

Fate & Transport Solution Set #2

CE/ESR 479/579

1. Chapter 1, Problem 17

I convert everything to ug, m, and s units:

$$Q = 200 \text{ gpm} - 756 \text{ L/min} = 12.6 \text{ L/s}$$

$$V = 1.5 \times 10^6 \text{ gal} = 5.67 \times 10^6 \text{ L}$$

$$d = 10 \text{ ft} / 3.25 \text{ ft/m} = 3.08 \text{ m}$$

$$A_s = V/d = (5670 \text{ m}^3) / (3.08 \text{ m}) = 1843 \text{ m}^2$$

$$MW = 133 \text{ ug/umol}$$

Mass balance: Inflows of TCA = Outflows of TCS and then solve for C_w , concentration in the waste:

$$Q_{in} C_w = m_{vol} + m_{bio} + Q_{out} C_{max}$$

$$C_w = \left[\begin{array}{l} (3 \times 10^{-4} \text{ ug cm}^{-2} \text{ s}^{-1})(1843 \text{ m}^2)(10^4 \text{ cm}^2 \text{ m}^{-2}) \\ + (6 \times 10^{-3} \text{ umol L}^{-1} \text{ d}^{-1})(5.67 \times 10^6 \text{ L})(133 \text{ ug umol}^{-1})(1 \text{ d} / 86,400 \text{ s}) \\ + (12.6 \text{ L s}^{-1})(10 \text{ ug s}^{-1}) \end{array} \right] / (12.6 \text{ L s}^{-1})$$

$$C_w = 450 \text{ ug/L}$$

2. Chapter 2, Problem 1 [3 pts total]

- a) [1 pts] The peak of the salt pulse travels at the mean velocity of the stream. It looks like peak arrives 20 m downstream (the sampling point) after about 750 s.

$$V = 20 \text{ m} / 750 \text{ s} = 0.027 \text{ m/s} = 2.7 \text{ cm/s} \text{ (a rather sluggish stream)}$$

- b) [2 pts] The curve is NOT expected to be exactly Gaussian because this time-series data rather than the spatial distribution. The cloud of salt continues to expand as it passes the sampling point so the part that arrives first will rise more quickly than the corresponding drop-off in the tail end of the cloud, which will appear to recede more gradually. However, since the skewness in the time distribution is not great here, we can assume the cloud is "frozen" as it passes and convert the C vs. t data to C vs. x data.

We want to get σ , which occurs at 0.61 times the peak C, which looks like 4600 micromhos.

$$0.61(4600) = 2800 \text{ micromhos}$$

Width of the distribution at the 2800 levels is about $500 \text{ s} = 2 \sigma$, so $\sigma = 250 \text{ s}$. Convert time interval to the width of the cloud by using the velocity. $\sigma = (250 \text{ s})(0.027 \text{ m/s}) = 6.75 \text{ m}$

$$D = \sigma^2 / 2t = (6.75 \text{ m})^2 / 2(750 \text{ s}) = 0.030 \text{ m}^2/\text{s} = 300 \text{ cm}^2/\text{s}$$

Notice this is over 10,000,000 times larger than molecular diffusion, even in this sluggish

stream.

3. Chapter 2, Problem 2 [4 pts total]

- a) [2 pts] Minimum flux would be purely *molecular diffusion*

$J = -D \frac{\partial C}{\partial z}$, where $D = 1 \times 10^{-9} \text{ m}^2/\text{s}$ and the vertical conc gradient is estimated from

Figure 2-7: DO drops from about 8 mg/L to zero in what looks to me like about 2 m, so:

$$J = -(1 \times 10^{-9} \text{ m}^2 / \text{s}) \left(\frac{8 \text{ mg} / \text{L}}{2 \text{ m}} \right) \left(\frac{1 \text{ L}}{10^{-3} \text{ m}^3} \right) = 4 \times 10^{-6} \text{ mg} / \text{s} - \text{m}^2 = 4.0 \text{ ng} / \text{s} - \text{m}^2$$

or about 0.35 mg/day-m². To put that in perspective, imagine a layer of water just below the thermocline that is 1 cm thick. One square meter of such a layer would contain 10,000 cm³ or 10 L of water. According to the above calculation, these 10 liters would receive only 0.35 mg of oxygen per day, or 0.035 mg/L per day. (By comparison the surface water has about 9 mg/L of DO). It's easy to see how microbial respiration in the hypolimnion rapidly depletes the DO because the supply of fresh DO is so slow.

- b) [1 pt] Actual flux is probably greater than this because the thermocline zone in reality is not 100% stagnant and there will always be a small amount of mechanical mixing across it. (For example if a fish swims through it or a boat propeller disturbs it).
- c) [1 pt] DO transport is not enough to keep up with respiration in the hypolimnion, as discussed above.