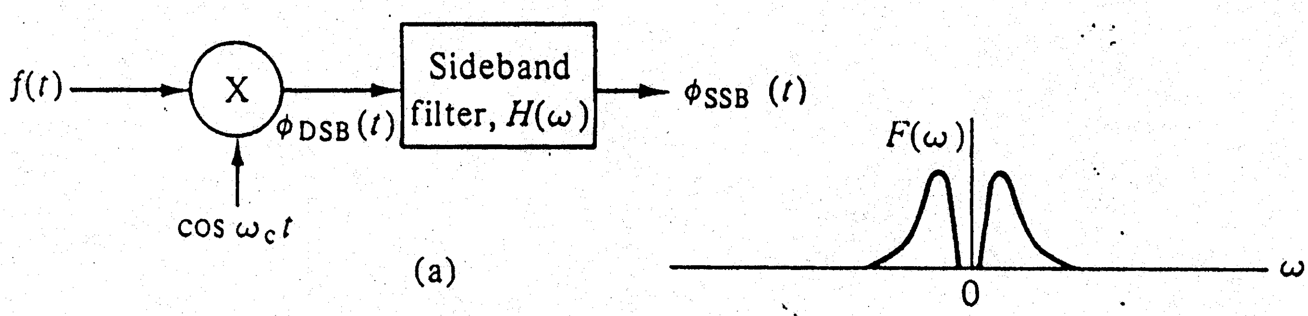
$$Z(\omega) = \begin{Bmatrix} 0 & \text{for } \omega < 0 \\ 2F(\omega) & \text{for } \omega > 0 \end{Bmatrix}. \quad (5.22)$$

$\hat{f}(t)$ can be found through inverse Fourier transform (note $sgn(\omega) \leftrightarrow \frac{j}{\pi t}$)

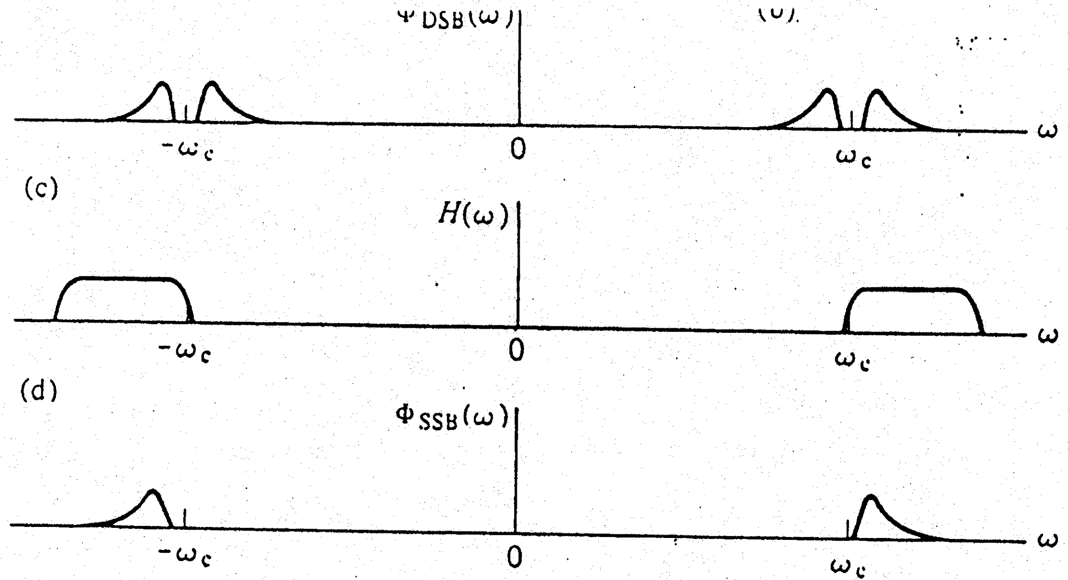
$$\hat{f}(t) = \mathcal{F}^{-1}\{\hat{F}(\omega)\} = f(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} d\tau. \quad (5.23)$$

- Analytic signal can be formed by adding $f(t)$ with $\hat{f}(t) = f(t) * \frac{1}{\pi t}$ in quadrature.
- Real part of analytic signal restores the original signal.

- Generation of SSB Signals:
- 1. Sideband Filtering (Selective Filtering): pass one sideband while suppress the other. The filter requires an ideal (or impossible) cutoff characteristeristic at ω_c.



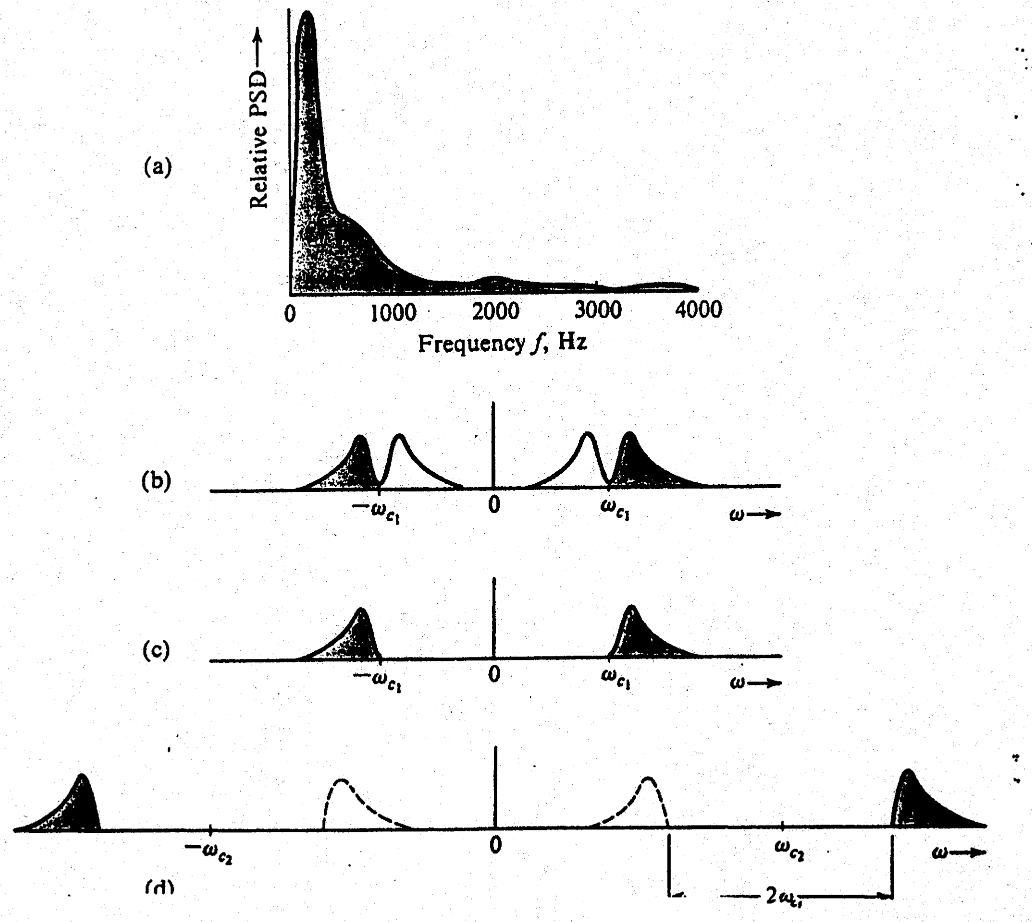
(a)



(e) SSB modulator using filtering for the upper sideband.

- The sideband filter can be used when the signal has unimportant low frequency component, e.g. for speech signal, component outside 300 – –3,500 Hz has no contribution to intelligibility.

- The sideband filter can be used at a translated frequency using heterodyning technique.



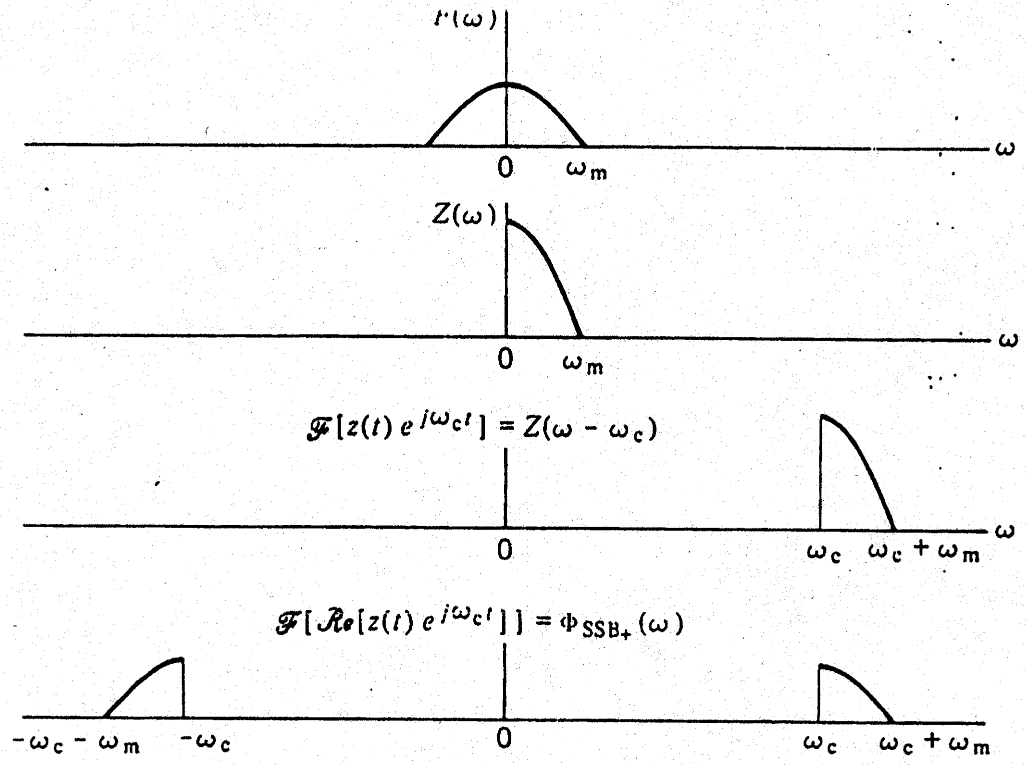


Figure Generation of SSB using analytic signals.

- 2. Phase-Shift method (Balanced Modulator): from

$$\Re e\{z(t)e^{j\omega_c t}\} = \Re e\{[f(t) + j\hat{f}(t)]e^{j\omega_c t}\}$$
$$= f(t)\cos\omega_c t - \hat{f}(t)\sin\omega_c t.$$

we can generate upper or lower single sideband signal as

$$\phi_{SSB\mp}(t) = f(t)\cos\omega_c t \pm \hat{f}(t)\sin\omega_c t.$$

The $\hat{f}(t)$ is simply a phase shift of $-\pi/2$ of $f(t)$ for all frequency. But the implementation of the phase-shift ~~will it~~ is unrealizable, or at best approximatable.

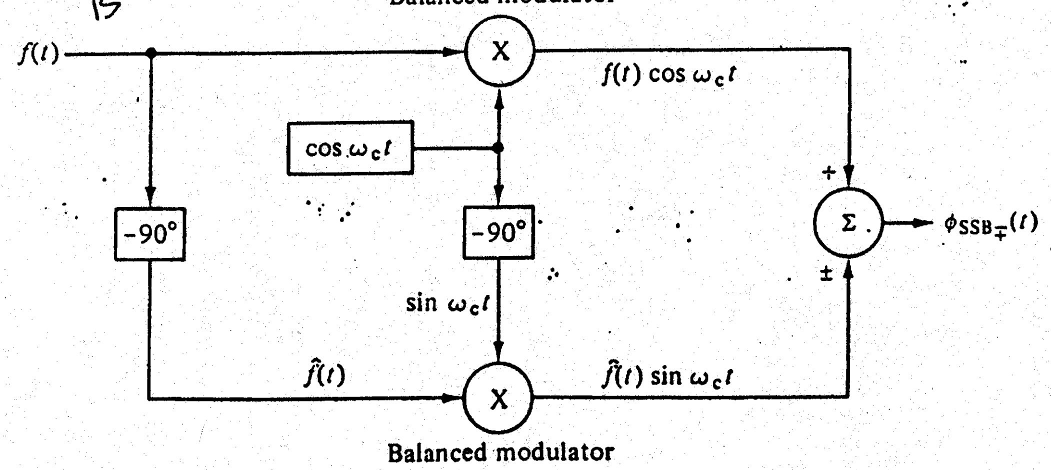
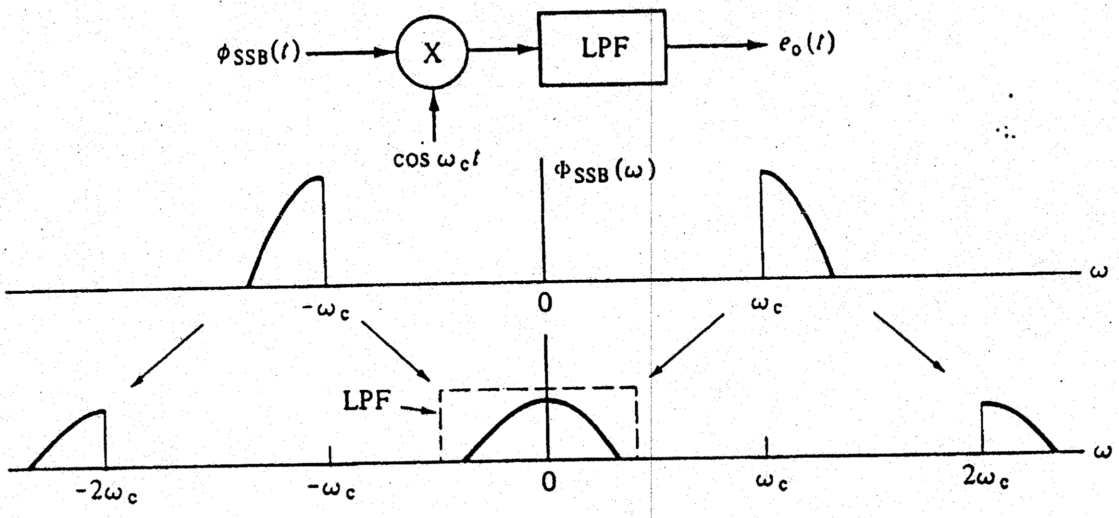


Figure Phase-shift method of generating SSB.

- Demodulation:
- 1. Synchronous detector for SSD-SC signal:

$$
\begin{aligned}
& \phi_{SSB\mp}(t)\cos\omega_c t \\
= \; & f(t)\cos^2\omega_c t \pm \hat{f}(t)\sin\omega_c t\cos\omega_c t \\
= \; & f(t)\frac{1+\cos 2\omega_c t}{2} \pm \hat{f}(t)\frac{\sin 2\omega_c t}{2}
\end{aligned}
$$



- 2. Envelope detector for SSD-LC signal:

$$\phi_{SSB\mp}(t) = A\cos\omega_c t + f(t)\cos\omega_c t \pm \hat{f}(t)\sin\omega_c t$$

the envelope

$$
\begin{aligned}
r(t) &= \sqrt{[A+f(t)]^2 + [\hat{f}(t)]^2} \\
&= A\sqrt{1+\frac{2f(t)}{A}+\frac{f^2(t)}{A^2}+\frac{\hat{f}^2(t)}{A^2}} \\
&\approx A\sqrt{1+\frac{2f(t)}{A}} \\
&\approx A+f(t). \qquad (5.24)
\end{aligned}
$$

For a good approximation, A needs to be very large!

- To avoid a large A while detect envelope of a SSB signal, we can use *Compatible Single-Sideband* (CSSB) signal as

$$\phi_(t) = \sqrt{f(t)}\cos[\omega_c t + k_p \log\alpha(t)]. \qquad (5.25)$$

This is beyond the scope of this chapter.

6.3 Sampling Theorem

Theorem 1: A band-limited (to W_N) signal $f(t)$ can be completed reconstructed from its sample values $f(nT_s)$ with

$$f(t) = \sum_{n=-\infty}^{\infty} T_s f(nT_s) \left\{ \frac{\sin\left[\frac{W_s(t-nT_s)}{2}\right]}{\pi(t-nT_s)} \right\}$$

If $\omega_s \geq 2W_N$

Proof:

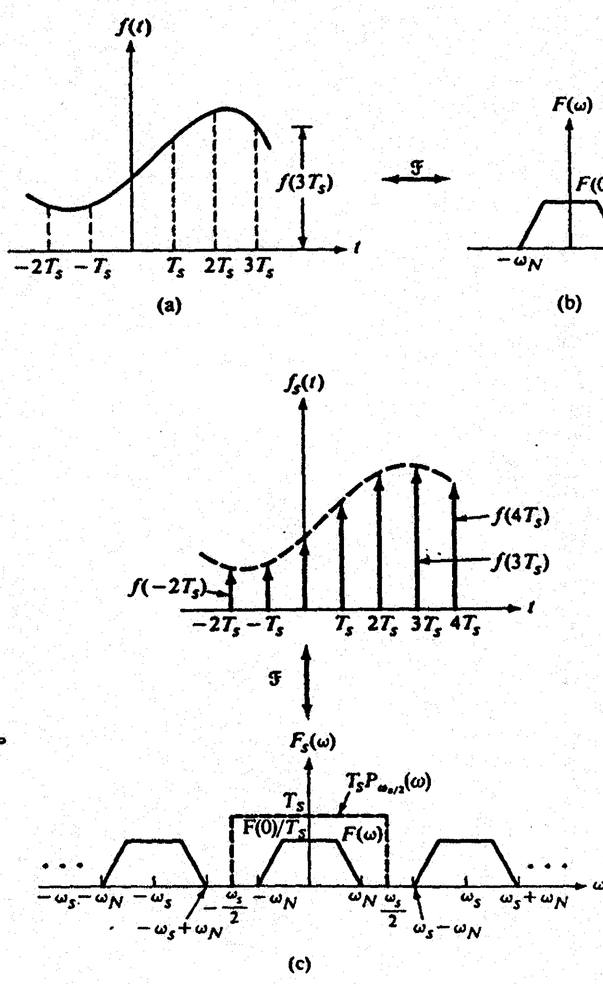
$$F(W) = P_{\frac{W_s}{2}}(\omega)\, T_s F_s(\omega)$$

$$= P_{\frac{W_s}{2}}(\omega)\, T_s \sum_{n=-\infty}^{\infty} f(nT_s) e^{-jn\omega T_s}$$

$$f(t) = \mathcal{F}^{-1}\{F(\omega)\}$$

$$= \mathcal{F}^{-1}\left\{P_{\frac{\omega_s}{2}}(\omega) T_s \sum f(nT_s) e^{-j\omega T_s}\right\}$$

$$= T_s \sum_{n=-\infty}^{\infty} f(nT_s)\, \mathcal{F}^{-1}\left\{P_{\frac{W_s}{2}}(\omega) e^{-j\omega nT_s}\right\}$$



Illustrations of the Sampling Theorem

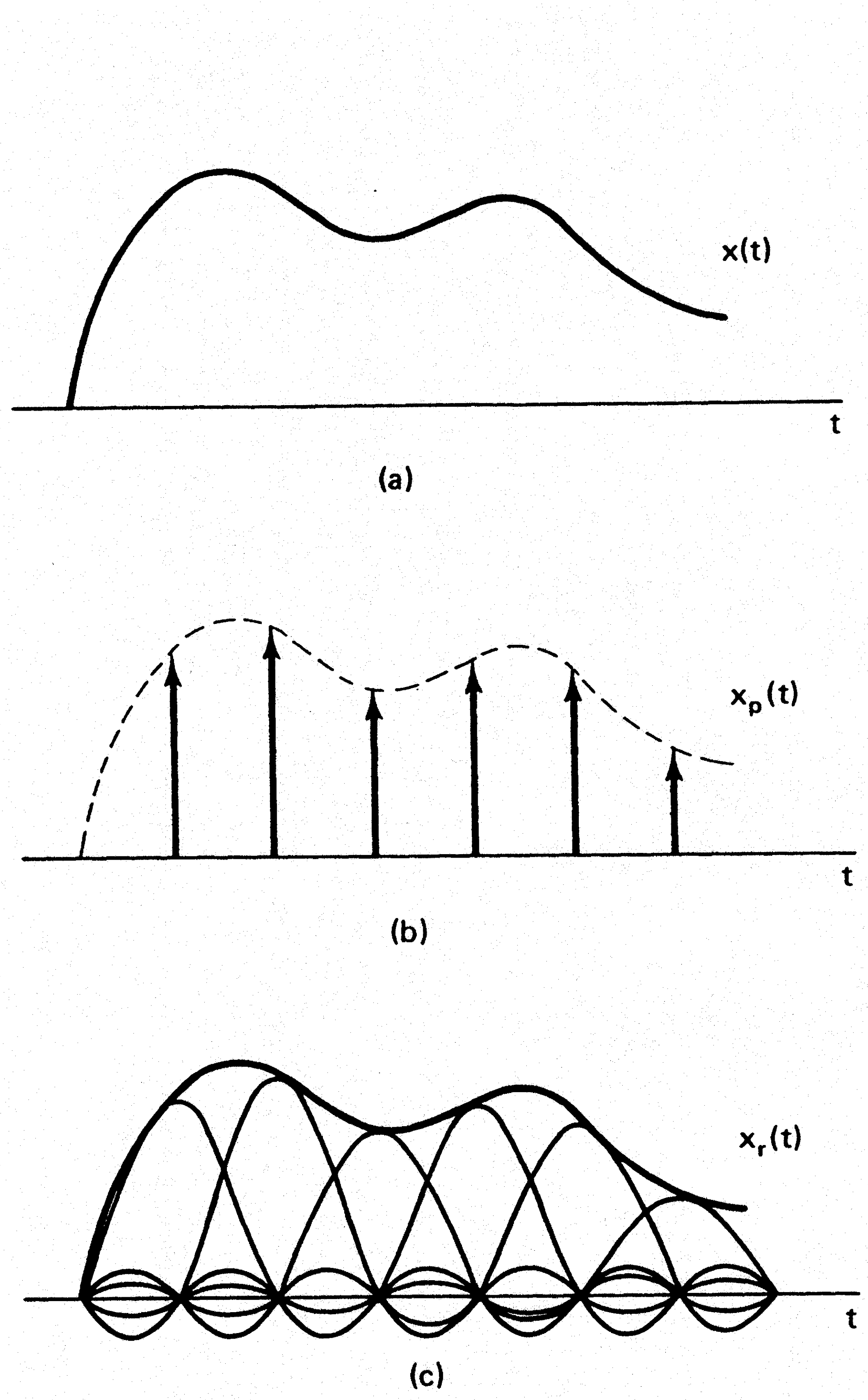


Figure 8.11
lation using t