# ECE 312 HW#2 Solution

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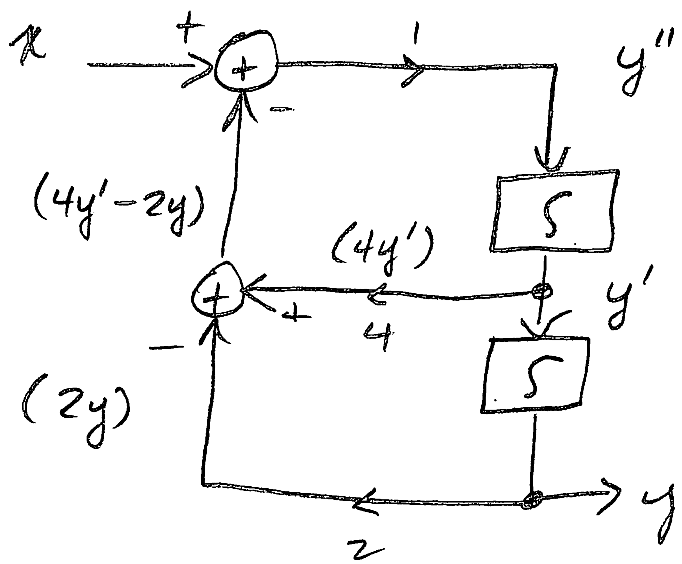


$$y'' = x - (4y' - 2y)$$
$$= x - 4y' + 2y$$

$\therefore$

$$y'' + 4y' - 2y = x$$

$$x \rightarrow \boxed{\int} \rightarrow y$$

(2)

then $y(t) = \int x(\tau)\,d\tau$

Suppose input is $K x(t)$. Then the output is

$$\int K x(\tau)\,d\tau$$
$$= K \int x(\tau)\,d\tau$$
$$= K y(t)$$

$\therefore \quad K x(t) \longrightarrow K y(t) \qquad \boxed{1}$

now, suppose $x(t) = x_1(t)$
then
$$y(t) = \int x_1(\tau)\,d\tau = y_1(t)$$
if $\quad x(t) = x_2(t)$
$$y(t) = \int x_2(\tau)\,d\tau = y_2(t)$$

if $x(t) = x_1(t) + x_2(t)$ then

$$y(t) = \int [x_1(\tau) + x_2(\tau)]\,d\tau$$
$$= \int x_1(\tau)\,d\tau + \int x_2(\tau)\,d\tau = y_1(t) + y_2(t)$$

$\therefore \quad x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t) \qquad \boxed{2}$

Since $\boxed{1}$ and $\boxed{2}$ hold, superposition holds and the system is therefore linear

(3)

(a) $y(t) = h(t) * x(t)$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_0^t e^{-2\tau} \cdot \left[ \frac{e^{j\omega_0(t-\tau)} + e^{-j\omega_0(t-\tau)}}{2} \right] d\tau$$

$$= \frac{1}{2}\int_0^t e^{-2\tau} \cdot e^{j\omega_0 t} \cdot e^{-j\omega_0 \tau}\, d\tau + \frac{1}{2}\int_0^t e^{-2\tau} \cdot e^{-j\omega_0 t} \cdot e^{j\omega_0 \tau}\, d\tau$$

$$= \frac{e^{j\omega_0 t}}{2}\int_0^t e^{-(2+j\omega_0)\tau}\, d\tau + \frac{e^{-j\omega_0 t}}{2}\int_0^t e^{-(2-j\omega_0)\tau}\, d\tau$$

$$= \frac{e^{j\omega_0 t}}{2} \cdot \left.\frac{e^{-(2+j\omega_0)\tau}}{-(2+j\omega_0)}\right|_0^t + \frac{e^{-j\omega_0 t}}{2} \cdot \left.\frac{e^{-(2-j\omega_0)\tau}}{-(2-j\omega_0)}\right|$$

$$= \frac{e^{j\omega_0 t}}{-4-j2\omega_0}\left( e^{-(2+j\omega_0)t} - 1 \right) + \frac{e^{-j\omega_0 t}}{-4+j2\omega_0}\left( e^{-(2-j\omega_0)t} - 1 \right)$$

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(b) $x(t) = \cos \omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \frac{1}{2}\left(x_1(t) + x_2(t)\right)$

$$y(t) = \int h(\tau)\, x(t-\tau)\, d\tau$$

$$= \frac{1}{2}\int h(\tau)\, e^{j\omega_0 (t-\tau)}\, d\tau + \frac{1}{2}\int h(\tau)\, e^{-j\omega_0 (t-\tau)}\, d\tau$$

Consider just the 1st integral

$$\frac{1}{2}\int h(\tau)\, e^{j\omega_0 (t-\tau)}\, d\tau = \frac{1}{2}\int h(\tau)\, e^{j\omega_0 t}\, e^{-j\omega_0 \tau}\, d\tau$$

$$= e^{j\omega_0 t} \cdot \underbrace{\left(\frac{1}{2}\int h(\tau)\, e^{-j\omega_0 \tau}\, d\tau\right)}_{=\lambda_1}$$

$$= \lambda_1 e^{j\omega_0 t}$$

$$= \lambda_1 x_1(t)$$

Similarly, for the 2nd integral

$$\frac{1}{2}\int h(\tau)\, e^{-j\omega_0 (t-\tau)}\, d\tau = e^{-j\omega_0 t} \cdot \underbrace{\left(\frac{1}{2}\int h(\tau)\, e^{j\omega_0 \tau}\, d\tau\right)}_{=\lambda_2}$$

$$= \lambda_2 e^{-j\omega_0 t}$$

$$= \lambda_2 x_2(t)$$

So, if $x(t)$ is a weighted sum of complex exponentials, the output is a weighted sum of them.