HOW MANY DIMENSIONS? (t)

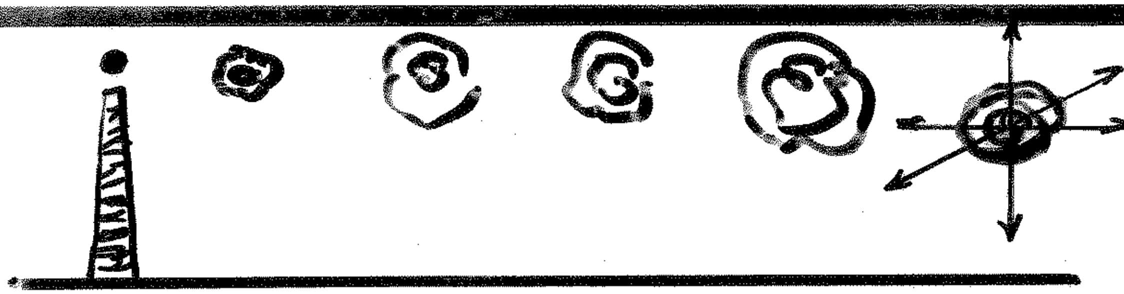
1D? 2D? 3D? steady?

Depends on

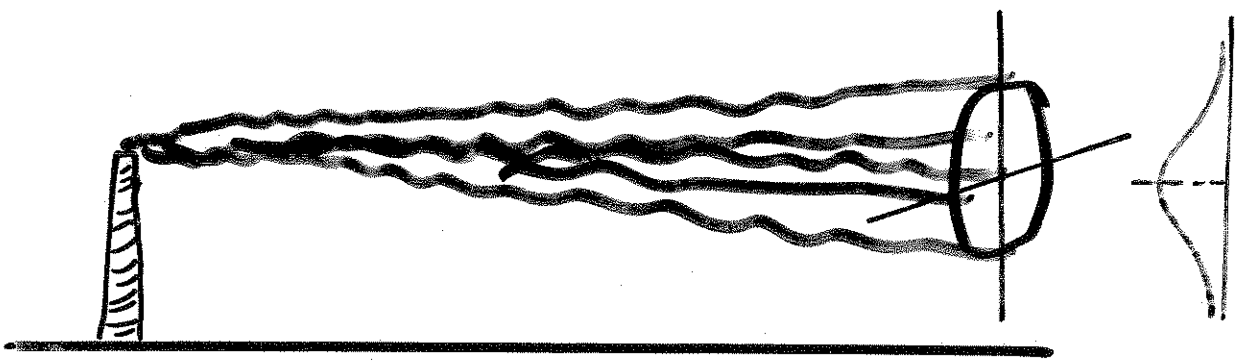
1. Geometry of system

* 2. Geometry of SOURCE

Ex. 1

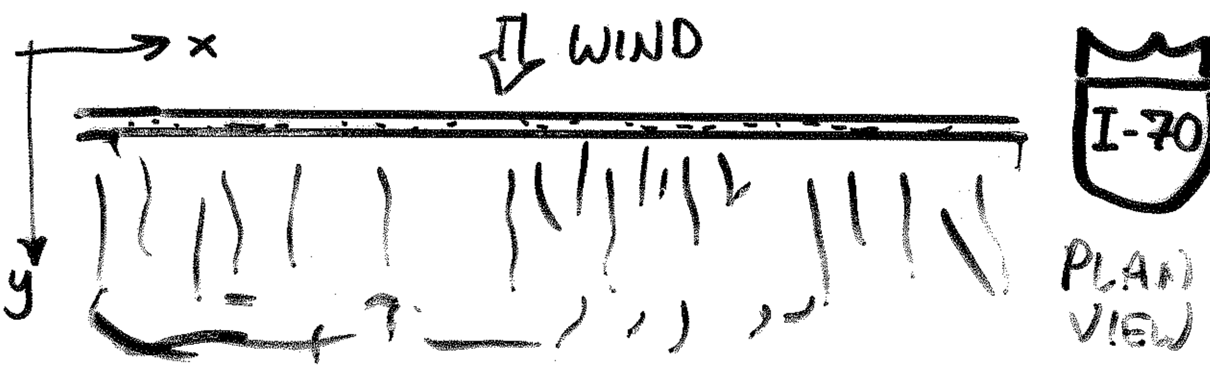


Ex. 2



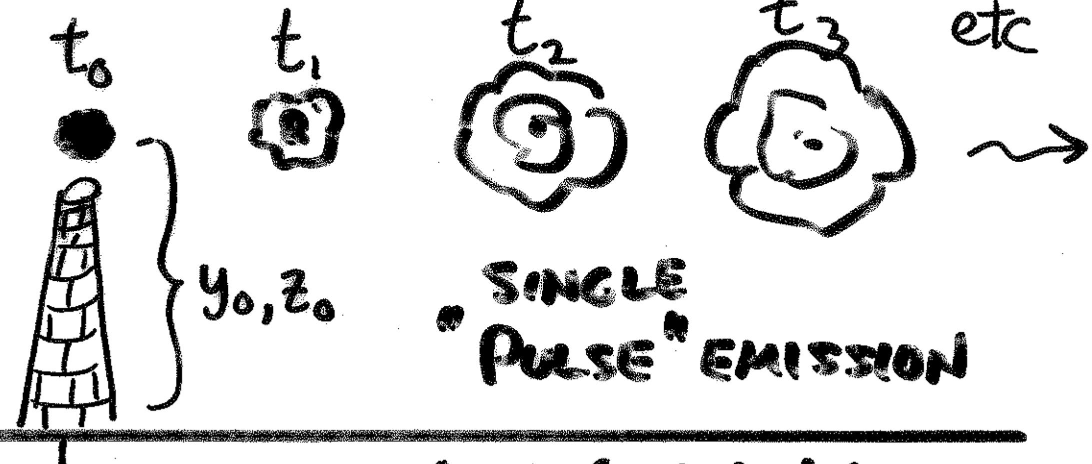
Ex. 3

Ex. 4



SPECIFIED - CONCENTRATION CONDITIONS (Initial or Boundary)

Initial:



x_0

At t_0 (Initial)

At (x_0, y_0, z_0)

$C = C_0$ OR $C = "\infty"$

And At $(x, y, z) \neq (x_0, y_0, z_0)$

$C = 0$

OR...

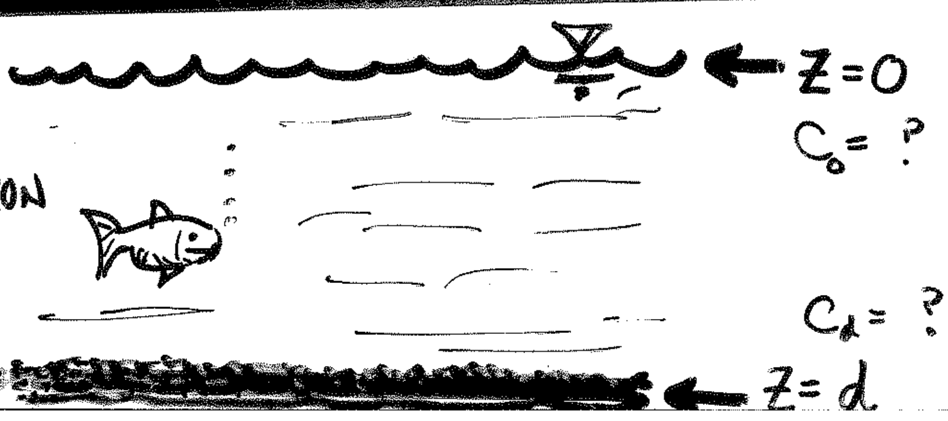
CONTINUOUS EMISSION

$C = C_0$

At $x_0 = 0$

$t =$ All $t > t_0$

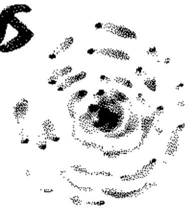
Boundary:



- RAPID EQUILIBRATION WITH AIR AT TOP, ANOXIC BOTTOM

"Method of Sources"

- **INSTANTANEOUS POINT SOURCE**



  - Infinite fluid (no boundaries)
  - Uniform, steady flow field
    - ↳ Velocity only in one direction
    - Velocity NOT a fcn of time
    - Turbulence probably UNIFORM (Homogeneous)

Governing Equation:

- $$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = E_x\frac{\partial^2 c}{\partial x^2} + E_y\frac{\partial^2 c}{\partial y^2} + E_z\frac{\partial^2 c}{\partial z^2} + kc$$

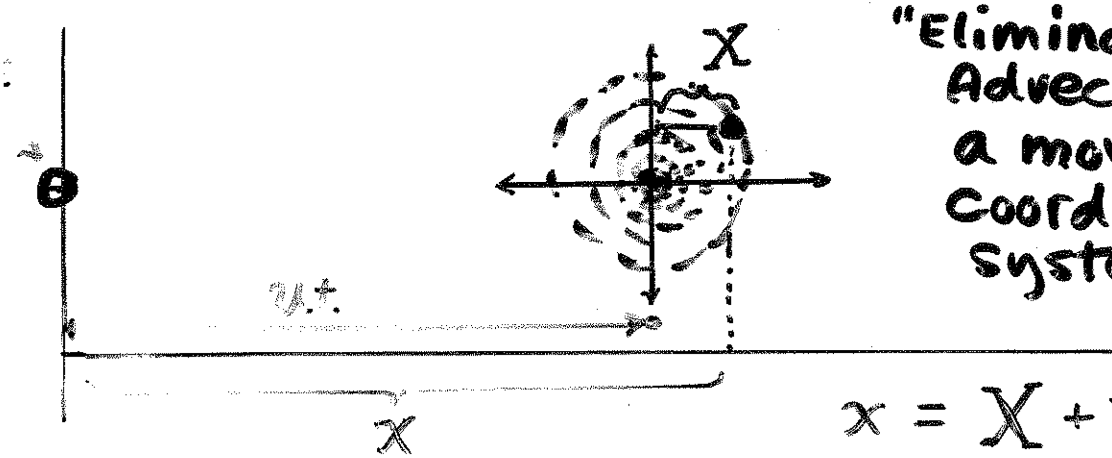
[illegible]

ADVECTION ONLY IN ONE DIRECTION

→ MIXING IS HOMOGENEOUS BUT STILL ALLOWED TO BE ANISOTROPIC

→ MIXING OCCURS IN ALL 3-D

[illegible]



"Eliminate" Advection with a moving coordinate system

$$x = X + Ut$$

SIMPLIFIED GOVERNING EQN:

- $$\frac{\partial \phi}{\partial t} = E_x \frac{\partial^2 \phi}{\partial \underline{X}^2} + E_y \frac{\partial^2 \phi}{\partial y} + E_z \frac{\partial^2 \phi}{\partial z}$$

STILL NOT STEADY but no explicit advection

"x"-direction now expressed w/r/t a moving frame of reference

y & z AXES are unchanged

No explicit Decay term

- **Find solution to this eqn., then go in and substitute to get solution in terms of original variables**

$$\left\{ \begin{array}{l} \phi = C e^{Kt} \quad \left(= \dfrac{C}{e^{-Kt}} \right) \\ X = x - Ut \end{array} \right.$$

$$C = \frac{M}{(4\pi t)^{\frac{3}{2}} (E_x E_y E_z)^{\frac{1}{2}}} \exp - \left\{ \frac{[(x - x_1) - Ut]^2}{4E_x t} + \frac{(y - y_1)^2}{4E_y t} + \frac{(z - z_1)^2}{4E_z t} + Kt \right\}$$

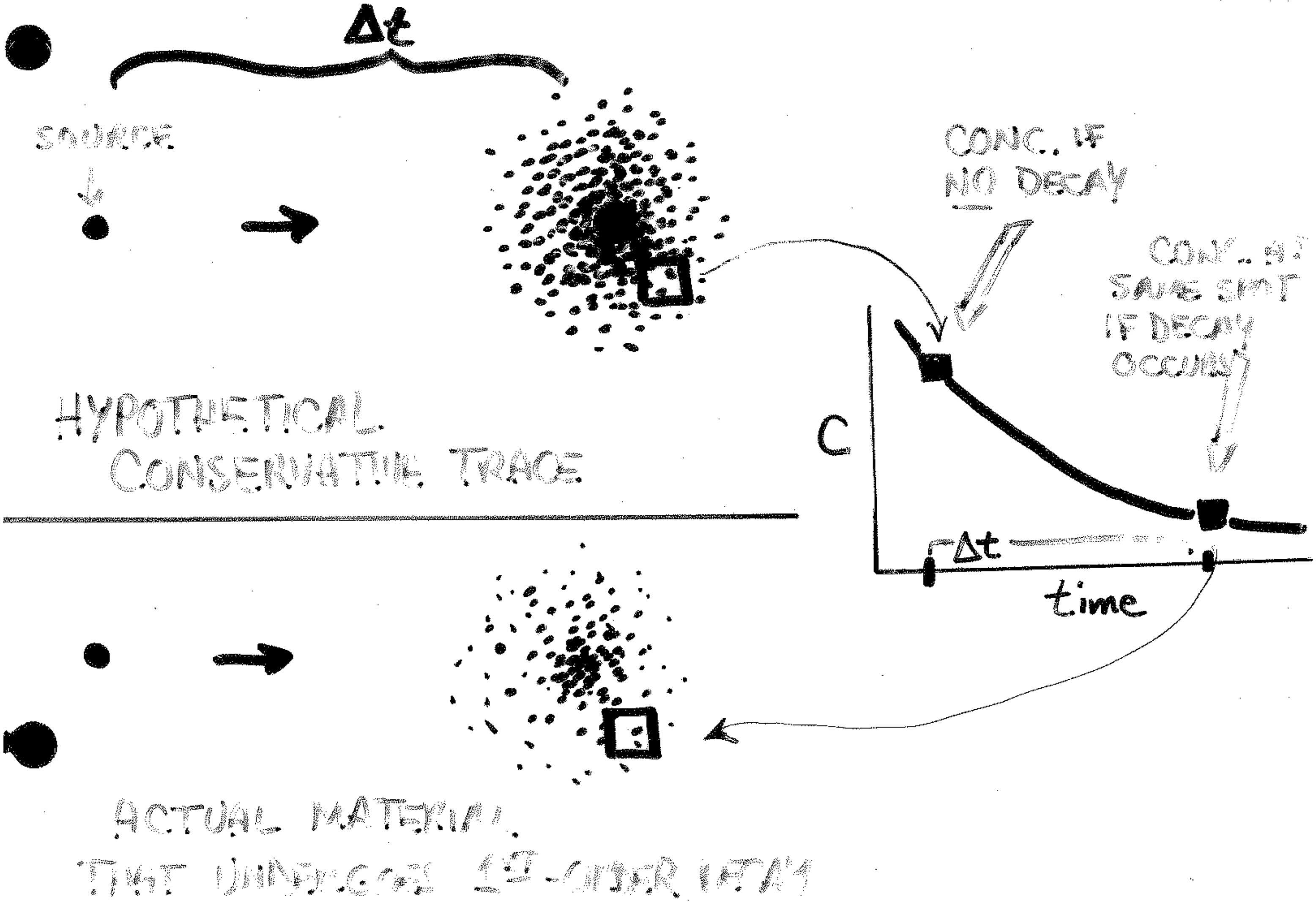
Mass M released at the point (x_1, y_1, z_1); Anisotropic

- **Or...**

$$C = \frac{M}{(4\pi D t)^{3/2}} \exp - \left[\frac{r^2}{4Dt} \right]$$

Mass M diffusing in a stagnant (isotropic) fluid.

LIKEWISE, "Eliminate" the DECAY TERM...



FIRST-ORDER DECAY

RATE OF DECAY $\equiv \dfrac{\partial C}{\partial t} = -KC$

$K \equiv$ UNITS OF $\dfrac{1}{\text{TIME}}$ (s^{-1})

$$\boxed{C = C_0 e^{-Kt}}$$

EXPONENTIAL DECAY

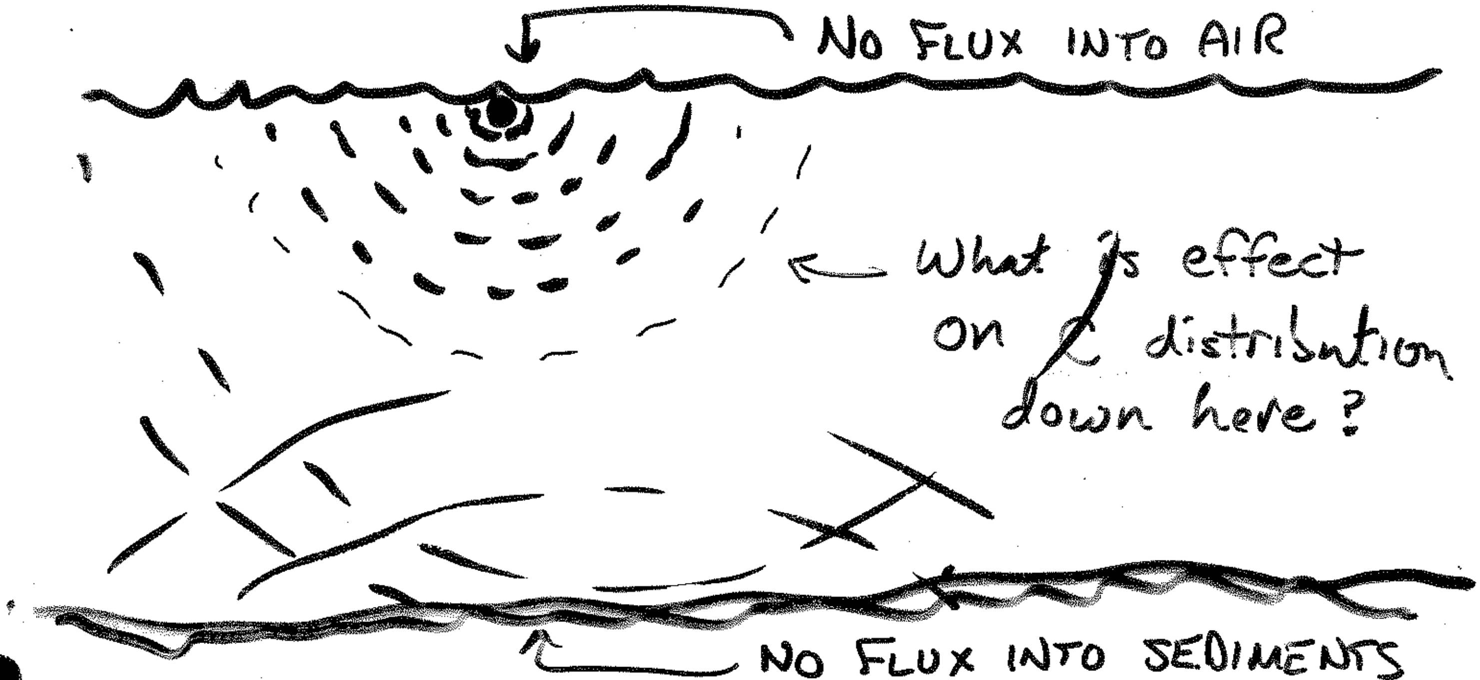
DEFINE ϕ AS THE CONC. OF HYPOTHETICAL NON-DECAYING TRACER.

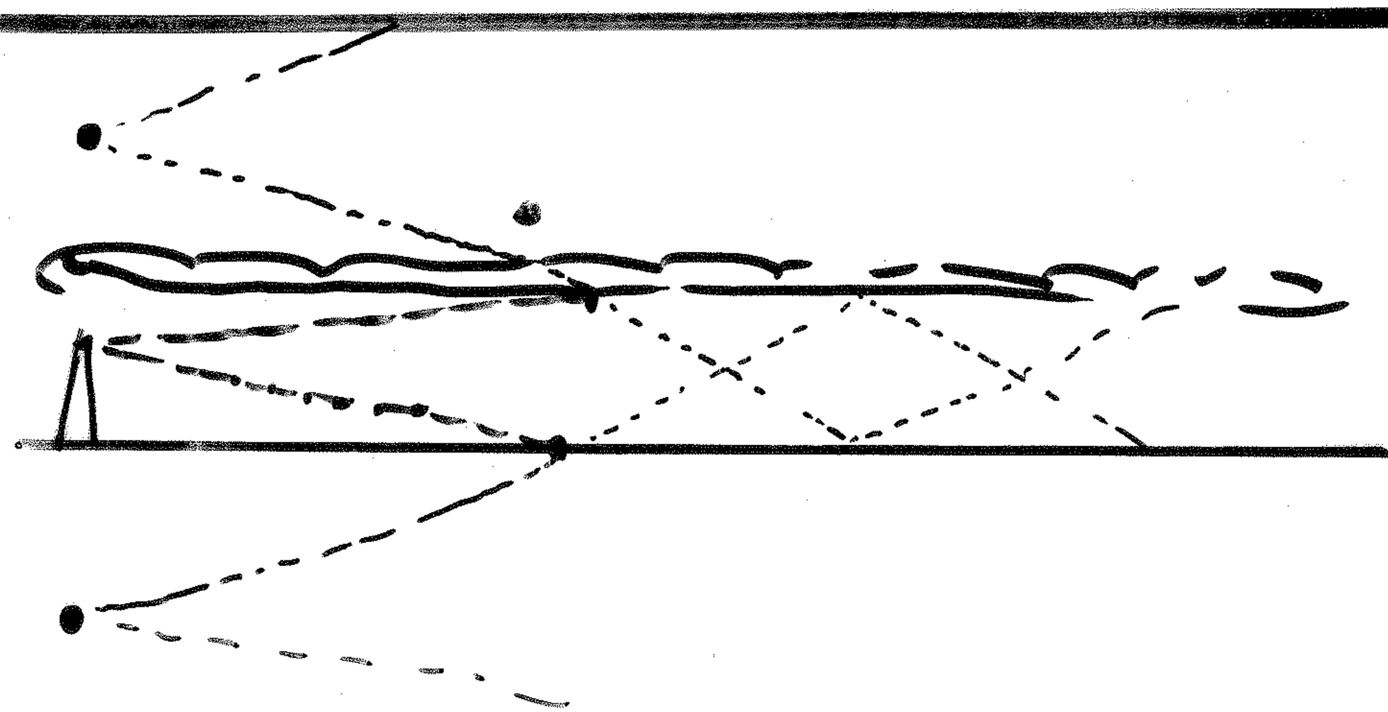
$$C = \phi e^{-Kt}$$

So, end up with a simpler governing equation.

"HYPOTHETICAL" STATIONARY, NONDECAYING, DIFFUSING PRT...

HANDLING NO-FLUX BOUNDARIES
"IMAGE" SOURCES





For a conservative tracer w/ no advection

- released RIGHT ON BOUNDARY

SIMPLY
DOUBLE the predicted conc. at each pt.

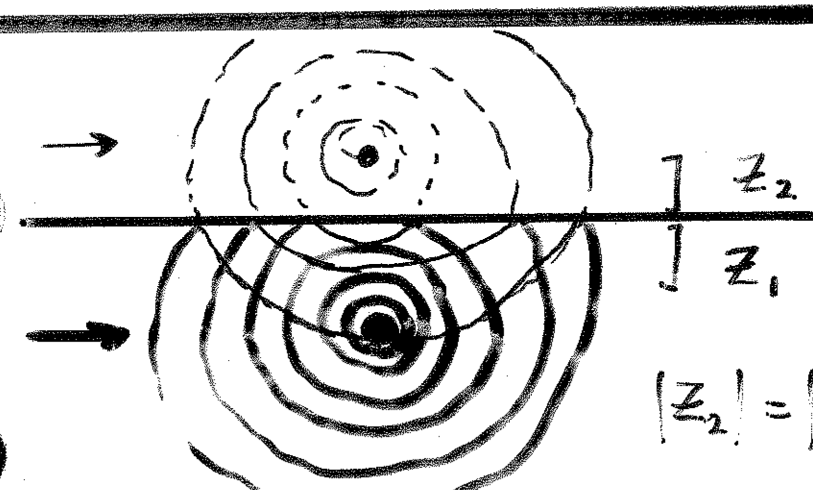
$$C = \frac{\boxed{2}M}{\rho(4\pi t)(E_xE_yE_z)^{\frac{1}{2}}} \exp -\left\{\frac{(x-x_1)^2}{4E_xt} + \frac{(y-y_1)^2}{4E_xt} + \frac{(z-z_1)^2}{4E_xt}\right\}$$

If nonconservative, just add in the $-kt$ term

- If advection, just put back the "$-ut$" term

IMAGE SOURCE (x_2, y_2, z_2)

TRUE SOURCE (x_1, y_1, z_1)



$|z_2| = |z_1|$

$x_1 = x_2 \quad y_2 = y_1$

- $$C = \left(\frac{M}{\rho(4\pi t)^{\frac{3}{2}}(E_xE_yE_z)^{\frac{1}{2}}} \exp - \left[\text{"}x_1\, y_1\, z_1\text{"}\right]\right) + \left(\frac{M}{\rho(\ldots)(\ldots)^{\frac{1}{2}}} \exp - \left[\text{"}x_2\, y_2\, z_2\text{"}\right]\right)$$

$$C = \frac{M}{\ldots}\left[\exp -\{\mathrm{I}\} + \exp -\{\mathrm{II}\}\right]$$

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