# On Modeling of A Mobile Multipath Fading Channel

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### ABSTRACT

Multipath fading is one of the major practical concerns in wireless communications. Multipath problem always exists in mobile environment, especially for mobile unit which is often embedded in its surroundings. A time-variant tapped line delay model has been used for multipath fading in a wide-band spread spectrum mobile system. In this paper, we proposed to use the detection and estimation techniques developed in spectrum analysis and array processing to determine the number of delay paths (or taps), to estimate the time delay of each path, and to estimate tap weight of each delay path based on chip rate channel information in a realistic mobile environment. Simulations show that the new approach outperforms the existing approaches.

#### 1. INTRODUCTION

Multipath fading is one of the major practical concerns in wireless communications. A multipath transmission takes place when a transmitted signal arrives at receiver by two or more paths of different delays. Such multiple paths may be due to atmospheric reflection or refraction, or reflections from buildings or other objects. In mobile environment, the source of multipaths normally attributes to the surroundings to the mobile unit. The different paths may consist of several discrete paths, or might consist of a continuum of paths [1] (see Figure 1). In a building-up area, there may not even be a line-ofsight path (direct path) from the vehicle-borne antenna to the base-station transmitter. Propagation is therefore mainly by way of scattering from the surface of the buildings and by diffraction over and/or around them.

In a multipath environment, the signals arrive from different directions each with a different attenuation and different time delay. They combine vectorially at receiver to give a resultant signal which fluctuates in its level. Without correction of such fading, multiple paths will generally cause severe degradation the quality of the mobile communications.

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Multipath fading channel is usually modeled as a time-variant tapped delay system [2, 3]. Based on this model, a diversity receiver can be used to detect the signal from the multipath fading channel. However, the number of delay taps, the time delay of each paths, and the tap weight for each delay path need to be determined.

### Figure 1: Typical Multipath Environment

The objective of the paper is to explore the possibility of using the advance signal processing algorithms to estimate the number of delay, the time delay of each path, and the tap weight for each delay path in multipath channels and to investigate the performance of the diversity receiver based on chip rate channel estimates in a realistic mobile environment.

Contributions included in this paper are:

- 1. A general description of diversity receiver based on chip rate channel estimate.
- 2. An "Information Theoretic Criterion" detection approach (AIC and MDL as described in [4]) to determine the number of delay paths.
- 3. A subspace-based estimation approach (MUSIC [5]) to determine the time delay of each path.
- 4. A least-square estimation approach to determine the tap weight of each delay path.
- 5. The performance of the diversity receiver based on information obtained above in a realistic mobile multipath fading channel [6, 7].
- 6. Simulation results on bit-error-rate (BER) as a function of  $E_b/N_0$  using BPSK modulation for the multipath fading channel.

# 2. MULTIPATH FADING CHANNEL

Multipath fading channel is usually modeled as a timevariant tapped delay system (see Figure 2). The signal

Figure 2: Tapped Delay Model for Multipath Fading

arrived at receiver can be expressed as

$$
r(t) = \sum_{n=1}^{L} c_n(t)u(t - \tau_n) + z(t) =
$$
  
\n
$$
\begin{bmatrix} u(t - \tau_1) & \cdots & u(t - \tau_L) \end{bmatrix} \begin{bmatrix} c_1(t) \\ \vdots \\ c_L(t) \end{bmatrix} + z(t) \quad (1)
$$

where  $\tau_n$  is the time-delay at each path,  $z(t)$  is the additive channel noise, and  $u(t)$  is the signal transmitted through the multipath fading channel. The time-variant tap weights  ${c_n(t)}$  are zero-mean complex-valued stationary processes statistically independent to each other [3], but for a slowly fading channel,  $\{c_n(t)\}\$ is a constant within a chip. Therefore, if we sample the received data  $K$  times, we have

$$
\mathbf{r} = \begin{bmatrix} r(t_1) \\ \vdots \\ r(t_K) \end{bmatrix} = \begin{bmatrix} u(t_1 - \tau_1) & \cdots & u(t_1 - \tau_L) \\ \vdots & \ddots & \vdots \\ u(t_K - \tau_1) & \cdots & u(t_K - \tau_L) \end{bmatrix} \times \begin{bmatrix} c_1 \\ \vdots \\ c_L \end{bmatrix} + \begin{bmatrix} z(t_1) \\ \vdots \\ z(t_K) \end{bmatrix} . \tag{2}
$$

We define the vector  $\mathbf{u}(\tau)$ 

$$
\mathbf{u}(\tau) = \begin{bmatrix} u(t_1 - \tau) & \cdots & u(t_K - \tau) \end{bmatrix}^H \qquad (3)
$$

for future use. Notice that  $\mathbf{u}(\tau)$  has a known form (known waveform in communications) and unknown delay  $\tau$ . Further, if same signal could be repeated N times (as signals in pilot channel, for example),  $c_n$  will be different each time, then we can form a date matrix

$$
\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \cdots & \mathbf{r}_N \end{bmatrix} \tag{4}
$$

where each  $\mathbf{r}_j$  has a same form as in (2) except  $c_n$  is different. In matrix form

$$
\mathbf{R} = \begin{bmatrix} u(t_1 - \tau_1) & \cdots & u(t_1 - \tau_L) \\ \vdots & \ddots & \vdots \\ u(t_K - \tau_1) & \cdots & u(t_K - \tau_L) \end{bmatrix} \times \begin{bmatrix} c_{11} & \cdots & c_{1N} \\ \vdots & \ddots & \vdots \\ c_{L1} & \cdots & c_{LN} \end{bmatrix} + \begin{bmatrix} z_1(t_1) & \cdots & z_N(t_1) \\ \vdots & \ddots & \vdots \\ z_1(t_K) & \cdots & z_N(t_K) \end{bmatrix} \tag{5}
$$

or equivalently

$$
\mathbf{R} = \mathbf{U}(\tau)\mathbf{C} + \mathbf{Z} \tag{6}
$$

The sample covariance matrix is

$$
\mathbf{Cov} = \frac{1}{N} \mathbf{R} \mathbf{R}^H
$$
 (7)

where superscript H stands for Hermitian conjugate. An eigenvalue (or SVD) decomposition can be performed one the sample covariance matrix

$$
Cov = E\Lambda E^H
$$
 (8)

where  $E$  and  $\Lambda$  are eigenvalue and eigenvector matrix, respectively.

# 3. DETECTION OF THE DELAY PATHS

Let eigenvalues  $\lambda_i$   $(i = 1, \dots, K)$  be decreasingly ordered on the diagonal of Λ. We can apply information theoretic criterion to determine the number of delay paths, that is to estimate  $\hat{L}$  by minimizing the following function over the possible number of delay paths  $l$ 

$$
\Phi(l, N) = N(K - l) \log \left[ \frac{\frac{1}{K - l} \sum_{i=l+1}^{K} \lambda_i}{(\prod_{i=l+1}^{K} \lambda_i)^{\frac{1}{K - l}}} \right] + \phi(l, N)
$$
\n(9)

where the penalty function

$$
\phi(l,N) = \begin{cases} l(2K-l) & \text{for AIC} \\ \frac{1}{2}l(2K-l)\log N & \text{for MDL} \end{cases}
$$
 (10)

# 4. ESTIMATION OF THE TIME DELAYS

The eigenvector matrix  $E$  can be partitioned into

$$
\mathbf{E} = [\mathbf{E}_s | \mathbf{E}_o] \tag{11}
$$

where eigenvectors in  $E_s$  are associated with L (or  $\hat{L}$ ) largest eigenvalues  $\lambda_i$   $(i = 1, \dots, L)$  and eigenvectors in  $E_o$  are associated with  $K - L$  smallested eigenvalues  $\lambda_i$  $(i = L+1, \dots, K)$ . From subspace theory, we can see  $\mathbf{E}_s$ is in the the column span of  $U(\tau)$  while  $E_o$  is orthogonal to the column span of  $\mathbf{U}(\tau)$ , i.e.  $\mathbf{U}(\tau)^H \mathbf{E}_o = 0$ . We can then use the subpace algorithms to estimation the delay  $\tau_i$ .

For MUSIC, we search for minimum of

$$
P(\tau) = |\mathbf{u}(\tau)^H \mathbf{E}_o|^2 \tag{12}
$$

over  $\tau$ .

For Min-Norm [8], we search for minimum of

$$
P(\tau) = |\mathbf{u}(\tau)^H \mathbf{d}|^2 \tag{13}
$$

over τ. The linear prediction error vector

$$
\mathbf{d} = \mathbf{E}_o \frac{\mathbf{e}_1}{\|\mathbf{e}_1\|^2} \tag{14}
$$

where  $\mathbf{e}_1^H$  is the first row of  $E_o$ .

# 5. ESTIMATION OF DELAY TAP WEIGHTS

Eq. (2) can be expressed in a vector form

$$
\mathbf{r} = \mathbf{U}\mathbf{c} + \mathbf{z}.\tag{15}
$$

As we proposed in [9]

$$
\hat{\mathbf{c}} = \mathbf{U}(\tau)^{\dagger} \mathbf{r} \tag{16}
$$

gives  $\{c_n\}$  a least-square error (LSE) estimate at a given symbol time. In (16), the left pseudo-inverse † is defined as

$$
\mathbf{U}(\tau)^{\dagger} = (\mathbf{U}(\tau)^T \mathbf{U}(\tau))^{-1} \mathbf{U}(\tau)^T \tag{17}
$$

 $\hat{c}_n$  can now be used in diversity receiver as in Figure 3. The estimation error covariance can be found as

$$
E_z[(\hat{c} - c)(\hat{c} - c)^T] = \sigma_z^2 [\mathbf{U}(\tau)^T \mathbf{U}(\tau)]^{-1}
$$
 (18)

#### 6. DIVERSITY RECEIVER

Based on the previous channel estimated for a tapped delay model, a Diversity receiver (as in Figure 3) [3] can be used to detect the signal from the multipath fading channel.

#### Figure 3: Diversity Receiver

In a realistic mobile environment, the wide-band spread spectrum technique can be used to provide effective diversity gain to improve system performance. In such a spread spectrum mobile communication system, when user information is transmitted through the *traf*fic channel, there is always a companion *pilot* channel used as reference signal for channel estimation. The estimated channel is used for diversity receiver to recover the multipath fading signal in the traffic channel.

#### 7. PERFORMANCE COMPARISON

In our computer simulations, we choose three paths with delays  $\tau = 1, 7, 20$  chips, respectively. The ratio of power attenuation of the second path over that of first path is −3dB, and of third path over the first path is −6dB. The fundamental frequency of the binary message signal  $m(t)$  is chosen to be 19.2 KHz, and the fundamental frequency of PN code sequence  $b(t)$  is  $64 \times 19.2$  KHz = 1.2288 MHz.

Figure 4 shows the detection probability (for correctly estimate the number of delay paths) versus energyper-bit-over-noise-density  $(E_b/N_0)$  using both AIC and MDL detection algorithms. Figure 5 shows the delay spectrum  $P(\tau)$  versus time at  $E_b/N_0 = 10$ dB using MU-SIC algorithm. Figures 6 shows the root-mean-squared error (RMSE) of the delay estimation for all three paths verse  $E_b/N_0$ .

The Bit-Error-Rate (BER) verse  $E_b/N_0$  of the diversity receiver is used to measure the performance, as shown in Figure 7. The diversity receiver using our channel estimates is plotted in line A and the traditional RAKE receiver using tap weight estimates described in [3] in plotted in line B. Clearly, our new approach proposed in this paper outperform the previous approach.

Figure 4: Delay Detection Probability verse  $E_b/N_0$ 

Figure 6: Delay Estimation RMSE verse  $E_b/N_0$ 

Figure 5: Delay Spectrum at  $E_b/N_0 = 10dB$ 

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Figure 7: Performance of BER verse  $E_b/N_0$ 

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