1 The forces of nature

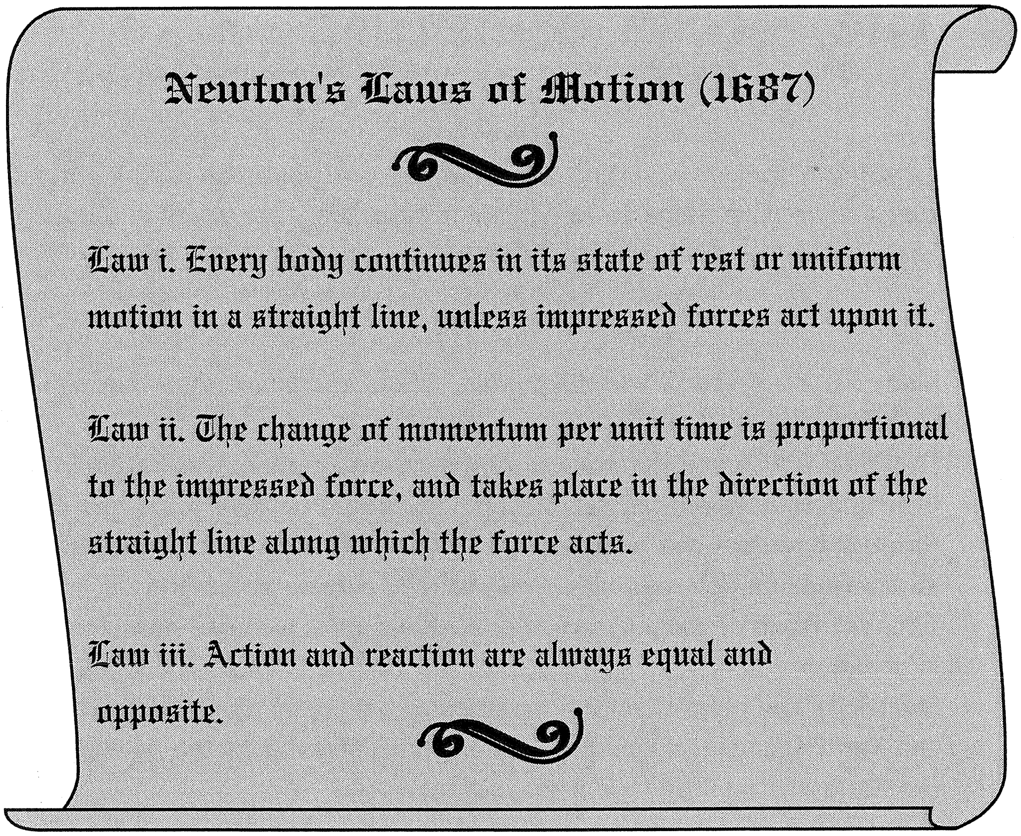
Landslides, tornadoes, falling raindrops and sedimentation are all natural systems that can be understood by Newtonian mechanics – the study of forces, momentum and motion. The following key concepts are covered in this chapter:

- **Many natural systems contain motion, forces and momentum that can be described by Newton's laws**
- **Frictional forces are important in any real-world motion, dissipating energy**
- **Gravity acts between any two bodies, dependent on their mass and distance apart**
- **Rotational motion can be described by laws and equations analogous to those for straight line motion, particularly relevant to climate and orbits**
- **Different types of wave observed in environmental systems have certain properties in common**
- **Electricity and magnetism are inextricably linked, as one induces the other**
- **The Earth's magnetic field provides a tool to investigate the geological history of the Earth and a subsurface surveying technique**

Newton's laws underpin areas such as gravity, friction and rotational dynamics, some of the key processes both in the natural environment and in a number of environmental control technologies. Many environmental systems contain waves – electromagnetic waves, sound waves, seismic waves in the Earth's interior or waves on the sea – which have certain properties in common. Electrical and magnetic fields and forces are important in several environmental applications, in particular related to the magnetic field of the Earth. This chapter deals with some of these fundamentals of physics that will be applied to a broad range of applications later in the book. Given limited space, the aim is to highlight key processes and concepts, such that the reader may seek further details elsewhere. For those who need help with rearranging of equations or basic mathematics, refer to Appendix A – Mathematical hints.

Fundamentals: Newtonian mechanics

- Why are blue whales bigger than elephants?
- When is someone going to invent a car that runs on air and water?
- Why don't we just bottle up CO_2 and shoot it into space?
- Why are skyscrapers taller than trees?



Newton's Laws of Motion (1687)

Law i. Every body continues in its state of rest or uniform motion in a straight line, unless impressed forces act upon it.

Law ii. The change of momentum per unit time is proportional to the impressed force, and takes place in the direction of the straight line along which the force acts.

Law iii. Action and reaction are always equal and opposite.

Figure 1.1 ***Newton's laws of motion.***

These questions and many more can be answered using mechanics, which concerns itself with forces, momentum, energy and motion. Mechanics is built upon the work of Newton, who, back in 1687, had a vision – that the jumble of phenomena we observe in the natural world can be broken down, simplified, special cases taken and provisions made, and then described precisely by mathematical equations. While the pure Newtonian model is now known to be a simplification, and has been superseded by quantum mechanics and relativity in some areas, it still provides the building blocks that underpin many other areas of physics. This chapter will start by looking at Newton's laws of motion.

Momentum and inertia

Newton's first law is a statement of the principle of **conservation of momentum**, which is one of six fundamental conserved properties in physics. All theory and evidence shows this to be true in all cases.

Momentum, the tendency to continue moving once doing so, is defined as mass multiplied by velocity, measured in kg m s^{-1} (kilogram metres per second).

Momentum is denoted *u*, thus $u = mv$ where *m* is mass and *v* is velocity. So the heavier something is and/or the faster it is moving, the more momentum it has.

The image of a world consisting of everything moving in straight lines without stopping is somewhat at odds with everyday experience – this is not because Newton was wrong, but because of that proviso 'unless impressed forces act upon it'. A force is anything that pushes or pulls – including air resistance, friction and gravity that are ever present.

An example of the conservation of momentum is the rebound of a gun when a bullet is fired. The bullet has forward momentum, and as the total momentum of bullet and gun is initially zero (assuming it is stationary when being fired), it must still be zero after being fired. The bullet's momentum is balanced by the gun's momentum in the opposite direction. The amount of rebound thus depends on the relative masses of gun and bullet, and the velocity of the bullet.

Momentum is of particular importance in atmospheric processes, as collisions between molecules and the exchange of their momentum control diffusion, heat flow and many fluid dynamic properties (see Chapter 4). Collisions are also relevant to a range of environmental processes, especially erosive forces such as the break-up of rocks into smaller pebbles and sand, and movements in sediments where collisions between the particles are important.

When two objects collide, momentum is conserved – because of the law of conservation of momentum. The total momentum after the collision is the same as that before, and changes in velocity may be calculated. A collision can be **elastic** or **inelastic**, or – like most real collisions – somewhere in between the two. In an elastic collision, the kinetic energy of the two objects stays the same, whereas in an inelastic collision some kinetic energy is converted into heat or sound. In elastic collisions the objects bounce off each other, while in inelastic collisions they may coalesce.

Inertia describes the same concept, the tendency for something to continue in motion or at rest unless it is pushed or pulled.

Forces

A force is simply a push or a pull, but it can be more closely defined than this. The second of Newton's laws states that rate of change of momentum is proportional to the force applied. In other words, the heavier something is and the faster it is moving, the more difficult it is to stop it or change its direction. A large heavy animal such as a horse or a wildebeest finds it more difficult to speed up and to turn than a light one such as a cheetah, which can accelerate and twist and turn easily because of its lower weight. Mathematically, the force is equal to the rate of change in momentum given by:

$$f = \Delta u \,/\, \Delta t$$

where f is the force, u is momentum, t is time and the symbol Δ means a change in the following quantity.

The definition of a force as exchange of momentum can be visualised in a jet engine. Air is drawn in at the front by a fan and ejected together with exhaust gases in a jet at the back. The momentum of the moving air is transferred to the aeroplane or boat, exerting a forwards force on it. Rocket engines work purely on this principle, ejecting exhaust gases to provide momentum, as in space there is nothing to push against.

This can be taken further, as momentum is mass times velocity. Velocity is defined as the change in distance over time. When something speeds up, the change in velocity over time is given by acceleration – if it slows down acceleration is negative. Mathematically:

$$v = \text{distance/time} = \Delta s/\Delta t$$

$$a = \text{velocity/time} = \Delta v/\Delta t$$

where s is distance, v is velocity, a is acceleration, t is time, and again Δ means a change. Now as momentum is mass times velocity, $u = mv$, the equation for a force can be given as:

$$\begin{aligned} f &= \Delta(mv)/\Delta t \\ &= m\Delta v/\Delta t \end{aligned}$$

As mass doesn't change, the change in momentum over time is the same as the mass multiplied by the change in velocity over time. But the change of velocity over time is acceleration, so:

$$f = ma \tag{1.1}$$

This definition of a force, as mass multiplied by acceleration, is an important equation in mechanics. By rearranging this equation, if the mass and the strength of the force are known, acceleration can be calculated.

Action and reaction

The third law introduces the concept of action and reaction. If an object is lying on a table, there is a force, due to gravity on the object, exerted on the table. The reason the object does not fall through the table on to the floor is that the table is also exerting a force upwards on the object. This force of reaction is, by the third law, equal to the force of gravity exerted by the object, in the opposite direction.

A car, or a rowing boat, is propelled by this principle – a force by the tyre against the ground or the oar against the water produces a reactive force, which drives the car or boat forwards.

The three laws together describe a simple model of forces and motion. For instance consider a tennis serve. As the racket moves towards the ball it has momentum, which depends on its weight and the speed at which the player strikes the ball. At the moment of impact, the racket exerts a force on the ball (action), and the ball exerts an equal and opposite force on the racket (reaction). The ball accelerates according to its mass and the size of this force, and likewise the racket decelerates. Momentum is transferred from racket to ball due to these forces, resulting in their changes in velocity. Mathematically it all adds up – momentum is conserved overall, the force depends on how fast this momentum is transferred from racket to ball, and the force of reaction allows the racket to decrease its momentum accordingly. If you measured the speed of the racket head just before and just after the moment of impact, and the masses of the racket and the ball, you could work out how fast the serve was.

Motion

If the velocity that an object is travelling at is constant, it can be calculated from time and distance travelled using:

$$\text{velocity} = \text{distance}/\text{time}$$

or:

$$v = \Delta s/\Delta t$$

If the object's velocity varies, this is no longer true. For the special case of motion in a straight line when acceleration is constant (i.e. an object subject to a constant force), the distance, velocity and acceleration are linked by three simple equations. Given the conventional notation, call distance s, velocity u at the start, when time t = zero, velocity v thereafter, and acceleration a. The three equations of motion are:

$$v = u + at \tag{1.2}$$

$$s = ut + \tfrac{1}{2}at^2 \tag{1.3}$$

$$v^2 = u^2 + 2as \tag{1.4}$$

Equations (1.2) and (1.3) can be demonstrated graphically, while Equation (1.4) can be derived from the other two by simple substitution. The three equations of motion can be used to calculate distance, velocity, acceleration, or time taken if any two of these are known. Together with Equation (1.1) defining a force, $f = ma$, this provides the basis for general solutions of problems concerning forces and motion, as described in any basic physics text.

SI units

It is important in physics to use a consistent set of units – otherwise the equations will no longer hold. The equation speed = distance/time is true if speed is in metres per second, distance in metres and time in seconds. It obviously cannot be true if speed is in km/h or distance is in miles, without allowing for converting units. The SI system (Système Internationale) provides a standardised unit system, which must be stuck to, to get the right answers. An answer, or any number, is meaningless without a unit.

All SI units are derived from kilograms, metres, seconds, and coulombs (for electric charge). Very large and small quantities use the prefixes kilo, mega, micro and so on, listed in Appendix B.

In SI units, as distances are measured in metres and time in seconds, hence velocity is in metres per second, $\mathrm{m\ s^{-1}}$ (the minus sign in the superscript means 'per'). Acceleration, the increase in velocity per second, is in metres per second per second, or $\mathrm{m\ s^{-2}}$. The SI unit of force is the Newton, defined as $1\ \mathrm{N} = 1\ \mathrm{kg\ m\ s^{-2}}$, i.e. the force required to accelerate a mass of 1 kg by $1\ \mathrm{m\ s^{-2}}$.

In any equation, the units on one side must be the same as those on the other. You cannot add apples and oranges, and likewise you cannot add metres to seconds. This gives a handy method of checking the validity of an equation – if the units are wrong, the equation is wrong. For instance, as force is given by $f = ma$ in Equation (1.1), the unit of force must be the same as that of mass multiplied by acceleration, which means kg multiplied by $\mathrm{m\ s^{-2}}$, hence $\mathrm{kg\ m\ s^{-2}}$. To simplify things this is called a **Newton**, denoted N.

Some quantities do not have a unit, because they are either a proportion or a **dimensionless** quantity. For instance, the porosity of stone is the proportion of air spaces to total volume. The unit is thus volume divided by volume, so it has no units – it is the same whether volumes are measured in $\mathrm{m^3}$ or $\mathrm{mm^3}$. Another dimensionless quantity is the **Reynolds number**, a quantity constructed to characterise fluid flow (pp. 162–3), representing a ratio of viscous to inertial forces, where viscosity is a measure of how 'treacly' the fluid is, and density is the mass of a substance per unit volume (p. 12). The Reynolds number is defined as $\rho vl/\mu$, where a fluid of density ρ (unit: $\mathrm{kg\ m^{-3}}$) and viscosity, μ (unit: $\mathrm{kg\ m^{-1}\ s^{-1}}$) flows at velocity v (unit: $\mathrm{m\ s^{-1}}$) relative to a solid of length l (unit: m). The unit of the Reynolds number would thus be:

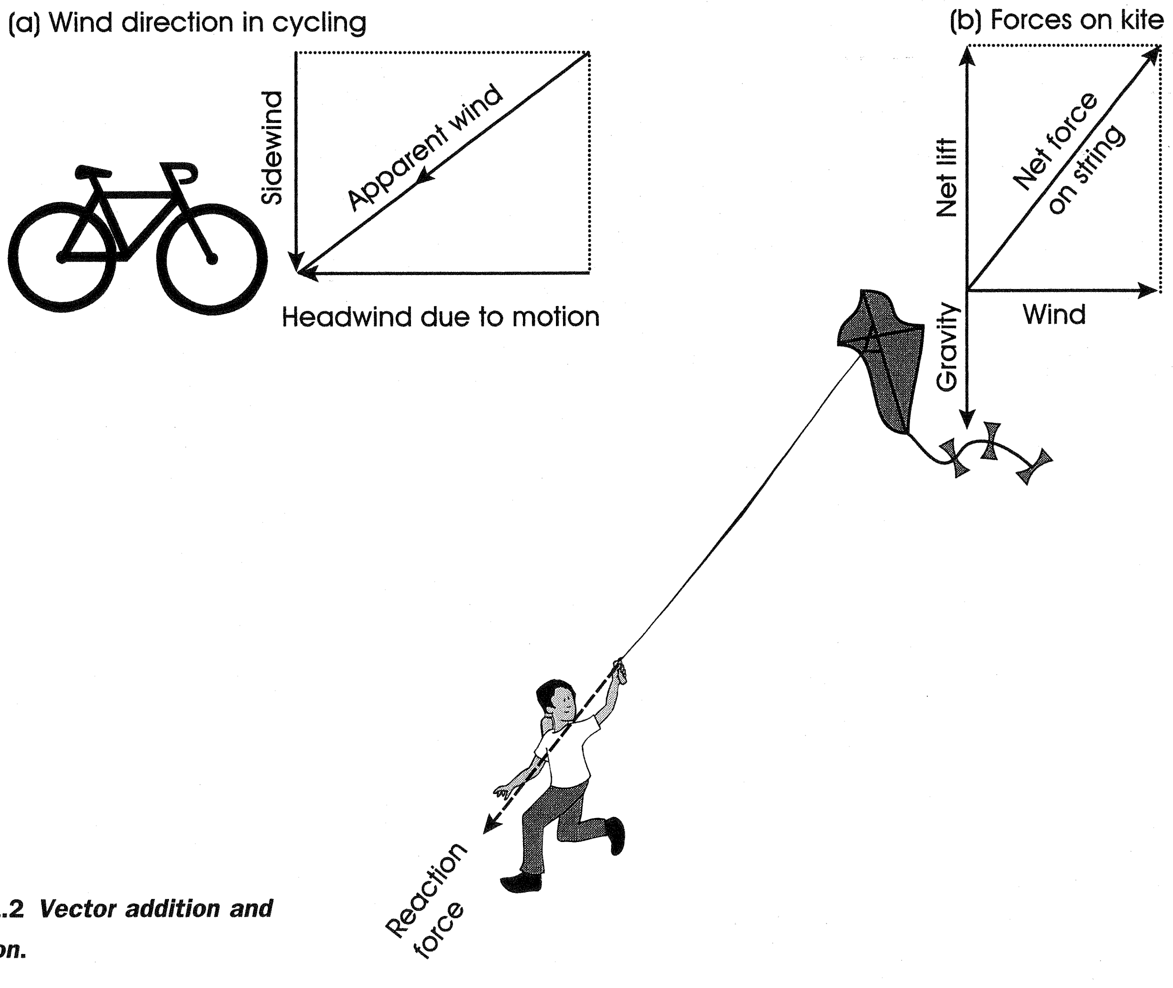
$$\begin{aligned}\text{unit} &= (\mathrm{kg\ m^{-3}\ m\ s^{-1}\ m})/\mathrm{kg\ m^{-1}\ s^{-1}}\\ &= \mathrm{kg\ m^{-1}\ s^{-1}}/\mathrm{kg\ m^{-1}\ s^{-1}}\end{aligned}$$

which cancels out, so it is dimensionless and has no units.

Scalars and vectors

Force, velocity and acceleration are all **vector** quantities, defined as those that have a direction associated with them as well as a magnitude. By contrast mass is a **scalar** quantity, which has magnitude but no direction. Other examples of scalar quantities could include time, the size of an object, or electric charge.

The key difference is that scalars can be added up directly (1 kg of lemons plus 2 kg of lemons makes 3 kg), while for vectors this does not apply, as the direction must also be taken into account. If a cyclist is travelling at 20 km h^{-1}, and subjected to a wind from the side at 15 km h^{-1}, adding the two numbers to 35 km h^{-1} is meaningless. To find the apparent wind, the two velocity vectors may be added either graphically as shown in Figure 1.2 (a), or algebraically. The cyclist will be subject to a wind of 25 km h^{-1}, coming at an angle. Likewise for forces, for instance a kite is subject to a horizontal force due to the wind and a vertical aerodynamic lift force, reduced by the downwards pull of gravity. This results in a net force along the direction of the kite string, with its equal but opposite reaction given by the person holding the string, as in Figure 1.2(b). The converse is that


(a) Wind direction in cycling

(b) Forces on kite

Figure 1.2 ***Vector addition and resolution.***

forces can be resolved back into their component parts. If the force and angle of the kite string are known, they could be used to calculate the wind speed and the lift force at the kite.

Friction and air resistance

Why do objects in common experience generally not continue moving in straight lines forever, as Newton's first law predicts they should? The answer lies in friction and air resistance. Friction is a force between two objects due to roughness of their touching surfaces, reducing their motion relative to one another, while air resistance results from frictional forces between an object and the air, slowing down any movement through it.

Friction has two forms – static friction and dynamic (or kinetic) friction. Static friction is when the two objects are still with respect to one another, and results in a larger frictional force than dynamic friction, when they are moving. Any moving object is subject to frictional forces between its moving parts, tending to slow it down, and creating heat in the process. Friction can be useful, for instance rock types used for road surfacing materials are chosen for their friction coefficients, with particular grades of stone with the highest friction coefficients sometimes being used near pedestrian crossings, allowing vehicles to stop safely in less time.

The degree of friction depends principally on the roughness of the surfaces, but will be affected by lubricants such as oil or water that flow into the tiny gaps in between solid surfaces allowing them to move more easily. In other words for a wet surface the frictional force is less, so for instance a vehicle may skid on a wet road when it brakes – the reaction force of friction is less than the force created by its rate of change of momentum in slowing down. Weight or pressure can increase friction, pressing two objects more tightly together and increasing their cohesion.

The force due to air resistance is known as a **drag** force, and is of great importance in aerodynamic design of vehicles (and birds), and in terminal velocity (pp. 17–18). The drag force from wind on natural surfaces such as leaves, water, and rock is relevant to processes such as wind pollination, resistance of plants to wind stress, and erosion. Drag depends on the object's shape, size, surface characteristics and speed, and will be least for a smooth, aerodynamic object with a relatively small surface area, moving slowly.

Drag forces can take two forms depending on flow characteristics (which will be considered in more detail on pp. 161–3). For a small object moving slowly, such as an airborne pollution particulate, or for movement in a viscous fluid (like oil or treacle) **skin drag** predominates, due to viscous forces between fluid particles around the object's surface. For faster moving objects in less viscous fluids, such as large objects in air, **form drag** is more important. This is to do with the change in momentum of the fluid as it moves out of the way, which is dependent on the object's shape.

In the nineteenth century, Stokes showed that the force on a sphere due to skin drag is:

$$F_{\mathrm{d}} = 6\pi r\mu v \tag{1.5}$$

where F_{d} is drag force, r is the radius of the sphere, μ is viscosity and v is velocity.

For form drag, the equivalent force is given by:

$$F_{\mathrm{d}} = \frac{1}{2}\rho_{\mathrm{a}} C\pi r^2 v^2 \tag{1.6}$$

where ρ_{a} is the density of air (1.2 kg m^{-3}) and C is a drag coefficient that depends on the shape of the object, with the value 1.2 for a sphere. C will be highest for a flat plate perpendicular to the movement, and lowest for an aerodynamic 'teardrop' shape. Note that this force increases proportionally to the radius squared, i.e. it depends upon the surface area of the object opposed to the motion, and to velocity squared, so it predominates over skin drag at high velocities.

Gravity

Earthbound creatures live with gravity constantly, and it affects many natural environmental processes, together with being useful in certain environmental technologies and in understanding the Earth through gravity surveying. The gravitational fields of the Moon and the Sun also control the Earth's tides, which occur in the atmosphere and in the Earth's crust in addition to the better known ones in the ocean.

Newtonian gravity

While sitting under his famous apple tree, Newton postulated that gravity is an attraction between masses, and that anything falling under gravity accelerates at the same rate, no matter how heavy it is. Motion under gravity is governed by gravitational acceleration, which at the Earth's surface has the value $g = 9.8$ m s^{-2}.

The force due to gravity, f, can be given by $f = mg$, which is just Equation (1.1) with acceleration equal to g. In other words the force increases in proportion to the mass, so the acceleration remains the same for any mass.

How fast do things fall? For an apple dropping from a tree 4.5 m high into Newton's lap sitting below, 0.5 m high, the distance fallen is 4 m at an acceleration of 9.8 m s^{-2}. Putting $s = 4$, $a = 9.8$, $u = 0$ into Equation (1.2) gives:

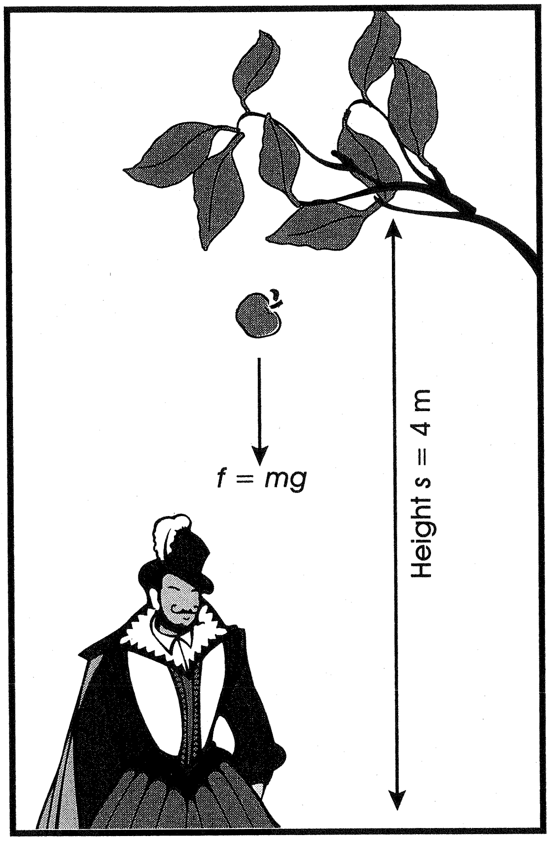


Figure 1.3 ***Newton's apple.***

$$s = (0) + \tfrac{1}{2}at^2$$

rearranging:

$$\begin{aligned} t &= \sqrt{(2s/a)} \\ &= \sqrt{(2 \times 4/9.8)} \\ &= 0.9 \text{ seconds} \end{aligned}$$

The velocity of the apple would be:

$$\begin{aligned} v &= u + at \\ &= 0 + 9.8 \times 0.9 \\ &= 8.9 \text{ m s}^{-1} \end{aligned}$$

Mass, weight and density

Mass can be seen as 'reluctance to be moved by a force'. While mass is an intrinsic quality of matter, the weight of an object is the force on it due to gravity. Weight is therefore measured in Newtons, not kg. In outer space, an object will have weight of zero, but it will have the same mass as on the Earth, and the momentum associated with it.

On the Earth, weight w is given by Equation (1.1), $f = ma$:

$$w = mg$$

So a 10 kg mass weighs $10 \times 9.8 = 98$ N.

On the moon, gravity is about one-sixth of that on the Earth. So moon gravity could be called $g_m = 9.8/6 = 1.6 \text{ m s}^{-2}$. The weight of the same 10 kg mass is now:

$$\begin{aligned} w &= mg_m \\ &= 16 \text{ N} \end{aligned}$$

The mass of an object stays the same wherever in the Universe it is.

The **density** of a substance indicates how heavy a given sized quantity is. Density is defined as mass per unit volume, measured in kg m^{-3} (kg per cubic metre). It is usually denoted by the Greek letter ρ (rho), given by $\rho = M/V$.

The density of water is 1,000 kg m^{-3}. **Relative density** of a substance means its density relative to water. For instance, the density of steel is 8,500 kg m^{-3}, so its relative density is 8.5. Any substance with relative density less than 1 will float on water.

Box 1.1

Landslides and slope stability

Gravity has been termed 'the great leveller', dragging downwards on everything at the surface of the Earth. Erosive agents such as wind, water and glaciers help greatly in moving large amounts of material, but even without these, considerable mass movements take place. These vary from minor soil slippage that may barely disturb vegetation, to major mudslides, rockslides and avalanches that can destroy large areas of crops or forest and any roads or villages in their path.

A landslide will occur if the force of gravity down a slope is greater than the frictional forces that support the soil, or in the case of rocks the shear strength of the rock involved (see Chapter 4). Because dynamic friction is generally less than static friction, once the slope starts to move it is likely to continue to do so and to accelerate, until it reaches the bottom of the slope.

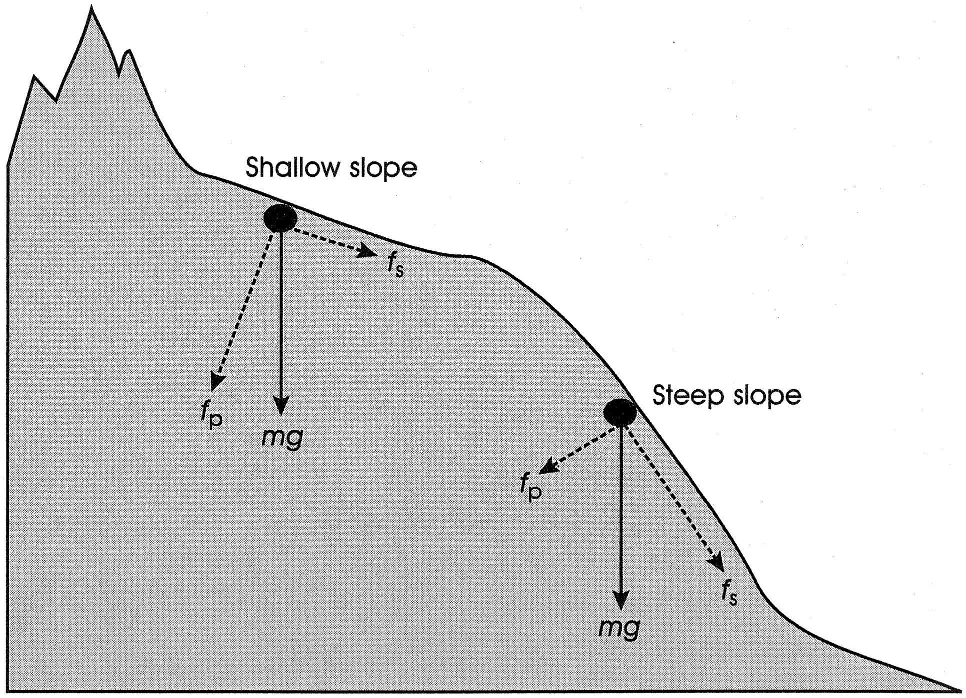


Figure 1.4 ***Forces acting in landslides.***

Factors that affect the likelihood of landslides include the angle of the slope, weight of overlying material or buildings, and anything that affects frictional forces. The gravitational force on a parcel of soil, mg, can be resolved into components f_s parallel to the slope and f_p perpendicular to the slope (Figure 1.4). Slope is important because the steeper the angle, the greater the component of gravitational force acting down the slope, f_s. Thus all other things being equal, the steeper the slope, the more likely a landslide is to occur. The maximum incline that a specific material can rest at and still be stable is known as the **angle of repose**. Angle of repose depends principally upon the material – different soil types will vary in the frictional forces between particles, as the size and

continued

shape of the soil particles varies, and also the level of compaction of the soil. Spoil tips must be constructed with these factors in mind, at a maximum slope angle depending on the material concerned. In rocks, the strength depends not only on the rock type but the angle of bedding planes and the existence of faults or weaknesses.

Water is important for several reasons. The sheer weight of water in soil increases gravitational forces, while it also generally reduces friction by lubricating the soil, especially when saturated, although very dry soil can also become weaker and less cohesive. Under extremely wet conditions, previously solid soil can become wet enough to flow like a liquid, due to water preventing friction between soil particles. Water can also cause cracks through expansion and contraction in clays, and freeze-thaw action. Changes in drainage or increases in moisture from irrigation or household drains can therefore be causes of landslides. Perhaps more commonly very heavy rainfall can saturate a slope that was stable under normal conditions, causing sudden large and rapid movements that can be devastating. This was the trigger for the disastrous collapse of unstable minewaste heaps in Aberfan in Wales, in the 1960s, engulfing the village school and causing the deaths of dozens of children.

Vegetation plays a part, as plant and tree roots hold together soil (like reinforcing rods in concrete), and increase its capacity to hold water. Vegetation of spoil tips and tree planting in roadside cuttings can thus not only improve their appearance but also their stability. Deforestation in mountainous areas can cause landslides because of the removal of this reinforcement, making stabilisation difficult. In November 1998, Hurricane Mitch struck Honduras and Nicaragua with torrential rain and wind, creating disastrous mudslides that caused the loss of thousands of lives and destruction of the homes and livelihoods of hundreds of thousands. This natural disaster was believed to have been greatly exacerbated by deforestation of the mountainous areas of these countries over the previous years, destabilising soil and creating the preconditions for major erosion and collapse.

Plate 1.1 ***Landslide on cliff top at Lyme Regis, UK.***
Source: photo by courtesy of Dr J. S. Griffiths, University of Plymouth

In general solids are more dense than their liquid form. This does not always hold however – water is unusual in that it expands (i.e. becomes less dense) when it freezes. Ice has a relative density of 0.9, and so it floats on water.

Universal gravity – big *G* and little *g*

Newton's gravitational acceleration g ('little g') is only constant at or near the Earth's surface. For a more generalised case, gravity is a force between any two objects, depending on their masses and the distance between them. The force increases proportionally to the two masses attracting, and inversely proportionally to the square of distance between them.

$$f=\frac{GMm}{R^2} \tag{1.7}$$

where f = force, M and m are the two masses, R is the distance between them, and G ('big G') is the universal gravitational constant, with the value $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Hence as you travel away from the Earth the force becomes weaker. If you imagine a gravitational field radiating out from the Earth like rays from the Sun, the strength depends on how far apart the rays are. At a distance R the rays will have spread out to cover a large sphere with an area proportional to R^2, so the force decreases accordingly. This is known as an **inverse-square law**, and there are similar examples from several different branches of physics, including electrical forces and the dissipation of sound.

At the Earth's surface this same force is given by $f = mg$, and so the relation between big G and little g can be shown:

$$g=\frac{GM}{R^2} \tag{1.8}$$

where M is the Earth's mass and R its radius. As the Earth's radius is about 6,400 km, its mass can be calculated from this equation:

$$\begin{aligned} M &= \frac{gR^2}{G} \\ &= \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} \\ &= 6 \times 10^{24}\ \text{kg} \end{aligned}$$

Gravity is a very weak force, so it is only noticeable from very large masses such as the Earth. Although in theory any two masses attract, such as two apples, the force would be so tiny it is virtually immeasurable.

Box 1.2

Gravitational anomalies and surveying

One method of studying the interior of the Earth is by measuring very small variations in the strength of gravity at the Earth's surface. These variations in gravity are termed **gravitational anomalies**.

If you measure the constant *g* in an aeroplane, it will be lower than at the Earth's surface – because you are further away from Earth's centre of gravity, and gravity decreases with distance (the inverse-square law). For this reason, gravitational anomalies are corrected for the height above sea level.

If you happen to be flying over the top of Mount Everest when you measure *g*, you would expect it to be slightly higher than its average, corrected value for the height. This is because of the additional mass of the mountain below you. Gravity measurements can thus also be adjusted for topography – the additional mass of visible features such as mountains.

However, when gravity measurements are made at the tops of mountains, and corrected for topography, they are found to be lower than normal – they show a negative anomaly.

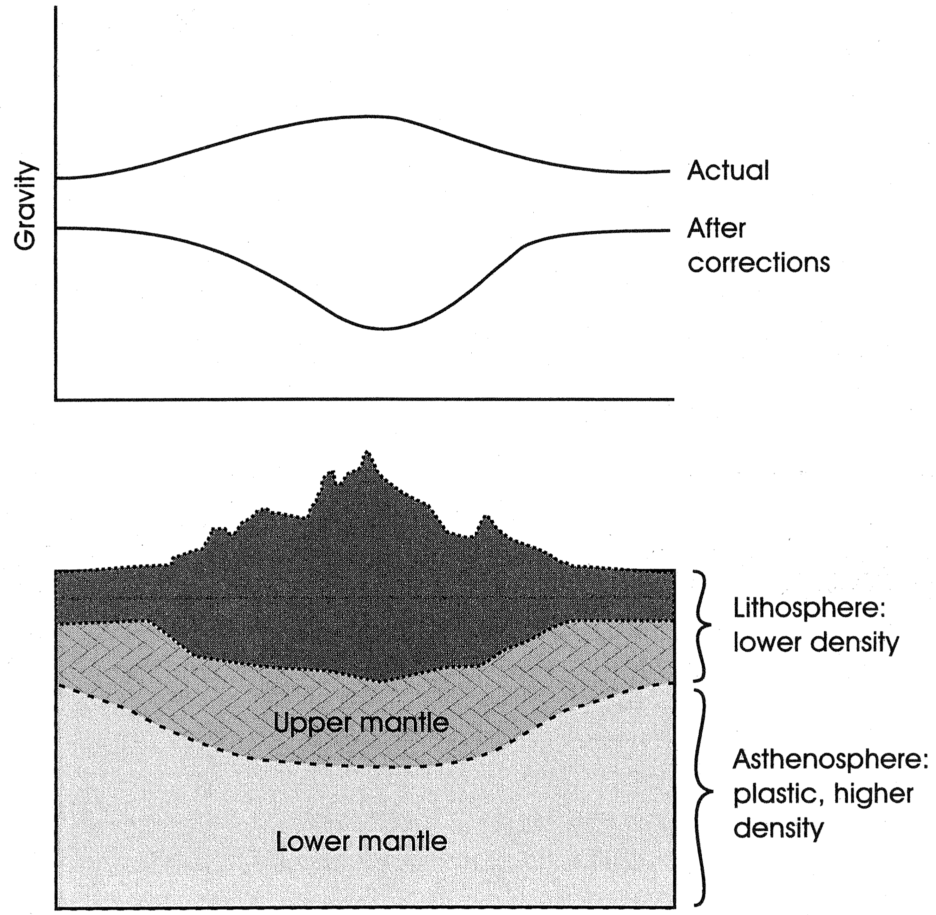


Figure 1.5 ***Gravitational anomaly over a mountain range.***

This is because of features below the surface: below a mountain range lies an area of thicker crust, which has a lower density than the mantle rocks below. Thus this crustal rock 'root' has lower mass, reducing gravity very slightly. This is due to **isostatic compensation** (see Ritter 1986 pp. 38–41), caused essentially by the weight of the mountains gradually pressing the crust and solid outer layer of the mantle down, as if they were floating on the plastic asthenosphere below (Figure 1.5; see also Box 4.1).

In a similar manner, gravitational surveying can be used to explore other natural features below the Earth's surface – the thickness of the crust, areas of rocks with higher density, or plumes of denser material upwelling from the lower mantle. They can also be used as an aid to mineral prospectors and hydrographers, to explore subsurface features that may indicate the presence of ore deposits, water or other resources. Man-made features can also be studied, for instance gravitational surveys have been used to assess the extent of old landfill sites and mineworkings, where historical records are inadequate (Sharma 1997).

Terminal velocity and settling velocity

The idea of constant gravitational acceleration is true in 'idealised' circumstances, but in real life air resistance plays a part. A feather and a brick will not fall at the same rate, but each will reach a maximum speed known as their **terminal velocity**. Terminal velocity is relevant to many environmental applications involving gravity, affecting falling rain, animal characteristics, and settling of air pollution particulates. In water, it is more often known as the settling velocity, important in water treatment and sedimentation in rivers and lakes.

When falling, an object will accelerate under gravity until the drag force is equal but opposite to the force of gravity, when speed will reach its maximum and remain constant. For a human being (without a parachute), terminal velocity is about 65 m s^{-1} (230 km h^{-1}), reached after about 10 seconds, when falling from around 300 m. Smaller creatures such as spiders have relatively much greater surface area compared to their weight than humans, and so have relatively higher drag and much lower terminal velocity. Hence they reach terminal velocity very quickly (and also have less momentum), and can withstand falls from great heights without damage.

Higher terminal velocity means large particles settle more rapidly, leading to the kind of sorting by size seen in pebbles on the beach, or in estuarine sediments, which affect the composition of sedimentary rocks. In addition, denser particles also settle rapidly, so for instance in developing countries where lead is still commonly used as a petrol additive, lead-rich dust from vehicle exhausts may be found closer to the roadside than lighter road dust or hydrocarbon particulates of the same size.

An expression for terminal velocity can be derived from the equilibrium between the force of gravity and the drag forces. For a small sphere where viscous forces predominate (such as a dust particulate settling in air), the skin drag is given by

Equation (1.5). Equating this to the force due to gravity gives:

$$mg = 6\pi r \mu v$$

For mass m given by the volume of a sphere ($4/3\pi r^3$) times density ρ,

$$\tfrac{4}{3}\pi r^3 \rho_s g = 6\pi r \mu v$$

$$v_t = \frac{2g\rho_s r^2}{9\mu} \tag{1.9}$$

where v_t is terminal velocity, g is acceleration due to gravity; ρ_s is the density of the sphere; μ is viscosity of the fluid; r is the radius of the sphere. This is known as Stokes' law, illustrating that settling velocity is highest for a larger, dense object in a less viscous medium.

For larger objects and more rapid flow a different expression applies, as drag forces are now described by Equation (1.6). Equating with gravitational force as before gives:

$$\tfrac{4}{3}\pi r^3 \rho_s g = \tfrac{1}{2}\rho_a C \pi r^2 v^2$$

$$v_t = \sqrt{\frac{8\rho_a r g}{3\rho_a C}} \tag{1.10}$$

where ρ_a is the density of air and C is drag coefficient. This equation can be used to describe the speed of falling raindrops. Entering numerical values into (1.10) gives the terminal velocity of a raindrop as $v_t \approx 4.3\sqrt{r}$, where r is the radius of the drop in mm. The largest drops of around 6 mm diameter fall at 7.5 m s^{-1}, while drizzle with droplets of 0.5 mm diameter falls at 2 m s^{-1} or slower. Thus a raindrop falling from a cloud at a height of 1,500 m could take from 200 seconds ($3\frac{1}{2}$ minutes) to 780 seconds (13 minutes) to reach the ground. Smaller droplets may also be lifted back up in updraughts, until they have grown to a size where their terminal velocity is higher than the speed of the updraught. It is evident that the finer rain will have more time as well as a larger surface area to absorb pollutants from the air during its fall, greatly affecting the impact of acid precipitation and related pollution problems.

Rotational dynamics and angular momentum

The physics that describes rotation and spin is useful to understand many natural systems, such as vortices in winds and water, and the relation between climate systems and the Earth's rotation. It is also used in the laboratory centrifuge, and certain pollution control technology.

Box 1.3

Settling chambers

These are among the simplest air pollution control devices, consisting of a large chamber in which flue gases slow down allowing particulates to drop out under gravity. They are used in many energy intensive industries, such as smelters or glassworks, where large coal-fired combustion plant are required. For a chamber of height h, length l, and gas velocity u, the gas will take l/u seconds to traverse the chamber, while a particle with settling velocity v_s will settle in h/v_s seconds. So any particle with diameter large enough such that $h/v_s < l/u$ will settle out, together with a proportion of smaller sized particles. Rearranging Stokes' law, Equation (1.9), gives a particle radius for 100 per cent settling of:

$$r = \sqrt{\frac{9\mu uh}{2\rho gl}}$$

From this formula the size of particulate that will be removed and the cleaning efficiency at smaller particle sizes can be calculated. For instance for flue gas flowing at 4 m s^{-1} through a chamber of size 0.5 m by 10 m, with particulate density of 1,350 kg m^{-3}, with gas viscosity at the operating temperature of 1.75×10^{-5}N s m^{-2}, the size for 100 per cent settling will be:

$$r = \sqrt{\frac{9 \times 1.75 \times 10^{-5} \times 4 \times 0.5}{2 \times 1350 \times 10 \times 10}}$$

$$= 34.1 \times 10^{-6} \text{ m or } 34\ \mu\text{m}$$

Settling chambers provide a cheap method to remove large particles but are ineffective for finer particulates, requiring large areas and/or very slow gas flows. They are generally used as a primary treatment before a more expensive and efficient device such as a scrubber or electrostatic precipitator (Box 1.6).

The same principle is used in primary water treatment, to allow larger suspended solids to settle out before further treatment.

Angular momentum is the tendency for something that is spinning to continue to spin, whether it is a planet or a bicycle wheel. Like linear momentum, it is conserved – that is, unless external forces act, something that spins will continue to do so forever.

Rotational movement can be described by a series of equations analogous to those for linear motion and forces, but with distance, velocity, momentum and mass replaced by their rotational counterparts: angle turned, angular velocity, angular momentum and moment of inertia respectively. Linear forces are replaced by turning force termed **torque**. Angular velocity is given the symbol ω, and is equal to the tangential (i.e. straight line) velocity divided by the radius of the circle being rotated around.

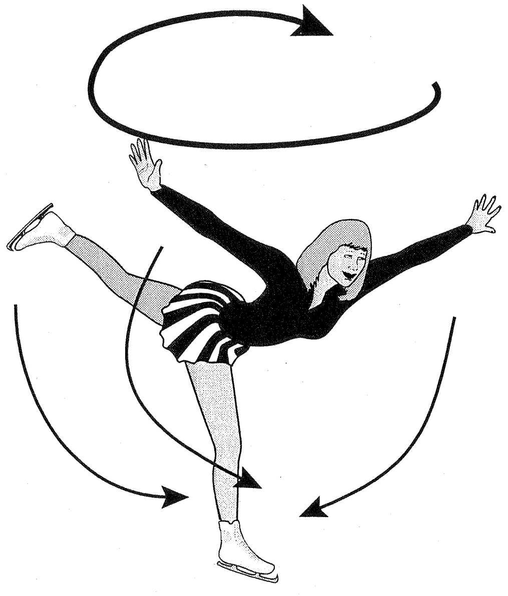


Figure 1.6 *Moment of inertia and an ice-skater. As she draws her arms and legs in and down, she will spin faster.*

Moments of inertia

The amount of angular momentum something has depends on how fast it is spinning, its mass, and how that mass is distributed. The distribution of that mass around the axis is represented by the moment of inertia, I. If an ice-skater is turning slowly with their arms and legs outstretched, then crouch down and hold their limbs in to their body, they will spin faster, as shown in Figure 1.6. Similarly, high divers control the rate of their spin by stretching out to slow down, curling up to spin faster.

When ice-skaters hold their limbs in to the body, they reduce their moment of inertia, as this means that the mass is closer to the axis of rotation. The increase in the rate of spin is an effect of the conservation of angular momentum. Angular momentum L is given by the product of the moment of inertia and the angular velocity:

$$L = I\omega \tag{1.11}$$

where ω is the angular velocity. This is analogous to linear momentum, given by mass times linear velocity. As this product is constant, reducing the moment of inertia must cause an increase in angular velocity.

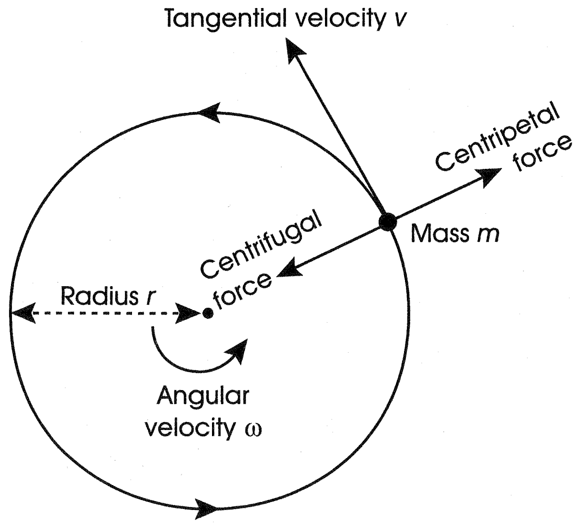


Figure 1.7 *Central forces and rotation.*

Central forces

When an object spins, it tends to continue in a straight line, at a tangent to the direction it is spinning, because of conservation of linear momentum. This results in a **centripetal force** pulling away from the centre of rotation. To continue spinning the centripetal force must have an equal but opposite force of reaction, pulling into the centre, which is the **centrifugal force**. If you whirl a lasso the centrifugal force is provided by the tension in the rope and the force of you holding it; in the case of the Earth going round the Sun, it is provided by gravity. The magnitude of the centripetal force is given by: