Contemporary Communication Systems

Chapter 4

Amplitude Modulation

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What is Modulation?

- Signal processing by which a message or information-bearing signal $s(t)$ is transformed into another signal to facilitate transmission over a communication channel (e.g., cellular, satellite, twisted wire pair [TWP])
- The message signal $s(t)$ is transmitted through the communication channel by impressing it on a **carrier** signal

$$c(t) = A\cos\left(2\pi f_c t + \theta\right)$$

Amplitude **Frequency** **Phase**

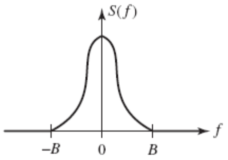
  - **Amplitude Modulation –** amplitude of the carrier varied in accordance with the message signal
  - **Frequency Modulation –** frequency of the carrier varied in accordance with the message signal
  - **Phase Modulation –** phase of the carrier varied in accordance with the message signal

Why Modulate?

- **Frequency translation** – Transfer the message signal $s(t)$ to a new frequency slot depending upon the intended frequency of transmission
  - The frequency slot is determined by the frequency of the carrier
- **Channelization** – Enable **sharing** of a single communication (usually wideband) channel by several lower bandwidth signals/users
- **Practical equipment design** – Higher the transmitted signal frequency, the smaller the antenna size required
  - Narrowband electronics easier to realize
- **Noise performance improvement** – Increase the noise immunity in transmission by expanding the bandwidth of the transmitted signal

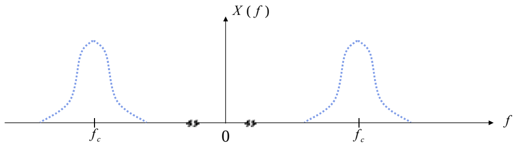
Low-pass Signals

- **Low-pass (LP) signal** – spectral energy clustered around the DC or zero frequency
- All practical LP signals have a frequency above which their spectral components may be considered negligible
  - This frequency, denoted by B, is called the **bandwidth** of the LP signal



Bandpass Signals

- **Bandpass (BP) signal** – spectral components concentrated in the vicinity of a frequency f_c, which is usually much higher than the bandwidth of the signal



- **Amplitude-phase representation of a BP signal**

$$x(t) = A(t)\cos[2\pi f_c t + \psi(t)]$$

Amplitude or Envelope **Phase**

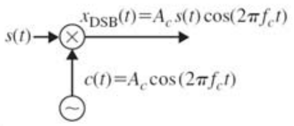
Types of Amplitude Modulation

- Double-sideband, suppressed-carrier (DSB-SC) amplitude modulation (AM)
- Conventional AM
- Single-sideband AM (SSB-AM)
- Vestigial-sideband AM (VSB-AM)

Double-Sideband Suppressed-Carrier AM

- A double-sideband suppressed-carrier (DSB-SC) AM signal is obtained by multiplying the message signal *s*(*t*) with the carrier signal

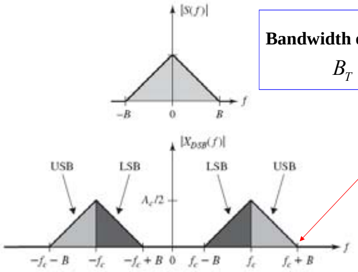
$$x_{DSB}(t) = s(t)c(t) = A_c s(t)\cos(2\pi f_c t)$$



Spectrum of the DSB-SC AM Signal

$$X_{DSB}(f) = S(f)\otimes C(f) = \frac{A_c}{2}S(f)\otimes[\delta(f-f_c)+\delta(f-f_c)]$$

$$= \frac{A_c}{2}[S(f-f_c)+S(f-f_c)]$$



Bandwidth of DSB-SC AM signal

$$B_T = 2B$$

Power Content of DSB-SC AM Signal

- Normalized average power $P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x_{DSB}^2(t)dt$

$$= \lim_{T\to\infty}\frac{A_c^2}{T}\int_{-T/2}^{T/2} s^2(t)\cos^2(2\pi f_c t)dt$$

$$= \frac{A_c^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s^2(t)[1+\cos(4\pi f_c t)]dt$$

$$= \frac{A_c^2}{2}P_s$$

- P_s = Average power in the baseband message waveform *s*(*t*)
- $\frac{A_c^2}{2}$ = Unmodulated carrier power

Peak Envelope Power (PEP)

- Average power supplied by the modulator to a 1 ohm load during one carrier frequency cycle at the crest of the modulation envelope

PEP_{DSB} = Power supplied by a sinusoidal waveform with amplitude $A_c\left[\max_t s(t)\right]$

$$= \frac{A_c^2\left[\max_t s(t)\right]^2}{2}$$

- Sinusoidal modulating signal: $s(t) = A_m\cos(2\pi f_m t)$

$$P_x = \frac{A_c^2 A_m^2}{4} \qquad \Rightarrow PEP_{DSB} = 2P_x$$

$$PEP_{DSB} = \frac{A_m^2 A_c^2}{2}$$

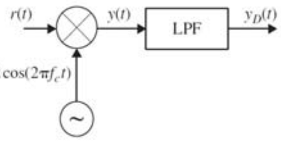
Coherent Demodulation of DSB-SC AM Signals

- Multiply the received signal *r*(*t*) by a locally generated carrier that is frequency- and phase-locked to the carrier used for modulation at the transmitter.
- Recover the message signal by passing the product through an ideal LP filter having a bandwidth *B*. The demodulation scheme is called **coherent**

$$y(t) = r(t)\times 2\cos(2\pi f_c t) = x_{DSB}(t)\times 2\cos(2\pi f_c t)$$

$$= A_c s(t)\cos(2\pi f_c t)\times 2\cos(2\pi f_c t)$$

$$= A_c s(t) + \underbrace{A_c s(t)\cos(4\pi f_c t)}_{\text{Double carrier frequency term. Filtered out.}}$$



Demodulation with Phase/Frequency Error

- Assume a frequency error Δf and phase offset ϕ in the locally generated carrier at the demodulator. The multiplier output is

$$y(t) = r(t)\times 2\cos\left[2\pi\left(f_c+\Delta f\right)t+\phi\right]$$

$$= A_c s(t)\cos\left(2\pi\Delta f t+\phi\right) + \underbrace{A_c s(t)\cos\left[2\pi\left(2f_c+\Delta f\right)t+\phi\right]}_{\text{Double carrier frequency term. Filtered out.}}$$

- The demodulated output is

$$y_D(t) = s(t)\cos\left(2\pi\Delta f t+\phi\right)$$

- For $\Delta f = 0$, the desired signal is scaled by a factor that depends on the phase offset cos(ϕ)
  - If cos(ϕ) = 45°, the amplitude of the desired signal is reduced by √2
  - If cos(ϕ) = 90°, the desired signal component vanishes

Conventional AM

- In the conventional AM, a portion of the sinusoidal carrier is added to the DSB-SC AM signal, which greatly simplifies the demodulation process
- The transmitted signal is given by

$$\begin{aligned} x_{AM}(t) &= A_c\cos(2\pi f_c t) + s(t)\cos(2\pi f_c t) \\ &= [A_c + s(t)]\cos(2\pi f_c t) \end{aligned}$$

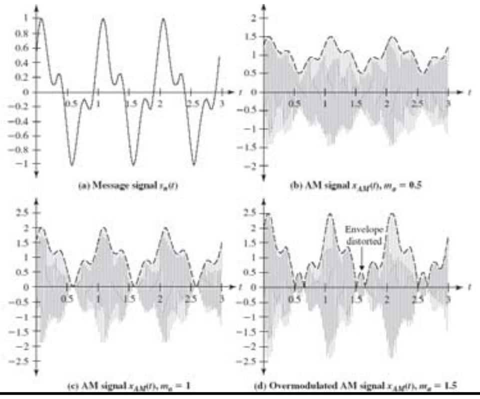
- We can express the conventional AM signal as

$$x_{AM}(t) = A_c[1 + m_a s_n(t)]\cos(2\pi f_c t)$$

where **Normalized message signal**

$$s_n(t) \square \frac{s(t)}{\left|\min_t s(t)\right|}, \qquad \left|\min_t s(t)\right| \neq 0$$

Conventional AM Waveforms



(a) Message signal $s_n(t)$ (b) AM signal $x_{AM}(t)$, $m_a = 0.5$

(c) AM signal $x_{AM}(t)$, $m_a = 1$ (d) Overmodulated AM signal $x_{AM}(t)$, $m_a = 1.5$

Modulation Index

- The parameter m_a determines the extent to which the carrier has been amplitude-modulated. It is called the **modulation index** and is defined as

$$m_a \square \frac{\left|\min_t s(t)\right|}{A_c}$$

- We observe from the figure that the envelope of the modulated signal $x_{AM}(t)$ is always positive, and hence retains the shape of the message signal $s_n(t)$ if

$$m_a \le 1$$

- Therefore, the message signal $s_n(t)$ can be easily recovered from $x_{AM}(t)$ by using a simple envelope detector

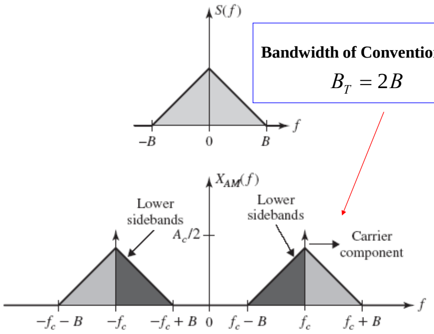
Spectrum of the Conventional AM Signal

- The spectrum of the AM signal $x_{AM}(t)$ is given by

$$\begin{aligned} X_{AM}(f) &= \Im\{A_c[1 + m_a s_n(t)]\cos(2\pi f_c t)\} \\ &= \Im\{A_c\cos(2\pi f_c t)\} + \Im\{A_c m_a s_n(t)\cos(2\pi f_c t)\} \\ &= \frac{A_c}{2}[\delta(f - f_c) + \delta(f - f_c)] + \frac{A_c m_a}{2}[S_n(f - f_c) + S_n(f - f_c)] \end{aligned}$$

- Observe impulses at $\pm f_c$ indicating presence of the carrier component in the modulated signal spectrum. It is wasted power because it does not carry any information.
- The message signal information is conveyed in the sidebands of the AM signal spectrum.
- Note that the spectrum of a conventional AM signal occupies twice the bandwidth of the message signal

Spectrum of the Conventional AM Signal (contd)



Bandwidth of Conventional AM signal

$$B_T = 2B$$

Power Content of Conventional AM Signal

- Normalized average power of a conventional AM signal

$$P_x = \frac{A_c^2}{2} + \frac{A_c^2 m_a^2}{2} P_{s_n}$$

Power in the carrier component **Signal power**

$$\eta = \frac{\text{Signal power}}{\text{Total Power}} = \frac{(m_a A_c)^2 P_{s_n}/2}{A_c^2/2 + (m_a A_c)^2 P_{s_n}/2} = \frac{m_a^2 P_{s_n}}{1 + m_a^2 P_{s_n}} \le 0.5$$

Power or modulation efficiency

- Peak envelope power (PEP) of a conventional AM signal

$$PEP_{AM} = \frac{A_c^2}{2}\left(\max_t [1 + m_a s_n(t)]\right)^2$$

Envelope Detection

- Suitable for conventional AM signals
- Does not require generation of a coherent carrier at the receiver
- Simple hardware: diode, resistor, capacitor
- Will work with suppressed carrier modulation systems if the receiver inserts a carrier

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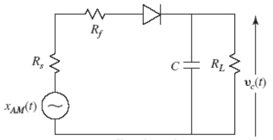
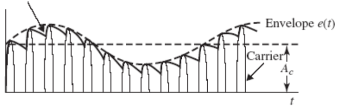
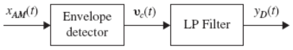
Demodulation of Conventional AM Signal



(a) Envelope detector



(b) Envelope detector output with conventional AM signal



(c) Demodulation of conventional AM signal

Hilbert Transform

- The **Hilbert transform** $\hat{x}(t)$ of the signal $x(t)$ is defined as the signal whose frequency components are all phase shifted by $-\pi/2$ radians

$$\hat{x}(t) = \mathcal{H}\{x(t)\}$$

- **Examples**

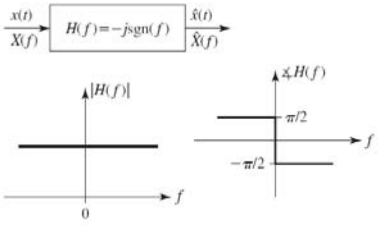
$$x_1(t) = A\cos(2\pi f_c t + \phi) \xleftrightarrow{\mathcal{H}} A\cos\left(2\pi f_c t + \phi - \frac{\pi}{2}\right) = A\sin(2\pi f_c t + \phi)$$

$$x_2(t) = A\sin(2\pi f_c t + \phi) \xleftrightarrow{\mathcal{H}} A\sin\left(2\pi f_c t + \phi - \frac{\pi}{2}\right) = -A\cos(2\pi f_c t + \phi)$$

- For an arbitrary signal, $\hat{x}(t)$ is obtained by passing $x(t)$ through a filter with transfer function

$$H(f) = -j\,\mathrm{sgn}(f)$$

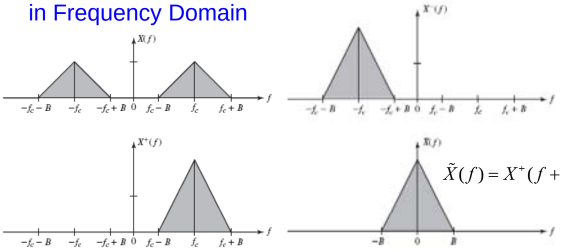
Hilbert Transform in Frequency Domain



FT of $\hat{x}(t)$

$$\hat{X}(f) = X(f)H(f) = -j\,\mathrm{sgn}(f)X(f) = \begin{cases} -jX(f), & f > 0 \\ 0, & f = 0 \\ jX(f), & f < 0 \end{cases}$$

Alternative BP Signal Representations in Frequency Domain



$$\tilde{X}(f) = X^+(f + f_c)$$

$$X^+(f) = X(f)[1 + \mathrm{sgn}(f)] = \begin{cases} 2X(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

$$X^-(f) = X(f)[1 - \mathrm{sgn}(f)] = \begin{cases} 0 & f > 0 \\ 2X(f) & f < 0 \end{cases}$$

Alternative Representations of BP Signals

- **Quadrature carrier representation**

$$x(t) = I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)$$

$$\text{In-phase component:} \quad I(t) = A(t)\cos\psi(t)$$

$$\text{Quadrature component:} \quad Q(t) = A(t)\sin\psi(t)$$

$$A(t) = \sqrt{I^2(t) + Q^2(t)}, \qquad \psi(t) = \tan^{-1}\left(\frac{Q(t)}{I(t)}\right)$$

- **Complex envelope representation**

$$\tilde{x}(t) \triangleq I(t) + jQ(t)$$

Complex envelope of $x(t)$

  - Equivalent LP (“phasor”) representation for the BP waveform $x(t)$ in terms of its in-phase (I) and quadrature (Q) components

Alternative Representations of BP Signals (contd)

- **Analytic signal representation**

$$x^{+}(t) = \tilde{x}(t)e^{j2\pi f_c t}$$

- It is easy to prove that

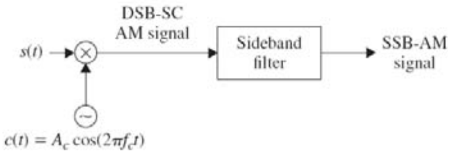
$$x(t) = \mathrm{Re}\left\{x^{+}(t)\right\}$$
$$\hat{x}(t) = \mathrm{Im}\left\{x^{+}(t)\right\}$$
$$x^{+}(t) = x(t) + j\hat{x}(t)$$
$$x^{-}(t) = [x^{+}(t)]^{*} = x(t) - j\hat{x}(t)$$

- Also

$$\tilde{x}(t) = x^{+}(t)e^{-j2\pi f_c t}$$

Single-Sideband AM

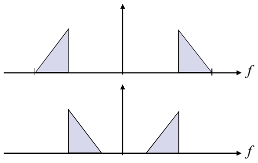
- Both DSB-SC and Conventional AM systems require a channel bandwidth of $B_T = 2B$ Hz
- The transmission of either sideband is sufficient to reconstruct the message signal $s(t)$ at the receiver.
- We may reduce the transmitted bandwidth to B Hz by filtering out either the upper or the lower sideband

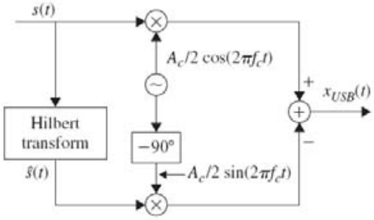


Phasing Method

$$x_{USB}(t) = \frac{A_c}{2}\{s(t)\cos(2\pi f_c t) - \hat{s}(t)\sin(2\pi f_c t)\}$$

$$x_{LSB}(t) = \frac{A_c}{2}\{s(t)\cos(2\pi f_c t) + \hat{s}(t)\sin(2\pi f_c t)\}$$





Phasing Method in Frequency Domain

- We can express a USB-AM signal as

$$x_{USB}(t) = \frac{A_c}{2}\mathrm{Re}\left\{s^{+}(t)e^{j2\pi f_c t}\right\} = \frac{A_c}{4}\left\{s^{+}(t)e^{j2\pi f_c t} + s^{+*}(t)e^{-j2\pi f_c t}\right\}$$
$$= \frac{A_c}{4}\left\{s^{+}(t)e^{j2\pi f_c t} + s^{-}(t)e^{-j2\pi f_c t}\right\}$$

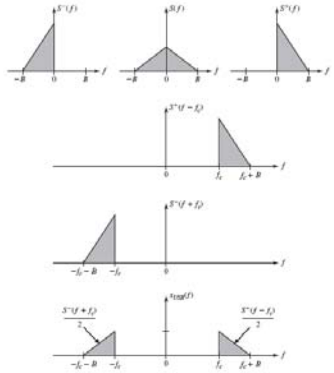
- Taking FT of both sides, the spectrum of can be expressed as

$$X_{USB}(f) = \frac{A_c}{2}\left\{\frac{S^{+}(f-f_c)}{2} + \frac{S^{-}(f+f_c)}{2}\right\}$$

Positive frequency portion of $S(f)$ shifted in frequency by f_c

Negative frequency portion of $S(f)$ shifted in frequency by f_c

Phasing Method in Frequency Domain (contd)

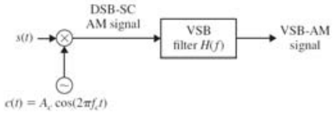


Vestigial-sideband (VSB) AM

- VSB-AM relaxes the requirement of eliminating the second sideband
  - Allows a portion ("vestige") of the unwanted sideband to appear at the output of the modulator
- A VSB-AM signal is generated by partially suppressing one of the sidebands of a DSB-SC signal by a sideband-shaping filter

$$x_{VSB}(t) = x_{DSB}(t)\otimes h(t) = \left[A_c s(t)\cos(2\pi f_c t)\right]\otimes h(t)$$

$$X_{VSB}(f) = \frac{A_c}{2}\left[S(f-f_c) + S(f+f_c)\right]H(f)$$



Coherent Demodulation of VSB-AM

- Multiply the VSB signal by the coherent carrier $2\cos(2\pi f_c t)$

$$y(t) = x_{VSB}(t) \times 2\cos(2\pi f_c t)$$

- In the frequency domain, the output signal can be expressed as

$$Y(f) = [X_{VSB}(f - f_c) + X_{VSB}(f + f_c)]$$

- Substituting for $X_{VSB}(f)$ yields

$$Y(f) = \frac{A_c}{2}\left\{ \begin{array}{l} S(f)[H(f-f_c)+H(f+f_c)] \\ \qquad +[S(f-2f_c)H(f-f_c)+S(f+2f_c)H(f+f_c)] \end{array} \right\}$$

- The LP filter in the demodulator removes the message signal terms in multiplier output translated to frequencies $f = \pm 2f_c$

VSB-AM (contd)

- The demodulator LP filter passes through the message signal spectrum $S(f)$ without any distortion if the VSB filter $H(f)$ satisfies the property

$$H(f - f_c) + H(f + f_c) = C \qquad |f| \le B$$

- This is called **vestigial symmetry condition**
- Figure displays a frequency response of a VSB filter that truncates the lower sideband of the DSB-SC signal
- Observe that the VSB filter roll off characteristic exhibits *odd* symmetry in the transition width of $2f_v$ ($f_v << B$) around the carrier frequency f_c
- **VSB+C** – a variant of VSB where a carrier component is added to the VSB signal. It can now be demodulated using an envelope detector like a Conventional AM signal

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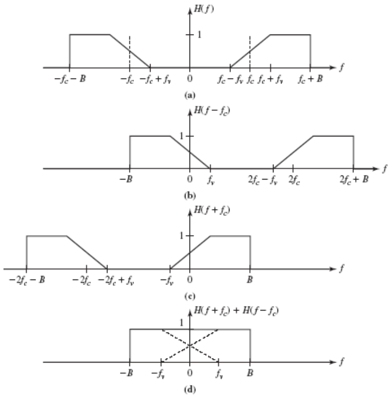
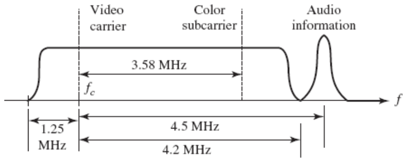
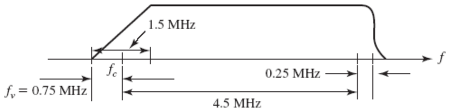
VSB-AM Filter



Analog TV Broadcasting Format



(a) Spectrum of the transmitted TV signal

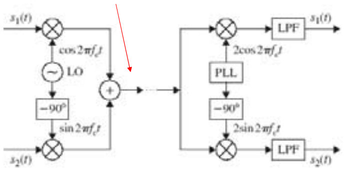


(b) Magnitude response of the VSB filter in the TV receiver

Quadrature Carrier Multiplexing

- Transmit two message signals in the *same* frequency slot by using quadrature (orthogonal) carriers
  - $s_1(t)$ modulates **in-phase** carrier $\cos(2\pi f_c t)$ to produce the DSB signal $s_1(t)\cos(2\pi f_c t)$
  - $s_2(t)$ modulates **quadrature** carrier $\sin(2\pi f_c t)$ to produce the DSB signal $s_2(t)\sin(2\pi f_c t)$

$$x_{QAM}(t) = s_1(t)\cos(2\pi f_c t) + s_2(t)\sin(2\pi f_c t)$$



Coherent Demodulation

- Multiply $x_{QAM}(t)$ with $2\cos(2\pi f_c t)$ and recover the message signal $s_1(t)$ by LP filtering the resulting signal

$$x_{QAM}(t) \times 2\cos(2\pi f_c t) = s_1(t)[1 + \underbrace{\cos(4\pi f_c t)] + s_2(t)\sin(4\pi f_c t)}_{\text{Double frequency terms − Filtered out}}$$

- Similarly, the message signal $s_2(t)$ is recovered by multiplying with $2\sin(2\pi f_c t)$ and then LP filtering the output
- Thus two baseband signals, each of bandwidth B Hz, can be transmitted simultaneously without any distortion over the same frequency channel of bandwidth $2B$ Hz by using orthogonal carriers
- Quadrature-carrier multiplexing, therefore, achieves the bandwidth efficiency of SSB-AM.

Multiplexing

- Process of combining multiple user signals into a composite signal such that individual signals can be separated at the receiving end without any distortion
- There are several common methods for signal multiplexing:
  - **Frequency division multiplexing (FDM)**
  - **Time division multiplexing (TDM)**
  - **Code division multiplexing (CDM)**
  - **Spatial multiplexing**
    - **Antenna direction**
    - **Signal polarization**
- TDM and CDM schemes are used in the transmission of digital signals
- FDM and Spatial multiplexing may be used for the transmission of either analog or digital signals

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Frequency Division Multiplexing (FDM)

- The total system bandwidth is divided into nonoverlapping frequency slots, called **channels**
  - Each user is assigned a unique channel to prevent interference during simultaneous signal transmissions. Tradeoff between adjacent channel interference versus # of users assigned to share the frequency band
  - **Guard bands** = spacing between users
- For example, commercial AM broadcasting
  - The standard AM radio signal occupies 10 kHz in 535 – 1605 kHz band.
  - Multiplexing allows to carry multiple radio signals (voice and music programming) simultaneously over the AM band

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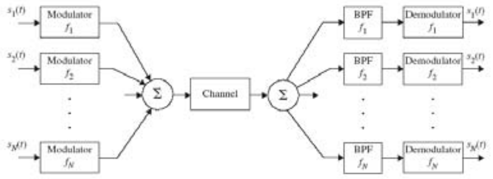
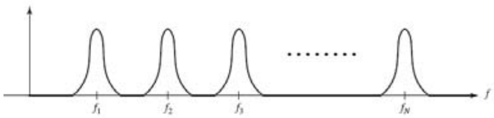
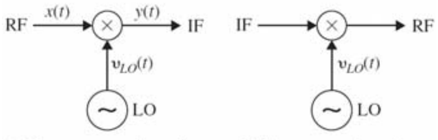
FDM (contd)





Frequency Translation

- Frequency translation – move a signal from one carrier frequency to another
  - A necessary step in the design of communication transmitters and receivers
  - Performed by a multiplier (called **mixer**) that multiplies the input BP signal by a fixed amplitude sinusoidal output from a **local oscillator (LO)**



(a) Down-conversion mixer **(b) Up-conversion mixer**

Down-conversion Mixer

- Let

  RF or High Frequency input: $x_{RF}(t) = A(t)\cos[2\pi f_c t + \psi(t)]$

  LO output: $\upsilon_{LO}(t) = V_o \cos(2\pi f_{LO} t)$

- The mixer output is

$$y(t) = x_{RF}(t) \times \upsilon_{LO}(t)$$

$$= \frac{A(t)V_o}{2}\left\{\cos\left[2\pi f_{IF} t + \psi(t)\right] + \underbrace{\cos\left[2\pi(f_c + f_{LO})t + \psi(t)\right]}_{\text{Filtered out by IF filter}}\right\}$$

- Note that the mixer translates the input signal at frequency f_c to the intermediate frequency (f_{IF})

Down-conversion Mixer (contd)

- **Low-side injection –** LO frequency below the RF or carrier frequency

  $f_{LO} = f_c - f_{IF}$

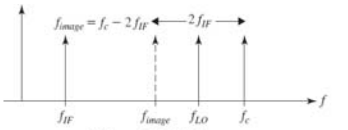
- **High-side injection –** LO frequency above the RF or carrier frequency

  $f_{LO} = f_c + f_{IF}$

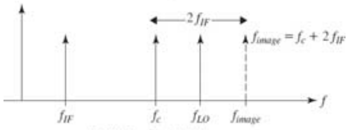
- For the low-side injection, for a given choice of f_{IF}, the input frequency $f_c - 2f_{IF}$ is also converted to the same IF frequency
- Similarly, for the high-side injection, the input frequency $f_c + 2f_{IF}$ is also converted to the same IF frequency.

Image Frequencies

- These frequencies that are separated from the desired frequencies by $2f_{IF}$ are called **image frequencies**



(a) Low–side injection



(b) High–side injection

Image Rejection

- Images cause interference in the reception of the desired signal
  - Noise and interference at the image frequency is also transferred to IF thereby corrupting the desired signal
- To avoid the corruption of the desired signal, place an image-reject filter immediately before the mixer

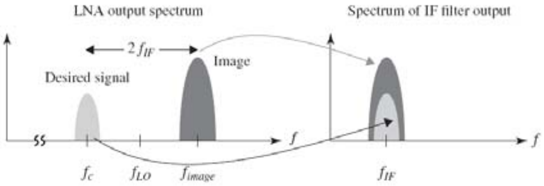
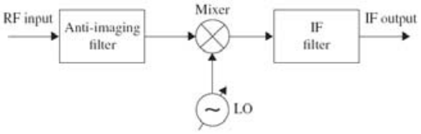
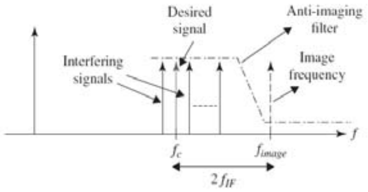


Image Rejection (contd)





Communication Receivers

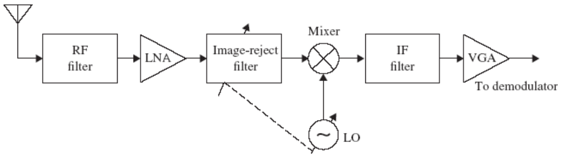
- Extract the desired signal in the presence of noise and interfering signals. Key functions include:
  - **Reception/amplification.** Low-noise amplification in the front end for improved sensitivity
    - **Sensitivity** is a measure of a receiver's ability to receive weak signals in the presence of noise with an acceptable signal-to-noise ratio
  - **Channel or signal selection.** Tuning of the desired signal (frequency slot) from the received signal that may contain other signals in addition to noise
    - **Selectivity** is the measure of the ability of a receiver to select a particular frequency or a particular band of frequencies and reject all other unwanted frequencies.
  - **Demodulation.** Recovering the original baseband message signal

Types of Receivers

- Superheterodyne receivers
- Direct-conversion receivers
- Low IF receivers

Superheterodyne Receiver

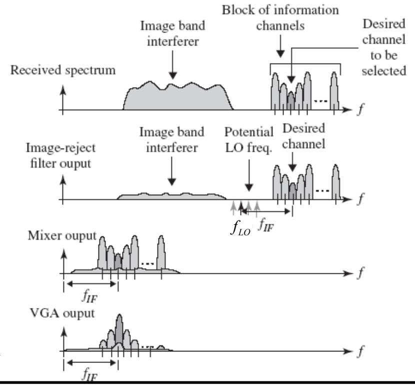
- Most popular type of communication receiver
  - Used for AM/FM & TV broadcasting, cellular & satellite systems, radars, GPS, etc.
- **Main idea** – downconvert RF signal to some fixed lower IF, then amplify it and demodulate



Superheterodyne Receiver

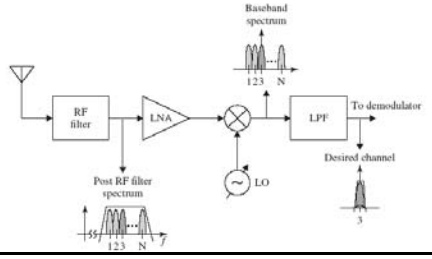
- **Low-noise amplifier (LNA)–** amplifies a weak RF signal coming out of the antenna. Rejects the image frequency. Bandwidth much wider than the signal bandwidth
- **Image-reject Mixer –** together with the local oscillator downconverts the RF signal to the IF
- **Local oscillator –** allows tuning the receiver to a desired channel
- **IF amplifier –** amplifies the IF signal significantly (up to 10^6) and rejects adjacent channel signals and interference (frequency selectivity). Bandwidth same as the signal's
  - Provides **automatic gain control (AGC) –** adjusts the IF amplifier gain according to the signal level (keeps the average signal amplitude almost constant)
- **Detector (demodulator) –** demodulates (recovers) the message signal

Spectra at various stages in a superhet receiver



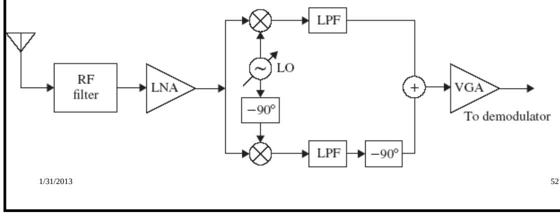
Direct-Conversion or zero-IF (ZIF) Receiver

- Down-convert RF signal with a center frequency of many GHz to baseband in one step, hence the name **zero-IF (ZIF)**
- This approach removes the IF stage from the receiver and eliminates the need for image rejection

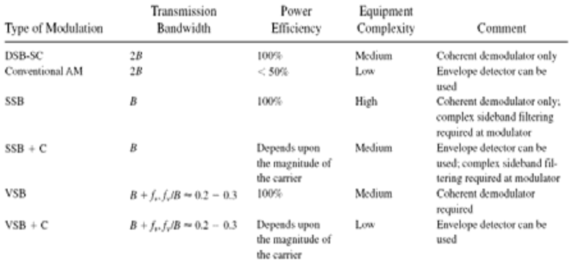


Low-IF Receiver

- DC offset problems in a zero-IF receiver eliminated
- Down-convert the desired RF signal to a low IF– one or two channel bandwidths away from DC
- Low-IF receiver is able to eliminate the off-chip IF SAW filter
- Implemented as RF CMOS ICs
  - Wireless LAN (WLAN), Bluetooth, and GSM



Comparison of Amplitude Modulation Schemes



Type of Modulation	Transmission Bandwidth	Power Efficiency	Equipment Complexity	Comment
DSB-SC	$2B$	100%	Medium	Coherent demodulator only
Conventional AM	$2B$	< 50%	Low	Envelope detector can be used
SSB	B	100%	High	Coherent demodulator only; complex sideband filtering required at modulator
SSB + C	B	Depends upon the magnitude of the carrier	Medium	Envelope detector can be used; complex sideband filtering required at modulator
VSB	$B + f_v, f_v/B \approx 0.2 - 0.3$	100%	Medium	Coherent demodulator required
VSB + C	$B + f_v, f_v/B \approx 0.2 - 0.3$	Depends upon the magnitude of the carrier	Low	Envelope detector can be used