A periodic signal $\mathrm{x}(t)$ with a fundamental period of 2 seconds is described over one period by

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$$\mathrm{x}(t)=\begin{cases}\sin(2\pi t) & , |t|<1/2 \\ 0 & , 1/2<|t|<1\end{cases}.$$

Plot the signal and find its CTFS description. Then plot on the same scale approximations to the signal $\mathrm{x}_N(t)$ given by

$$\mathrm{x}_N(t)=\sum_{k=-N}^{N}\mathrm{X}[k]e^{j2\pi kt/T_0}$$

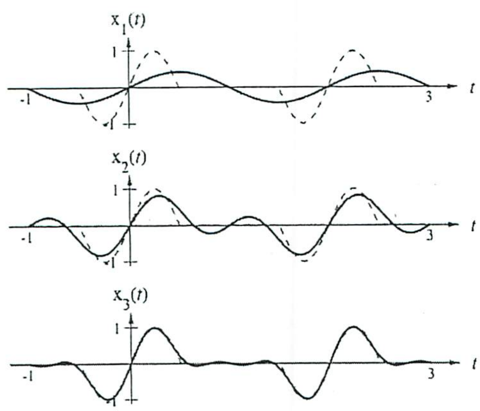
for $N=1$, 2 and 3. (In each case the time scale of the plot should cover at least two periods of the original signal.)

$$T_0=2 \;,\; f_0=1/2$$
$$\mathrm{x}(t)=\sin(2\pi t)\mathrm{rect}(t)*\delta_2(t)$$

The Fourier representation time is twice the period of $\sin(2\pi t)$, therefore the impulses in the harmonic function for $\sin(2\pi t)$ will be at $k=\pm 2$.

$$\mathrm{x}(t)=\sin(2\pi t)\mathrm{rect}(t)*\delta_2(t)\xleftrightarrow[2]{\mathcal{FS}}\mathrm{c}_\mathrm{x}[k]=(j/2)(\delta[k+2]-\delta[k-2])*(1/2)\mathrm{sinc}(k/2)$$

$$\mathrm{c}_\mathrm{x}[k]=(j/4)(\mathrm{sinc}((k+2)/2)-\mathrm{sinc}((k-2)/2))$$



②

$$x[0] = 0$$

$$x[-1] = x[1] = \frac{1}{2}$$

$$x[-2] = x[2] = 0$$

③

$$c_{\cancel{0}} = 0$$

$$c_1 = 2.5$$

$$c_2 = 0$$

$$c_3 = 0$$

$$c_4 = 2.5$$

④

$$\hat{x}[k] = \frac{c_k}{5}$$

$$\therefore \quad \hat{x}[k] = 0$$

$$\hat{x}[1] = 0.5$$

$$\hat{x}[2] = 0$$

$$\hat{x}[3] = 0$$

$$\hat{x}[4] = 0.5$$