Reading I.3

Excerpt from:

"Dynamic Hydrology"

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On Last page I prepared a Summary of the SYMBOLS & NOTATION Used in this chapter

3-3 RADIATION PHYSICS

The transport of thermal energy by the processes of conduction and convection has been shown above to be dependent upon the temperature *gradient* and is essentially independent of the absolute temperature. Thermal radiation, it may be recalled, is proportional to the *absolute temperature* of the radiator, however. We can thus generalize that at low temperatures the conduction-convection processes are of primary importance, while at high temperatures radiation predominates. At the temperature of the sun, 6000°K, radiation predominates, while at the earth's surface, 287°K, all processes are significant.

Radiation of thermal energy from the sun is important because some of this energy is transmitted by absorption to the atmosphere and to the earth. The process by which this is accomplished is by no means clean and simple, however, since the absorption of radiated solar energy by the atmosphere is selective with respect to wavelength, and since the wavelengths present in the spectrum of radiated energy depend upon the temperature of the radiator. That is, the wavelengths of the energy reradiated from earth are different from those of the incident solar energy. In addition, reflections of energy take place, producing a scattering which is also selective and which depends upon the relative size of the reflecting body and upon the wavelengths of the incident energy.

In order that we may get an overall feeling for what is happening, it is well to review some of the basic considerations of *thermal radiation*, the name which we shall use to describe radiant energy emitted as a result of the temperature of a body. The intensity of radiation is described in terms of a *power density* or energy received per unit time per unit area of surface. Meteorologists commonly use the units cal cm^{-2} min^{-1}. Engineers use Btu ft^{-2} hr^{-1}.

Kirchhoff's law

Two small bodies having surface areas A_1 and A_2 are placed in a large box which has been evacuated and is perfectly insulated externally. When the box and its contents have reached a common temperature (thermal equilibrium), the two bodies will be emitting thermal radiation at the respective total rates A_1W_1 and A_2W_2, where W_n represents the intensity of the radiation. Let the intensity of radiation received by the bodies from the box be W_B, of which they each absorb a fraction given by their respective *absorptivities* a_1 and a_2. Conservation of energy then requires

$$W_BA_1a_1 = W_1A_1 \qquad \text{and} \qquad W_BA_2a_2 = W_2A_2 \tag{3-40}$$

Dividing both equations by the respective body areas, solving for W_B, and equating, we find

$$\frac{W_1}{a_1} = \frac{W_2}{a_2} = \frac{W_n}{a_n} = W_B \tag{3-41}$$

Equation (3-41) is Kirchhoff's law and tells us that at thermal equilibrium the ratio of radiation intensity (often called *emissive power*, *emittance*, or *radiant-flux density*) to absorptivity is the same for all bodies. The upper limit to W_n as indicated by this law occurs when a_n has the value unity, i.e., when all incident energy is absorbed. Such a body is called a perfect radiator or a *blackbody*.

The ratio of emissive power of a surface to that of a blackbody is called the *emissivity* E of the surface and can be seen from Kirchhoff's law to equal the absorptivity under conditions of thermal equilibrium.

Planck's law

The spectral distribution of thermal radiation is given by Planck's distribution function $W_{B\lambda}$ for a blackbody. $W_{B\lambda}$ is the monochromatic emissive power at the wavelength λ, and is defined in terms of the spectrum such that $W_{B\lambda}\,d\lambda$ is the energy emitted from a surface through a hemispherical angle per unit time per unit area in the wavelength interval (bandwidth) λ to $\lambda + d\lambda$. Planck's law may be written

$$\frac{W_{B\lambda}}{T^5} = f(\lambda T) = \frac{2\pi hc^2\lambda^{-5}T^{-5}}{e^{ch/k\lambda T} - 1} \tag{3-42}$$

where c = velocity of light = 2.998×10^{10} cm sec^{-1}
h = Planck's constant = 6.624×10^{-27} erg-sec
k = Boltzmann's constant = 1.380×10^{-16} erg °K^{-1}
λ = wavelength, cm

and where

$$\int_0^\infty W_{B\lambda}\, d\lambda \equiv W_B \tag{3-43}$$

The energy spectrum for thermal radiation from a blackbody is given in Fig. 3-5, as reproduced from McAdams [5].

A *gray body* is one which, at a given temperature, emits a fixed proportion of the blackbody radiation at that temperature in all wavelengths.

From Fig. 3-5 the monochromatic emissive power can be seen to be zero at $\lambda = 0$ and $\lambda = \infty$ and to pass through a maximum at some intermediate wavelength, the value of which is dependent upon the absolute temperature of the radiator and is defined by the Wien displacement law to be

$$(\lambda T)_{max} = 0.2898 \text{ cm °K} \tag{3-44}$$

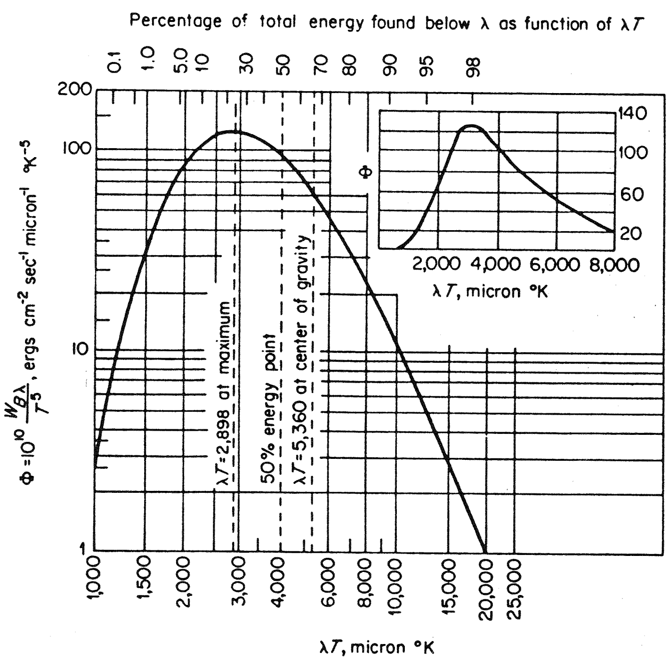

Fig. 3-5 Spectrum of thermal radiation from a blackbody. (*By permission from W. H. McAdams, "Heat Transmission," 3d ed., McGraw-Hill Book Company, New York,* 1954.)

Stefan-Boltzmann law

By introducing a change of variable and dividing both sides by T^4, Eq. (3-43) can be rewritten

$$\frac{W_B}{T^4} = \int_0^\infty \frac{W_{B\lambda}}{T^5}\, d(\lambda T) = \sigma \tag{3-45}$$

This is known as the Stefan-Boltzmann law[1] and demonstrates the direct proportionality (also demonstrable through the second law of thermodynamics) between the emissive power of a blackbody and the fourth power of the absolute temperature. By using Eq. (3-42) the constant σ can be evaluated as

$$\sigma = 0.826 \times 10^{-10} \text{ cal cm}^{-2} \text{ min}^{-1} \text{ °K}^{-4}$$

or

$$\sigma = 0.1713 \times 10^{-8} \text{ Btu ft}^{-2} \text{ hr}^{-1} \text{ °R}^{-4}$$

The unit cal cm^{-2} is known by meteorologists as the *langley* (ly).

Radiation spectrums for the sun and earth

Considering the sun and the earth as blackbodies radiating at 6000°K and at 287°K, respectively, we can use Fig. 3-5 to determine their individual spectrums. These are shown in Fig. 3-6 and represent the radiation unaffected by the presence of the earth's atmosphere. Note that there is a factor of 10^6 between the separate scales used to plot the solar and terrestrial radiation. Note also the Wien[1] displacement of the two spectrums because of the difference in body temperature of the earth and the sun. From Eq. (3-45) the total energy radiated per unit time and area is found to be

1. At surface of sun (T = 6000°K): W_B = 107,050 ly min^{-1}
2. At surface of earth (T = 287°K): W_B = 0.56 ly min^{-1}
3. At snow surface (T = 273°K = 32°F): W_B = 0.459 ly min^{-1}

Of course, only a very small fraction of that energy radiated by the sun is intercepted by the earth and its atmosphere. The average intensity of solar radiation received on a plane unit area normal to the incident radiation at the outer limit of the earth's atmosphere is referred to as the *solar constant.* This constant, the exact magnitude of which is subject to continuing revision in the light of measurements from artificial satel-

[1] Stefan determined this proportionality experimentally in 1879 and Boltzmann theoretically in 1894, while Wien found the displacement of spectral peaks theoretically in 1893. Planck's semiempirical law followed from quantum theory in 1900.

lites, is given the symbol W_{B0} and is generally accepted to have the value

$$W_{B0} = \text{solar constant} = 2.00 \text{ cal cm}^{-2} \text{ min}^{-1}$$

Hydrologists are primarily concerned with the intensity of direct solar radiation incident upon a horizontal surface, however. This quantity is called *insolation*. The insolation I_0 at the outer limit of earth's atmosphere is given by

$$I_0 = W_{B0} \sin \alpha \tag{3-46}$$

in which α is the angle of the radiation with the horizontal and is given, from spherical trigonometry, by

$$\sin \alpha = \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \tau \tag{3-47}$$

where δ is the declination, τ the sun's hour angle, and Φ is the local latitude. [See Appendix B for definitions and the derivation of Eq. (3-47).]

The daily amount of insolation received at the outer edge of the earth's atmosphere at some geographical location is readily determined from Eqs. (3-46) and (3-47) by utilizing the time variation of δ and τ. The results of this calculation as given by List [6] are shown in Fig. 3-7.

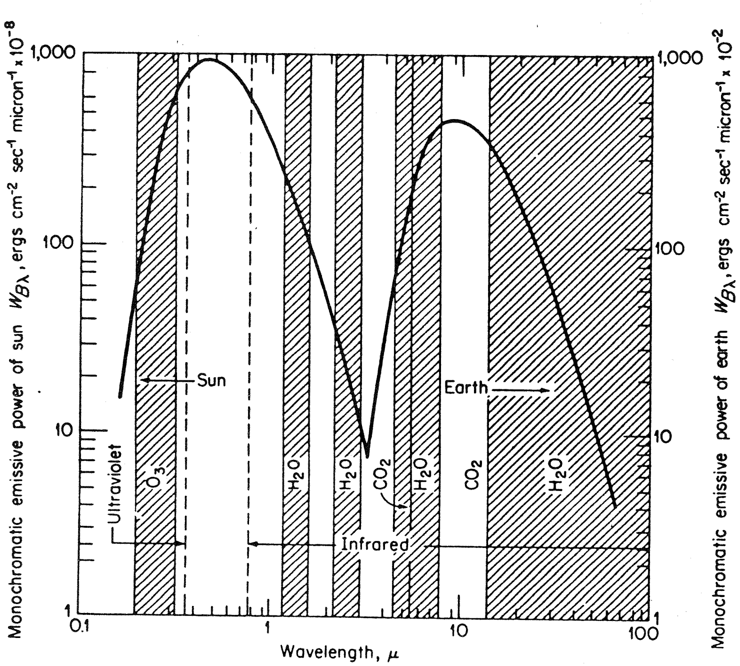

Fig. 3-6 Spectrums of thermal radiation by the sun and the earth with primary absorption bands. Shaded areas indicate primary bands of absorption by atmospheric constituents; solar and terrestrial radiation are shown to different scales.

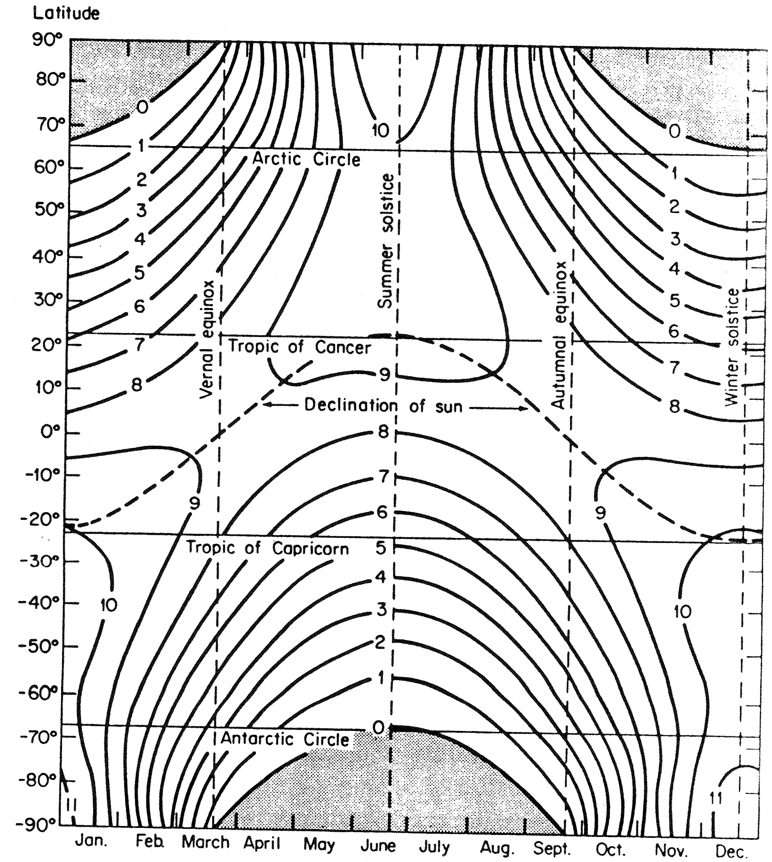

Fig. 3-7 Daily insolation amounts outside the earth's atmosphere (in hundreds of langleys). (*By permission from "Smithsonian Meteorological Tables," 6th rev. ed., prepared by R. J. List, Smithsonian Institution, Washington,* 1951.)

Effects of the atmosphere and the earth's surface

The spectrum of solar radiation shown in Fig. 3-6 is not the spectrum, either in intensity or in distribution, to be expected at the earth's surface. Radiation traversing the atmosphere suffers losses due to reflection, scattering, and absorption.

Molecular scattering is a selective refraction process which is roughly proportional to λ^{-4}. It can be evaluated analytically with considerable accuracy [7]. The blue of a clear sky is thought by many to result from a scattering of the short wavelengths (violet) by the air molecules.

Particulate scattering by macroscopic particles such as dust and water droplets (i.e., fog, haze, and "smog") is also a selective refraction process but is difficult to calculate. It is negligible for wavelengths much greater than the droplet size, becomes a maximum when the two are equal, and recedes to a lower asymptote as the wavelength becomes much smaller than the droplet. When seen through a cloud of large particles such as smoke or dust, things may appear red because the blue light has been reflected and the red light is being scattered.

Reflection of shortwave energy takes place from clouds as well, of course, as from sheets of snow, ice, and water and from surfaces of sand, soil, and vegetation. This phenomenon is effective for all wavelengths less than the particle size and is hence essentially nonselective. The reflection coefficient, reflected energy/incident energy, is commonly given the name *albedo*. Values of the albedo for various surfaces will be presented later. The albedo of clouds accounts for their pure white appearance when viewed in reflected sunlight, the water particles being larger than all significant wavelengths in the solar spectrum. Dark black clouds, on the other hand, merely indicate a lack of illumination (shadow) caused by the highly reflective clouds passing between the sun and the observer.

Molecular absorption is highly selective and thus is confined to lines (i.e., discrete frequencies) or to narrow spectral bands. It arises because of the excitation of the molecules of the atmospheric constituents in particular resonant modes, the latter being critically dependent upon frequency. The selectivity of this absorption with respect to wavelength, combined with the Wien displacement of the two spectrums, is what permits (in fact accounts for) life as we know it on this planet. Superimposed on Fig. 3-6 are the primary bands within which a portion of the radiation is absorbed by one or more of the atmospheric constituents. Their effect is to cause the atmosphere to be, in general, transparent to the shortwave radiation of the sun but also, largely because of water vapor and carbon dioxide, to cause it to absorb a large portion of the energy reradiated from the earth. This *greenhouse effect* serves to keep the earth warm and, as we shall soon see, to provide the energy for the circulation of the atmosphere and the oceans. The absorption bands shown represent only a few of those currently recognized. Goldberg [8] lists 127 bands in the spectrum between 0.3 and 24 μ.

Beer's law

The above atmospheric effects are approached analytically by considering the transfer of monochromatic radiation through a thin layer of effective thickness Δs. We shall first neglect the emission of radiation by the layer at the given wavelength due to its own thermal state. Using the definition sketch of Fig. 3-8 and the absorptivity a_λ, we write, for the

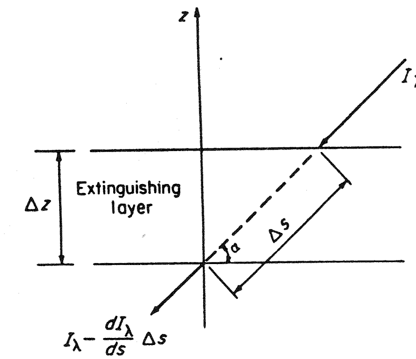

Fig. 3-8 Definition sketch for radiation extinction.

local monochromatic radiation intensity I_λ,

$$dI_\lambda = -a_\lambda I_\lambda = -\rho k_{\lambda a}\, ds\, I_\lambda \tag{3-48}$$

where ρ = mass density of medium
$k_{\lambda a}$ = its *absorption coefficient*

Integrating Eq. (3-48) over the distance s,

$$\frac{I_{\lambda s}}{I_{\lambda 0}} = \exp\left(-\int_0^s k_{\lambda a}\rho\, ds\right) \tag{3-49}$$

Equations (3-48) and (3-49) are forms of Beer's law for absorption. They take on the same form for extinction by scattering, but the coefficient $k_{\lambda s}$ is then a *scattering coefficient* with a different wavelength dependence.

When reemission from the layer is not negligible (in the infrared region, for example), we have *Schwarzschild's equation* of radiative transfer

$$dI_\lambda = a_\lambda(W_{B\lambda} - I_\lambda) \tag{3-50}$$

3-4 SHORTWAVE RADIATION

Cloudless sky

Because of the complex structure of the absorption spectrums of water and carbon dioxide, evaluation of the integral in Eq. (3-49) is very difficult. Similar problems arise for the corresponding integral in the case of scattering. Although Elasser [9] has presented a simplified rational method for handling the absorption problem, scattering accounts for the major loss of shortwave radiation, and we must approach this problem as follows:

Considering only scattering by the air molecules, we can integrate

Eq. (3-49) over the entire spectrum to obtain

$$\frac{I}{I_0} = \frac{\int_0^\infty I_{\lambda s}\, d\lambda}{\int_0^\infty I_{\lambda 0}\, d\lambda} = \exp\left(-\int_0^\infty k\rho\, ds\right) \tag{3-51}$$

and can approximate the integral by

$$\int_0^\infty k\rho\, ds = a_1 m \tag{3-52}$$

where m = relative thickness of air mass = cosecant of solar altitude α, when atmospheric pressure is 760 mm mercury.

a_1 = molecular scattering coefficient = $0.128 - 0.054 \log m$

Using a similar approach for absorption (a_2) and for particulate scattering (a_3), we have

$$\frac{I_c}{I_0} = \exp\left[-(a_1 + a_2 + a_3)\right] = \exp(-a_t m) \tag{3-53}$$

or

$$\frac{I_c}{I_0} = \exp(-n a_1 m) \tag{3-54}$$

where $n = a_t/a_1$ = *turbidity factor* of the air, which varies from about 2.0 for clear mountain air to as high as 4 or 5 for smoggy urban areas.

I_c = direct, cloudless sky insolation at earth's surface
I_0 = insolation at outer limit of earth's atmosphere

Cloudy sky

In the presence of clouds the surface insolation will be further reduced from the value I_c given by Eq. (3-53). In addition, the clouds will cause there to be significant diffuse shortwave sky radiation. The sum I'_s of the direct and diffuse radiation may be estimated [10] from an equation of the form

$$\frac{I'_s}{I_c} = 1 - (1 - k)N \tag{3-55}$$

where N is the fraction of the sky obscured by clouds (i.e., for an overcast sky $N = 1$), and I_c is given by Eq. (3-54). The coefficient k is the fraction of cloudless-sky insolation received on a day with overcast skies and varies with the altitude of the clouds according to the empirical relation [11]

$$k = 0.18 + 0.024z \tag{3-56}$$

where z is the cloud-base altitude in thousands of feet. Using a value $k = 0.22$ [which corresponds to a 1,700-ft cloud base using Eq. (3-56)],

TABLE 3-2 Solar radiation on a horizontal surface under average atmospheric conditions

Solar altitude α *deg*	*Average total insolation* I'_s *ly hr*$^{-1}$
5	3.6
10	9.7
15	17.2
20	25.0
25	32.8
30	40.6
35	47.7
40	54.7
45	61.1
50	67.2
60	77.5
70	85.3
80	89.7
90	91.4

SOURCE: Adapted from J. M. Raphael, Prediction of Temperature in Rivers and Reservoirs, *Proc. ASCE, J. Power Div.*, no. PO 2, paper 3200, July, 1962.

Hamon et al. [12] have developed useful charts for estimating daily surface insolation as a function of latitude and time of year as well as of fractional sunshine $S = 1 - N$. For calculations in which only an average surface insolation is needed, Table 3-2 is useful.

Effects of vegetation

In regions having a dense vegetal cover, the soil receives only a small fraction of the sun and sky shortwave radiation. If I'_s represents the total shortwave energy incident upon a horizontal surface at the top of the vegetation, and I'_{sg} represents that at the ground level, then the effect of various types of vegetation is given by the transmission coefficient $k_t = I'_{sg}/I'_s$. The transmission will, of course, vary widely with the density, type, and condition of the vegetation. Deciduous trees in particular show marked seasonal changes. Typical values of the transmission coefficient for grass are given in Table 3-3 and for a coniferous forest canopy, in Fig. 3-9.

TABLE 3-3 Typical extinction of insolation by grass

Height of grass	$k_t = I'_{sg}/I'_s$
1 m	0.18
50 cm	0.18
10 cm	0.68

SOURCE: From data of Sutton [13].

Reflection

The portion I''_s of the direct incident shortwave energy I'_s which is reflected is given by the *albedo* A of the surface, where

$$A = \text{albedo} = \frac{I''_s}{I'_s} \qquad (3\text{-}57)$$

The average albedo of the *planet* earth is given by Houghton [14] to be 0.34, but that of individual surface materials shows a wide variation. Some approximate values for natural surfaces are given in Table 3-4, compiled from Refs. 10, 15, 16, and 17.

A chart showing the effect of solar altitude and cloud cover on the albedo of a water surface is given in Fig. 3-10. For altitudes greater than

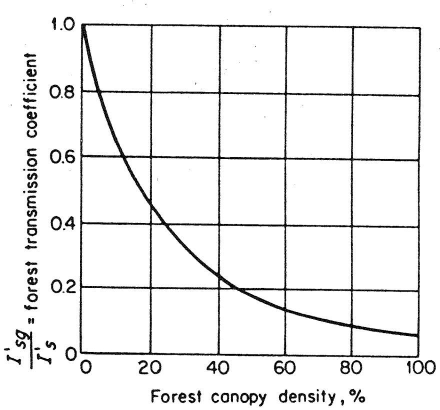

Fig. 3-9 Transmission of insolation by coniferous forest canopy. Canopy density is the percentage of forested area which is covered by a horizontal projection of the vegetation canopy. (*From "Snow Hydrology," U.S. Army Corps of Engineers, North Pacific Division,* 1956.)

TABLE 3-4 Albedo of natural surfaces

Surface	*Albedo A*	*Surface*	*Albedo A*
Water (see Fig. 3-10)	0.03–0.40	Spring wheat	0.10–0.25
Black, dry soil	0.14	Winter wheat	0.16–0.23
Black, moist soil	0.08	Winter rye	0.18–0.23
Gray, dry soil	0.25–0.30	High, dense grass	0.18–0.20
Gray, moist soil	0.10–0.12	Green grass	0.26
Blue, dry loam	0.23	Grass dried in sun	0.19
Blue, moist loam	0.16	Tops of oak	0.18
Desert loam	0.29–0.31	Tops of pine	0.14
Yellow sand	0.35	Tops of fir	0.10
White sand	0.34–0.40	Cotton	0.20–0.22
River sand	0.43	Rice field	0.12
Bright, fine sand	0.37	Lettuce	0.22
Rock	0.12–0.15	Beets	0.18
Densely urbanized areas	0.15–0.25	Potatoes	0.19
Snow (see Fig. 3-11)	0.40–0.85	Heather	0.10
Sea ice	0.36–0.50		

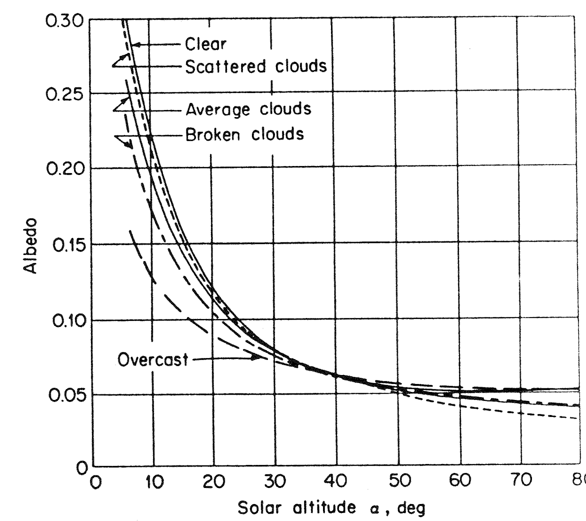

Fig. 3-10 Effect of solar altitude and cloud cover on albedo for a horizontal water surface. (*From J. M. Raphael, Prediction of Temperature in Rivers and Reservoirs, Proc. ASCE, J. Power Div., no. PO 2, paper* 3200, *July,* 1962.)

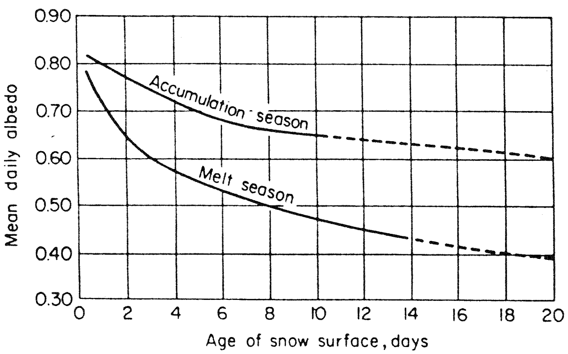

Fig. 3-11 Time variation in albedo of a snow surface. (*From "Snow Hydrology," U.S. Army Corps of Engineers, North Pacific Division,* 1956.)

15°, the curve for a clear sky can be shown to follow closely the well-known reflectivity formula of Fresnel, derived for plane surfaces and unpolarized light.

For snow surfaces the albedo is highly variable, being a function primarily of the state of snow metamorphosis (see Chap. 13). When the snow is fresh, the albedo is high, perhaps 80 percent; but as the snow ripens and becomes granular, the albedo decreases to as little as 40 percent. This is illustrated by the empirical curves of Figs. 3-11 and 3-12, in which the albedo is shown to decrease with time and with a cumulative temperature index, respectively.

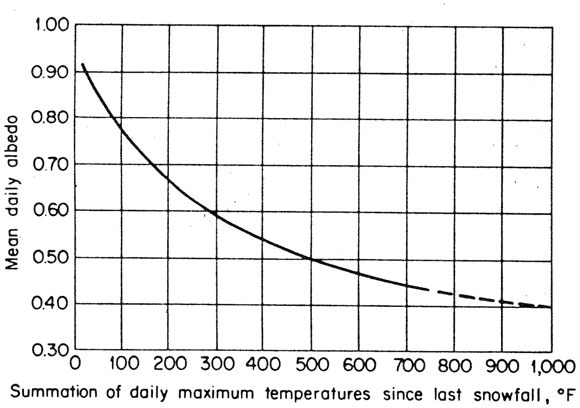

Fig. 3-12 Variation of albedo of a snow surface with accumulated temperature index. (*From "Snow Hydrology," U.S. Army Corps of Engineers, North Pacific Division,* 1956.)

Although snow behaves nearly as a diffuse reflector, there will be some increase of albedo with increase in the angle of incidence. Consideration of the resulting diurnal fluctuation in albedo is avoided through the use of a mean daily value; however, there will be a noticeable seasonal trend toward higher albedos in the winter than in the spring.

For engineering purposes it is often most meaningful to consider the effective incoming shortwave radiation I_s^*, which is given by

$$I_s^* = I_s' - I_s'' = I_s'(1 - A) \qquad (3\text{-}58)$$

A plot of these quantities versus solar altitude is given in Fig. 3-13. The upper curve in this figure represents the maximum (i.e., clear-sky) insolation I_c as given by Eq. (3-55) with $N = 0$, while the shaded band around the lower curve demonstrates the range of the effect of cloud cover on the reflected energy I_s'' from a water surface. From this it is clear that for a *water* surface, I_s'' is found with sufficient accuracy by using the average albedo curve of Fig. 3-10, while the primary effect of cloudiness is introduced through the reduction of I_c as given by Eqs. (3-55) and (3-56).

For snow surfaces, the effect of cloud cover is more pronounced. The albedo of snow is much larger than that of water, thus increasing

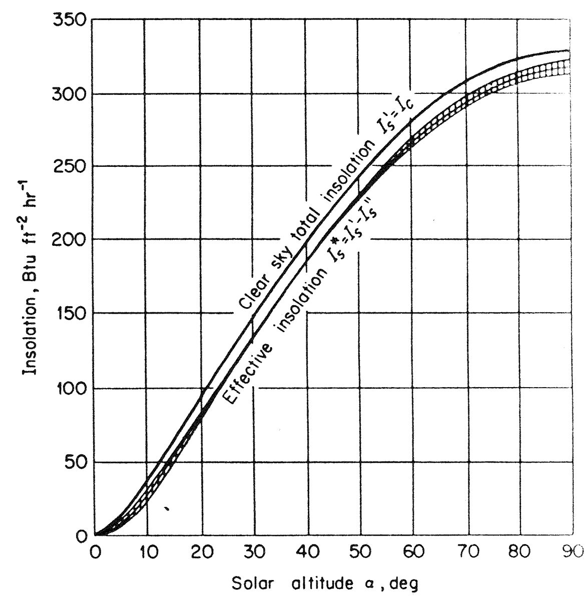

Fig. 3-13 Maximum effective insolation on a horizontal water surface. (*From J. M. Raphael, Prediction of Temperature in Rivers and Reservoirs, Proc. ASCE, J. Power Div., no. PO 2, paper* 3200, *July,* 1962.)

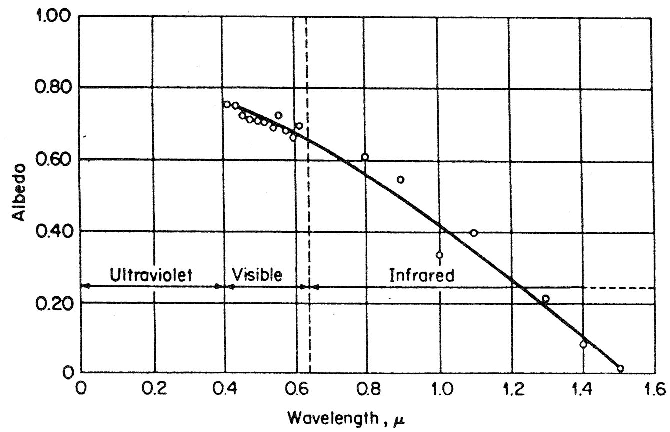

Fig. 3-14 Spectral distribution of albedo of a snow surface. (*From "Snow Hydrology," U.S. Army Corps of Engineers, North Pacific Division,* 1956.)

the reflected radiation. In addition, however, the albedo of snow is frequency-dependent (as shown in Fig. 3-14), in a manner which concentrates the reflected energy in the short wavelengths where it is subject to rereflection by the water droplets in the clouds. This increases the diffuse component and thus the total of I'_s considerably over that experienced with a given cloud cover without snow on the surface [Eq. (3-55)]. The value of k under these conditions will exceed that given by Eq. (3-56) and should be determined in each instance by measurement.

Extinction effect of water

The energy penetrating the sea surface is absorbed as it passes down through the water. This extinction is highly selective, with the long-wave energy being absorbed by the uppermost surface layers. Using published absorption coefficients for various wavelengths penetrating distilled water, Dake and Harleman [18] have calculated average absorption coefficients for the entire spectrum of natural light. These suggest that near-surface absorption of the longer wavelengths requires modification of Beer's law to

$$\frac{I'_{sz}}{I^*_s} = (1 - \beta)e^{-Kz} \qquad (3\text{-}59)$$

where, as before, I^*_s is the net shortwave energy incident upon a horizontal element of water surface, and I'_{sz} is the same quantity at location z beneath the surface. The values of K and β are highly variable because

TABLE 3-5 Extinction coefficients in water

Conditions	β	K, m^{-1}
Distilled water		
Natural light	0.75	0.029
Mercury-vapor lamps	0.62	0.60
Infrared lamps	0.75	0.60
Natural lake		
Clear (Lake Tahoe)	0.40	0.05
Turbid (Lake Castle)	0.40	0.27

SOURCE: From J. M. K. Dake and D. R. F. Harleman, An Analytical and Experimental Investigation of Thermal Stratification in Lakes and Ponds, *M.I.T. Dept. Civil Eng., Hydrodynamics Lab. Rept.* 99, September, 1966.

of variations in turbidity and plankton as well as variations in the actual incident spectrum. Some experimental values are given in Table 3-5.

Extinction effect of snow

For snowpacks having a homogeneous structure, the extinction of radiation penetrating the surface can be expressed in the more common form of Beer's law

$$\frac{I'_{sz}}{I^*_s} = e^{-Kz} \qquad (3\text{-}60)$$

Values of the extinction coefficient as a function of pack density are given in Table 3-6.

TABLE 3-6 Extinction coefficients in a snowpack

Snowpack density percent	*Extinction coefficient K* cm^{-1}
26.1	0.280
32.2	0.184
39.7	0.106
44.8	0.106

SOURCE: From *"Snow Hydrology,"* U.S. Army Corps of Engineers, North Pacific Division, 1956.

Summary

The disposition of shortwave radiation is summarized in Fig. 3-15.

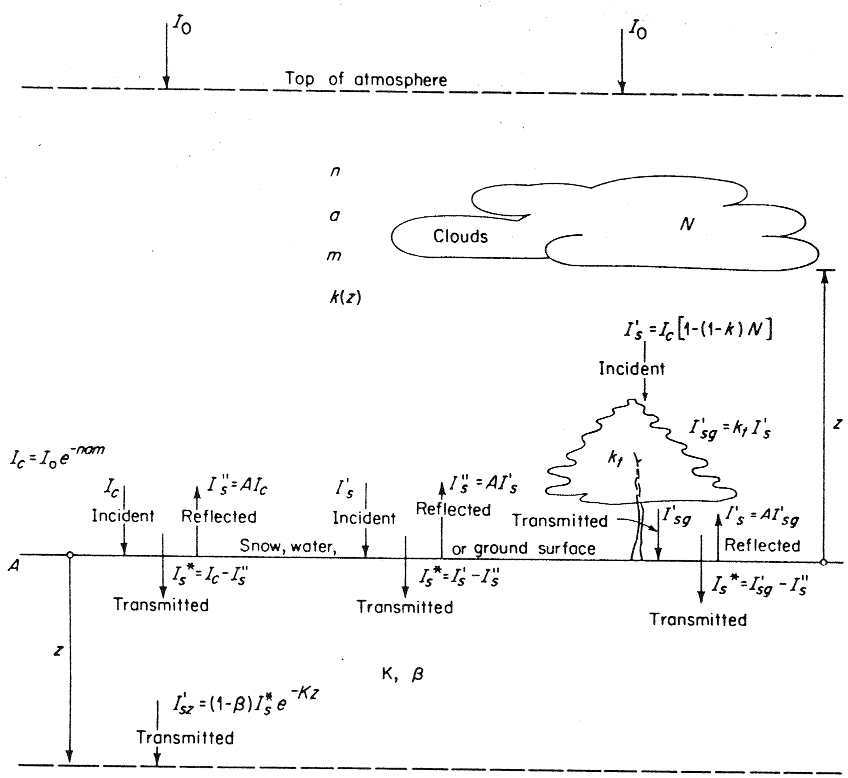

Fig. 3-15 Summary disposition of shortwave radiation.

3-5 LONGWAVE RADIATION

Atmospheric emissivity

There will also be some incoming longwave radiation which has been emitted by the various atmospheric constituents (chiefly water) and by forest cover. Since the atmosphere does not radiate as a blackbody or even as a gray body, it is necessary to develop an empirical relationship between this flux and the cloud and forest cover. From extensive measurements made over a water surface (effects of forest cover are therefore not included), Anderson [19] developed the following regression relation between incoming longwave radiation I'_l and the atmospheric vapor pressure e at ground level for *clear-sky* conditions:

$$\frac{I'_l}{W_B} = E_a = 0.740 + 0.0049e \qquad (3\text{-}61)$$

where W_B is the blackbody emissive power [Eq. (3-45)] evaluated at

ground-level air temperature, E_a is called the *atmospheric emissivity*, and e is measured in millibars.

Over snowfields the vapor pressure tends to remain close to that of the snowpack, which is generally between 3 and 9 mb.[1] Measurements [10] of incoming longwave radiation made over a snowpack having vapor pressures within this range indicate that for snow the clear-sky atmospheric emissivity is constant at $E_a = 0.757$; thus

$$\frac{I'_l}{W_B} = 0.757 \qquad (3\text{-}62)$$

again using the near-surface air temperature to evaluate W_B.

For a *cloudy sky* the atmospheric emissivity is a function of cloud thickness, density, type, and height, and it is difficult to generalize. From measurements over water, Raphael [20] presents an empirical relation for E_a as a function of cloud cover which is reproduced in Fig. 3-16. For zero cloud cover his relation very closely approximates Eq. (3-61). In other cases it must be regarded as only approximate.

In Fig. 3-6, we saw the peak of the terrestrial radiation spectrum to be at a wavelength of about 10 μ, where water vapor is not effective in

[1] The saturated vapor pressure at 32°F is 6.11 mb.

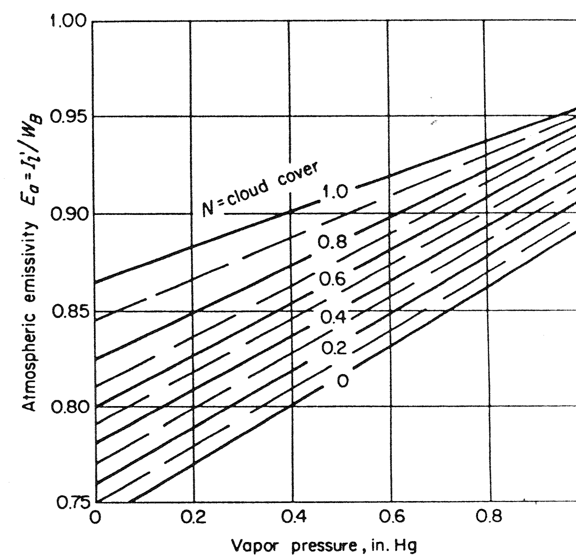

Fig. 3-16 Atmospheric emissivity for cloudy sky. (*From J. M. Raphael, Prediction of Temperature in Rivers and Reservoirs, Proc. ASCE, J. Power Div., no. PO 2, paper* 3200, *July*, 1962.)

absorption. The absorption by liquid water is a minimum at this wavelength too, but it is so effective an absorbent that only 1 percent of the incident 10 μ energy is transmitted through a 0.1-mm film. This is a small amount of water even for thin clouds; therefore, we can consider all clouds to radiate as blackbodies. Under overcast conditions, we can thus estimate I_l' from

$$I_l' = \sigma T_c^4 \tag{3-63}$$

in which T_c is the absolute temperature of the cloud base.

Reflection

Dunkle et al. [21] found the albedo of a water surface to be 0.03 for this longwave radiation, while that of the granular-type surfaces such as snow, sand, and earth is essentially zero. The net incoming longwave radiation I_l^* can thus be written

$$I_l^* = I_l' - I_l'' = (1 - A)E_a W_B \tag{3-64}$$

where I_l'' is the reflected longwave radiation. For water this becomes

$$I_l^* = 0.97\sigma E_a T_a^4 \tag{3-65}$$

and for other natural surfaces

$$I_l^* = \sigma E_a T_a^4 \tag{3-66}$$

where T_a is the absolute temperature of the surface air.

Surface emissivity

Raphael [20] reports that a water surface radiates as a gray body with emissivity

$$E_w = 0.97 \tag{3-67}$$

independently of water temperature and dissolved solids. Snow and other natural surfaces may be assumed to radiate as blackbodies [10] (that is, $E = 1.0$). The outgoing or *back* longwave *radiation* I_l from the surface is given by

$$\frac{I_l}{\sigma T^4} = E \tag{3-68}$$

where T is the surface temperature. This relation is plotted for various surface emissivities in Fig. 3-17.

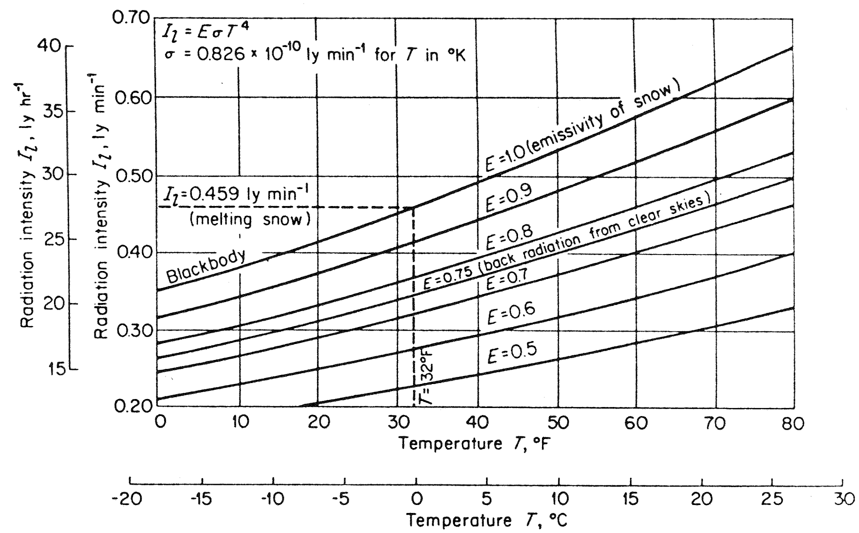

Fig. 3-17 Longwave back radiation as given by the Stefan-Boltzmann law. (*From "Snow Hydrology," U.S. Army Corps of Engineers, North Pacific Division,* 1956.)

Effective back radiation

The difference between the net radiant flux of longwave energy inward from the atmosphere, I_l^*, and that outward from the surface, I_l, provides the net longwave radiation exchange between the atmosphere and surface. The sign convention chosen makes this a negative quantity for surface-cooling processes such as evaporation and positive for surface-warming ones such as snowmelt. It is written

$$R_l = I_l^* - I_l \tag{3-69}$$

The effect of cloud cover on the longwave radiation exchange between the atmosphere and a surface may be approximated by an equation of the form

$$\frac{R_l}{R_c} = 1 - kN \tag{3-70}$$

in which R_c is the net longwave radiation exchange with clear skies, R_l is that with cloudy skies, and N is the fraction of the sky covered by clouds. The coefficient k has been determined empirically for snow surfaces [10] to be

$$k = 1 - 0.024z \tag{3-71}$$

where z is the elevation of the cloud base in thousands of feet. From Eqs. (3-64), (3-68), and (3-69) we can write

$$R_c = (1 - A)\sigma E_a T_a^4 - \sigma E T^4 \tag{3-72}$$

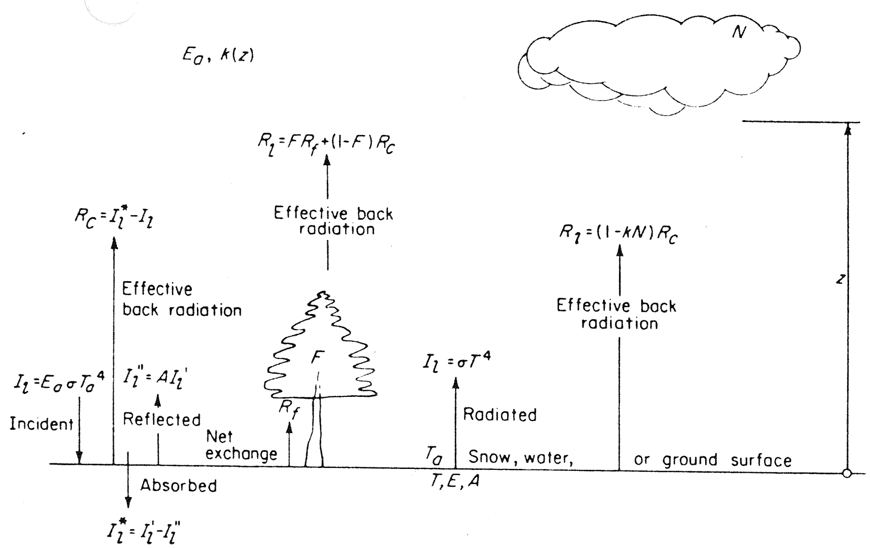

Fig. 3-18 Summary disposition of longwave radiation.

The effect of forest cover on this longwave radiation exchange can be approximated by treating a solid canopy as we did the overcast sky [Eq. (3-63)], i.e., radiation as a blackbody at the ambient air temperature. The net longwave exchange between the solid canopy and the surface is then

$$R_f = \sigma(T_a^4 - ET^4) \tag{3-73}$$

Proportioning the exchange according to the canopy density F,

$$R_l = FR_f + (1 - F)R_c \tag{3-74}$$

where F has a range from zero to one, with one representing a solid canopy. By using Eqs. (3-72) and (3-73) and assuming $A = 0$, this becomes

$$R_l = \sigma T_a^4 [F + (1 - F)E_a] - \sigma ET^4 \tag{3-75}$$

Summary

The disposition of longwave radiation is summarized in Fig. 3-18.

3-6 RADIATION MEASUREMENT

Shortwave solar radiation is measured with an instrument called a *pyranometer*[1] [22]. This device normally consists of a flat circular plate

[1] This term has been adopted by the World Meteorological Organization to replace the former term *pyrheliometer*.

mounted horizontally within a lime-glass bulb. The plate is divided into a central white spot, an intermediate black ring, and an outer white ring. A thermopile measures the temperature difference between the black and white areas, which, upon calibration, provides a unique measure of the radiation flux into the glass tube. Reflected solar radiation is measured in the same fashion by using a downward-facing *pyranometer*.

Total incoming (or outgoing) radiation is measured in a similar manner but without the filtering effect of the lime-glass. The device is called a *radiometer* and generally consists of a small flat plate mounted horizontally in the airstream of a blower. The plate has a black surface exposed to the radiation to be measured and a polished surface of the same material (usually aluminum) facing in the opposite direction. A layer of insulating material is placed between these two surfaces. A thermopile measures the temperature difference across this layer and hence the downward energy flow, while a separate thermopile measures the temperature of the black surface and consequently its back radiation. At equilibrium these two fluxes yield the incoming radiant flux. The blower acts to equalize convection effects at both surfaces of the plate (see Ref. 21).

PROBLEMS

3-1 Find the effective incoming shortwave radiation I_s^*, in langleys per minute, under the following set of conditions:

Latitude = 15°N
Date = July 10
Hour = noon
Clouds = overcast at 3,000 ft
Air turbidity = smoggy
Surface = grass-covered ground

3-2 Same as Prob. 3-1, except that the air is clear and there is a 30 percent forest canopy.

3-3 Estimate the effective incoming shortwave radiation received by a snow surface under the following conditions:

Latitude = 50°N
Date = October 25
Time = 2 P.M.
Clouds = none
Air = clear mountain
Snow age = 4 days

3-4 Calculate the clear-sky atmospheric emissivity at sea level, where the relative humidity is 75 percent and the air temperature is 90°F, and compare it with the value given by Fig. 3-16.

3-5 For the conditions of Prob. 3-4, calculate the clear-sky net longwave radiation exchange between the atmosphere and a water surface, where the temperature of the latter is 83°F.

3-6 Suppose the albedo of the planet earth is 0.34 with respect to solar radiation. Assume the earth to radiate as a blackbody, neglect atmospheric absorption of this terrestrial radiation, and calculate the earth's blackbody temperature. Compare this with the actual mean of 287°K, and comment on any difference observed.

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SUMMARY OF EAGLESON'S NOTATION

I_o Solar irradiance outside atmosphere (insolation)

I_c Direct, cloudless, insolation, AT EARTH'S SURFACE

I'_s Sum of DIRECT & DIFFUSE RADIATION from CLOUDY SKY

If we are interested in the insolation to the SOIL (for hydrology purposes; say, evapotranspiration rates) then can go a step further:

I'_{sg} = I'_s as further attenuated by vegetation cover.

I''_s Reflectance, as given by ALBEDO $A = \frac{I''_s}{I'_s}$

A Range:		
	Black, moist soil	A = 0.08
	Tops of fir trees	0.10
	White Sand	0.30
	Snow	up to 0.85

$$I^*_s = I'_s - I''_s = I'_s(1-A)$$

↳ Used in water penetration: $\frac{I'_{sz}}{I^*_s} = (1-\beta)e^{-kz}$