2.5 DISSOLVED OXYGEN MODELING IN SURFACE WATERS

Given that many organisms require oxygen for respiration, it is not surprising that the concentration of oxygen in a surface water is a critical attribute of the ecosystem. The suitability of natural waters for many types of organisms, including fish, is often characterized by the concentration of *dissolved oxygen* (DO). In a water body at equilibrium with the atmosphere, the concentration of DO is approximately 10 ppm; see Table 2-4 for oxygen solubility values as a function of temperature and salinity.

One of the first adverse effects of organic pollutants in surface waters to be recognized was the decrease in DO that resulted when sewage or other organic wastes were discharged to surface waters. In surface water, bacteria degrade and ultimately mineralize organic waste, consuming oxygen and releasing carbon dioxide in the process; see Eq. [2-52]. If oxygen is consumed at a rate that exceeds the replenishment rate of DO from the atmosphere or from photosynthesis, the DO concentration can decrease to less than a few milligrams per liter, and organisms such as fish suffocate. Therefore, reducing the severity of oxygen depletion in the vicinity of pollutant discharge points, such as sewage outfalls, was an early concern of environmental engineers, and models were devised to predict the amount of oxygen depletion that would occur under any given set of stream conditions and pollutant loading rates.

EXAMPLE 2-15

Upstream of a sewage outfall, a river contains 7 mg/liter DO. Some distance downstream of the outfall, however, DO has been diminished to 4 mg/liter due to organic waste decomposition by microbes. What is the approximate amount of organic matter ("CH_2O") that must have been degraded to account for this consumption of DO?

Neglecting O_2 diffusion into the water from the atmosphere and O_2 production by photosynthesis, and assuming no O_2 consumption by organisms other than the microbes, 3 mg/liter O_2 are consumed. This corresponds to

$$\frac{3\text{ mg }O_2}{1\text{ liter}}\cdot\frac{1\text{ mol }O_2}{32{,}000\text{ mg}}\cdot\frac{1\text{ mol }CH_2O}{1\text{ mol }O_2}\cdot\frac{30{,}000\text{ mg}}{1\text{ mol }CH_2O}=\frac{2.8\text{ mg }CH_2O}{1\text{ liter}}.$$

The degradation of 2.8 mg/liter of organic matter thus consumes 3 mg/liter of dissolved oxygen. Actually, some O_2 from the atmosphere will have dissolved into the stream, so 3 mg/liter is a minimum value for O_2 consumption.

Biochemical or *biological oxygen demand* (BOD) is a measure of the amount of oxygen required by bacteria to degrade the dissolved and suspended organic matter in a volume of water. Therefore, BOD is an indirect measure of the organic content of a water. [In ammonia-rich waters, some oxygen is also consumed by the oxidation of ammonia to nitrate in the process of *nitrification* (see Section 4.7.2).] Commonly, a 5-day test (BOD_5) is conducted, in which a water sample is fully aerated and then incubated over a 5-day period at 20°C. At the end of the test, the total amount of DO that has been consumed is measured. The duration and temperature of the test are historically based on the maximum travel time to the sea of some British rivers and the mean summer temperature of those rivers, respectively. BOD is expressed as milligrams of DO needed to oxidize the organic waste contained in 1 liter of water or wastewater (mg/L).

In the past, the primary goals of wastewater engineers were the lowering

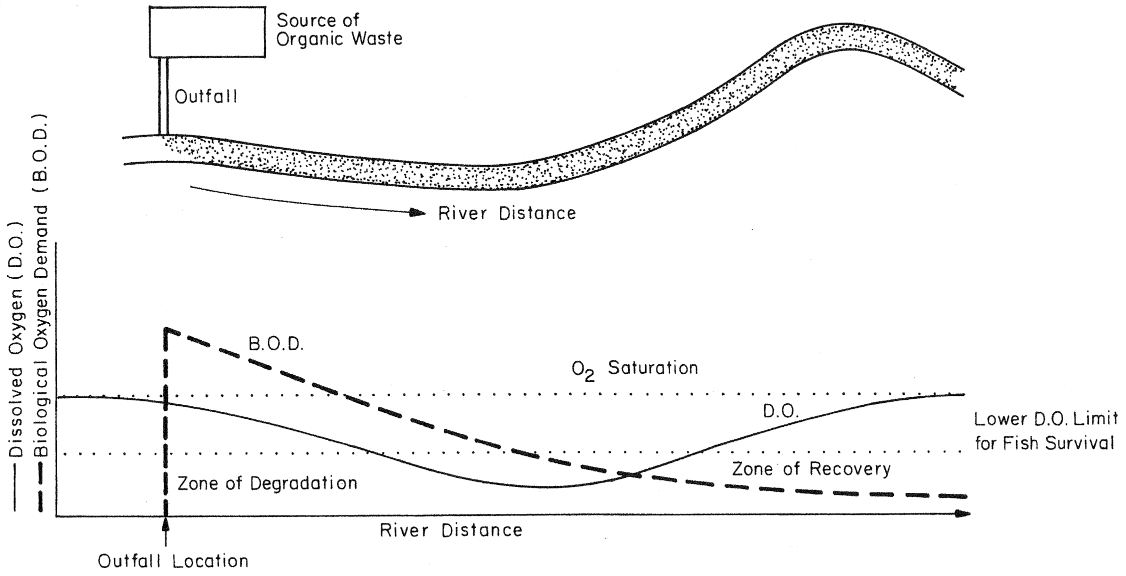

FIGURE 2-22 "DO sag" induced in a river by inputs of organic waste such as sewage. In the zone of degradation, oxygen is consumed more rapidly by biodegrading microorganisms than it can be replenished from the atmosphere. Sufficiently heavy loadings can cause oxygen concentrations to fall below the minimum required for many desirable forms of aquatic life (e.g., fish) or can even cause waters to become completely anoxic. Recovery begins downstream after much of the organic waste is degraded, and the river becomes reaerated.

of BOD to acceptable levels and the inactivation of disease-causing organisms *(pathogens)*. Later, more advanced techniques for the removal of nutrients as well as heavy metals and other pollutants in wastewater were developed, as more subtle effects of wastewater discharges on surface waters were discovered. In a river with a sewage outfall, a plot of DO versus distance downstream of the outfall (Fig. 2-22) is influenced by the DO consumption rate (which is maximum where BOD is maximum), the dissolution of oxygen from the atmosphere into the river, photosynthesis, and O_2 consumption in the bottom sediment *(benthic O_2 demand)*. The classic analysis of this situation is summarized in the *Streeter–Phelps* model, which is based on the mass balances of O_2 and BOD in a river or stream (Metcalf and Eddy, 1991).

The Streeter–Phelps model allows the estimation of the DO "sag" in the stream as a function of distance. This equation may be developed by considering a stream, initially saturated with oxygen, which receives a wastewater discharge containing BOD. As the water moves downstream, the BOD is assumed to decay at a first-order rate, K_{BOD}, typically on the order of 0.2/day:

$$\frac{d\,\mathrm{BOD}}{d\tau} = (-K_{\mathrm{BOD}})(\mathrm{BOD}), \qquad [2\text{-}60]$$

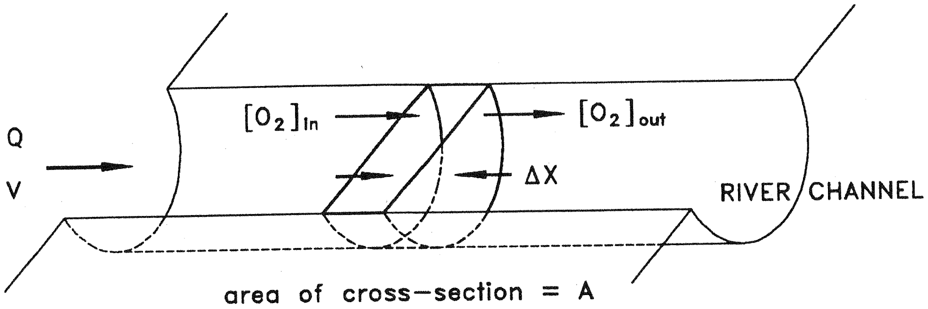

FIGURE 2-23 Definition of the control volume for which the DO mass balance is expressed in Eq. [2-62]. The control volume is a slice of thickness Δx and cross-sectional area A and is stationary in the river.

where τ is travel time. The solution to Eq. [2-60], comparable to the solution to Eq. [1-18] shown in Eq. [1-19], is

$$\mathrm{BOD} = \mathrm{BOD}_0 e^{-K_{\mathrm{BOD}}\tau}, \qquad [2\text{-}61]$$

where BOD_0 is the initial BOD at the location of the wastewater discharge.

To analyze the mass balance of DO, a useful control volume is a stationary segment of river Δx units long, as shown in Fig. 2-23. The steady-state mass conservation expression for oxygen in this slice is

$$\begin{aligned} &\text{Rate of } O_2 \text{ inflow} - \text{rate of } O_2 \text{ outflow} + \text{rate of } O_2 \text{ reaeration} \\ &\qquad - \text{rate of } O_2 \text{ consumption by BOD} = 0. \end{aligned} \qquad [2\text{-}62]$$

The first two terms of Eq. [2-62], advective inflow and outflow, can be written as

$Q c_i - Q c_o \approx Q \frac{\partial c}{\partial x} dx$

$$V \cdot A \cdot [O_2]_{\mathrm{in}} - V \cdot A \cdot [O_2]_{\mathrm{out}} \approx -V \cdot A \cdot \frac{d[O_2]}{dx} \cdot \Delta x, \qquad [2\text{-}63]$$

where V is river velocity [L/T], A is the cross-sectional area of the river [L^2], and Δx is the thickness of the river slice [L]. The flux density of oxygen from the atmosphere into the control volume, the third term in Eq. [2-62], is proportional to the O_2 deficit, i.e., the difference between the saturated O_2 concentration and the actual O_2 concentration, and the piston velocity, or gas exchange coefficient. Recall that the reaeration coefficient for oxygen is the piston velocity for oxygen divided by stream depth; thus, piston velocity equals the product of the reaeration coefficient and depth. Total flux of atmospheric O_2 into the control volume is

$$\{\text{Surface area}\} \cdot \{\text{piston velocity}\} \cdot \{O_2 \text{ deficit}\},$$

or

$$\{(\Delta x)(\text{width})\} \cdot \{(K_{O_2})(\text{depth})\} \cdot \{[O_2]_{\mathrm{sat}} - [O_2]\},$$

which equals

$$K_{O_2} \cdot ([O_2]_{\text{sat}} - [O_2])(A)(\Delta x), \qquad [2\text{-}64]$$

where K_{O_2} is the reaeration coefficient $[T^{-1}]$, $[O_2]_{\text{sat}}$ is the saturated oxygen concentration $[M/L^3]$, and $[O_2]$ is the actual oxygen concentration $[M/L^3]$.

The fourth term in Eq. [2-62], the rate of O_2 consumption by BOD, is equal to the BOD decay rate, $K_{\text{BOD}} \cdot \text{BOD}$, multiplied by the volume of water in the slice, $A\,\Delta x$. Thus, Eq. [2-62] can be rewritten as

$$\begin{aligned} -V \cdot A \cdot \frac{d[O_2]}{dx}\,\Delta x + K_{O_2} \cdot ([O_2]_{\text{sat}} - [O_2]) \cdot A\,\Delta x& \qquad [2\text{-}65] \\ - K_{\text{BOD}} \cdot \text{BOD} \cdot A\,\Delta x = 0.& \end{aligned}$$

Gov. Eqn

The corresponding differential equation is

$$-V\frac{d[O_2]}{dx} + K_{O_2} \cdot ([O_2]_{\text{sat}} - [O_2]) = K_{\text{BOD}} \cdot \text{BOD}. \qquad [2\text{-}66]$$

For the upstream conditions of BOD_0 and $[O_2]_{\text{sat}}$, Eq. [2-66] has the solution:

SOLVE

$$[O_2]_{\text{sat}} - [O_2] = \frac{K_{\text{BOD}} \cdot \text{BOD}_0}{K_{O_2} - K_{\text{BOD}}}\left(e^{-K_{\text{BOD}}\tau} - e^{-K_{O_2}\tau}\right). \qquad [2\text{-}67]$$

The minimum DO occurs at $\tau_{\max}$, the travel time at which the derivative of the preceding equation is zero:

$$\tau_{\max} = \frac{1}{K_{O_2} - K_{\text{BOD}}} \ln \frac{K_{O_2}}{K_{\text{BOD}}}. \qquad [2\text{-}68]$$

The minimum $[O_2]$ value, $[O_2]_{\min}$, is given by

$$[O_2]_{\text{sat}} - [O_2]_{\min} = \frac{K_{\text{BOD}}}{K_{O_2}} \text{BOD}_0 e^{-K_{\text{BOD}}\tau_{\max}}. \qquad [2\text{-}69]$$

This special case of the Streeter–Phelps model neglects photosynthesis, as well as oxygen consumption by the bottom sediments and by nitrification (see Section 4.7.2).

EXAMPLE 2-16

A stream is in equilibrium with atmospheric oxygen upstream of a waste outfall, which creates a BOD_0 of 20 mg/liter immediately downstream. K_{BOD} is 0.4/day, and K_{O_2} is 1.4/day. The stream temperature is 15°C. How far downstream, in terms of travel time, is the maximum DO sag, and what is the minimum DO in the river?

The travel time to the maximum DO sag can be estimated by Eq. [2-68]:

$$\tau_{\max} = \left(\frac{1}{1.4/\text{day} - 0.4/\text{day}}\right) \ln\left(\frac{1.4/\text{day}}{0.4/\text{day}}\right)$$

$$\tau_{\max} = (1\ \text{day})(1.25) = 1.25\ \text{days}.$$

The maximum O_2 sag, or deficit, which occurs at the location corresponding to the preceding travel time, is given by Eq. [2-69]:

$$[O_2]_{\text{sat}} - [O_2]_{\min} = \frac{0.4/\text{day}}{1.4/\text{day}} \cdot 20\ \text{mg/liter} \cdot e^{-(0.4/\text{day})(1.25\ \text{day})} \approx 3.5\ \text{mg/liter}.$$

From Table 2-4, $[O_2]_{\text{sat}}$ is approximately 10 mg/liter at 15°C. Therefore, $[O_2]_{\min}$ is approximately 6.5 mg/liter, which is probably enough oxygen to keep some hardy fish alive.

An alternative to BOD_5 is a test known as *chemical oxygen demand* (COD), which also measures the milligrams of oxygen needed to oxidize organic waste in 1 liter of water. This test takes only a few hours to perform on a water sample, but does not reflect natural conditions because a strong chemical oxidant is added to the water sample to cause the oxidation of organic matter. The relationship between BOD and COD varies from water body to water body.