TOPIC: ESR 320

DATE:

FILE UNDER: FLOW RELATIONSHIPS IN FLUIDS

1. **FLOW:** Mass Flow: $\dot{m}$ $\left[\frac{M}{T}\right]$ $\left\{ \begin{array}{l} g/day \\ kg/s \end{array} \right.$

Volumetric Flow: Q $\left[\frac{V}{T}\right] = \left[\frac{L^3}{T}\right]$ $\left\{ \begin{array}{l} m^3/s \\ L/min \end{array} \right.$

CONNECTION: $\rho = \frac{m}{V}$

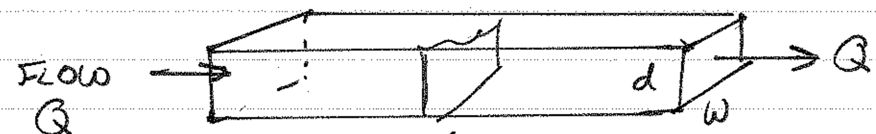
$\therefore \quad \dot{m} = \rho Q$

Ex: River w/ $Q = 1.5\ m^3/s$

$\rho_w = 1.0\ kg/L = 10^3\ kg/m^3$

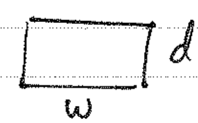
$\dot{m} = \left(1.5\ \frac{m^3}{s}\right)\left(10^3\ \frac{kg}{m^3}\right) = \underline{1.5 \times 10^3\ \frac{kg}{s}}$

2. **FLOW AND VELOCITY** Consider rectangular channel:



depth = d
width = w

Cross sectional area "$A_{x.s.}$"



$A_{xs} = wd$

$$\boxed{Q = v \cdot A_{xs}}$$

v = mean velocity of water

OR

$$\boxed{v = \frac{Q}{A_{xs}}}$$

3. Ex: Channel is 1.5 m deep, 4 m wide
Velocity is 0.80 m/s

$$Q = \left(0.80\ \frac{m}{s}\right)\underbrace{(1.5\ m)(4\ m)}_{A_{xs}} = \underline{4.8\ m^3/s} \text{ ANS.}$$

Ex: Flow is 500 L/s; $A_{xs} = 2.0\ m^2$ velocity = ?

$$v = \frac{(500\ L/s)(1/1000\ m^3/L)}{2.0\ m^2} = \underline{0.25\ m/s} \text{ ANS}$$

4. How about a "binary" fluid: Chemical A dissolved in B Fluid

$$C_A = \frac{m_A}{V_{A+B}} \approx \frac{m_a}{V_B} \leftarrow \text{Assume A is dilute, small part of system}$$

Ex: Mix 10 mg benzene ("A") into 500 L H_2O ("B")

$$C_A \text{ (benzene)} = \frac{10\text{ mg}}{500\text{ L}} = 0.02\text{ mg/L} \quad \text{(ppm)}$$

$$= 20\ \mu g/L \quad \text{(ppb)}$$

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5. What is the mass flow of benzene in a river w/ conc'n of 0.02 mg/L? (call benzene "A")

Flow: $v = 0.5$ m/s
$A_{xs} = 2.0\ m^2$

$$Q = vA_{xs} = (0.5\ m/s)(2.0\ m^2) = 1.0\ m^3/s$$

Mass Flow of A: $\dot{m}_A = QC_A$ $\quad \left[\frac{L^3}{T}\right]\cdot\left[\frac{M}{L^3}\right] = \left[\frac{M}{T}\right]$

So: $$\dot{m}_A = (1.0\ m^3/s)(0.02\ mg/L)\left(10^{-3}\ \frac{g}{mg}\right)\left(10^{3}\ \frac{L}{m^3}\right)$$

$$= \underline{0.02\ \frac{g}{s}}$$ ANS

NOTE: $\therefore\ 1\ \frac{mg}{L} = 1\ \frac{g}{m^3}$

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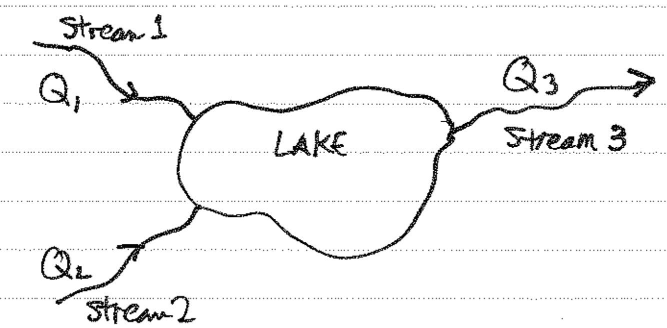
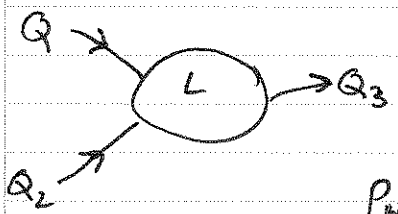
6. Why is mass flow useful? Can use to set up MASS BALANCES:

Ex:



7 Do Mass Bal. on <u>WATER</u>



MASS IN = MASS OUT   IF LAKE VOLUME STEADY

$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$\rho_w Q_1 + \rho_w Q_2 = \rho_w Q_3$ (div. by ρ_w)

OR $\quad Q_1 + Q_2 = Q_3$

So if $Q_1 = 1.5\ m^3/s$, $Q_2 = 3.5\ m^3/s$ $\rightarrow$ $Q_3 = 5.0\ m^3/s$

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8. MORE INTERESTING CASES:

Ex: Lake not necessarily steady volume.

$Q_1 = 1.5\ m^3/s$
$Q_2 = 3.5\ m^3/s$
$Q_3 = 4.5\ m^3/s$

$Q_1 + Q_2 \neq Q_3$ (?)

So: $Q_1 + Q_2 + Q_3 = \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$ RATE OF CHG. OF LAKE VOL.

(use (−) neg. sign on "out" flows)

$(1.5 + 3.5 - 4.5)\frac{m^3}{s} = 0.5\ m^3/s$

↑ RATE AT WHICH LAKE IS INCREASING IN VOLUME

MOVING ON:

9. Suppose Lake has area of 5 ha (hectares)
How is depth increasing? (dh/dt)

$V = hA$ $h = \frac{V}{A}$ IF A = constant then...

vol = depth × Area

$$\frac{dh}{dt} = \frac{1}{A}\frac{dV}{dt}$$

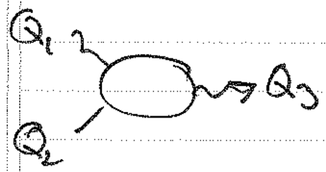
$$= \frac{0.5\ m^3/s}{50{,}000\ m^2} \leftarrow (1\ ha = 10{,}000\ m^2)$$

$$\frac{dh}{dt} = 1 \times 10^{-5}\ m/s$$ Doesn't sound like much...

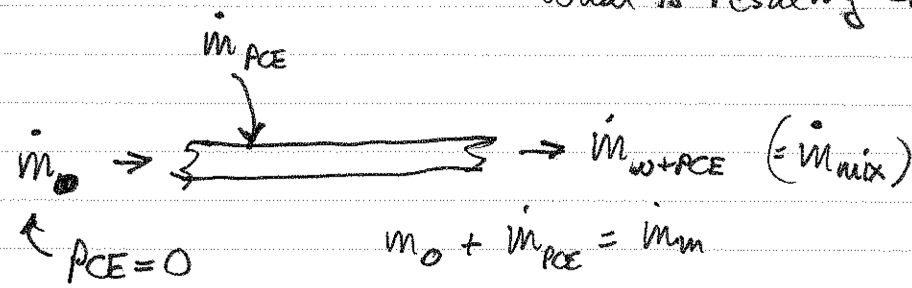
OR $(10^{-2}\ mm/s)(86{,}400\ s/d) = 864\ mm/d = 0.86\ m/d$

10. What about mass bal's for BINARY FLUIDS (A in fluid B?)

Go back to streams (#6.) $Q_1 = 1.5\ m^3/s$



STREAM 1: A storage tank leaks 15 g/s of PCE (cleaning fluid) into stream 1.
What is resulting conc?



$$m_0 + \dot{m}_{PCE} = \dot{m}_m$$

$$(0 + 15)\ g/s = \dot{m}_m = 15\ g/s$$

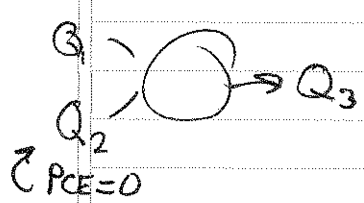
So?

11. But put the 10 g/s into the FLOW

$$\frac{15\ g\text{-}PCE/s}{1.5\ m^3/s} = 10\ \frac{g_{PCE}}{m^3} = \boxed{10\ \frac{mg}{L}}$$

$$\boxed{C_A = \frac{\dot{m}_A}{Q_T}} \qquad \left[\frac{M}{L^3}\right] = \left[\frac{M/T}{L^3/T}\right]$$

12 What about C_A^{LAKE}?



$$\dot{m}_1^P + \dot{m}_2^P = \dot{m}_3^P$$

~~10 mg/L ... = 10 mg/L~~

$$15\ \frac{g}{s} + 0\ \frac{g}{s} = 15\ \frac{g}{s}$$

$$C_A^3 = C_A^L = \frac{\dot{m}_P^3}{Q_3} = \frac{15\ g/s}{4.5\ m^3/s} = 3.3\ \frac{g}{m^3} = \boxed{3.3\ \frac{mg}{L}}$$

ANS