PRESSURE & T vs Atm Altitude

REVIEW LAST TIME

(1) Water: · Density varies w/ temp.

- Min. Density @ 4°c. NOT @ 0°

∴ Ice & colder water less dense (buoyant) compared to 4° water

- Density does NOT vary w/ pressure

→ Water is "incompressible"

Acts like a solid. (blow ~~cork~~ bottom out of Coke bottle w/ cork)

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(2) AIR Nearly an ideal gas.

∴ $pV = nRT$ works well

Use: $\rho = \frac{n}{V} = \frac{p}{RT}$ $\quad T\uparrow\ \rho\downarrow$ $\quad p\uparrow\ \rho\uparrow$ Makes sence

Q: where do we find value of R?

Q: What units do you use for R?

Only catch: Air is not incompressible, unlike w

E.g. Double the pressure & density ~~drops~~ doubles also

$\rho = \left(\frac{1}{RT}\right) p$

Complicates relationship of p, ρ, T & altitude

* (1b) Hydrostatic Pressure $\quad \left.\begin{matrix} p \\ \rho \end{matrix}\right\}$ all $f(h)$ All should decrease with h. But how?

$F_p = F_g^h = m_T^h g$ ~~weight~~ Force = Weight of water colum = mass of water × g (9.8 m/s²)

m_T^h = Volume × Density

$= \rho V = \rho h A$

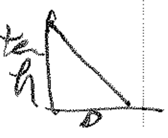
$F_p = \rho h A g$ Dive

$F_p / A = p = \rho g h$

$p = \rho g h$

g = const.

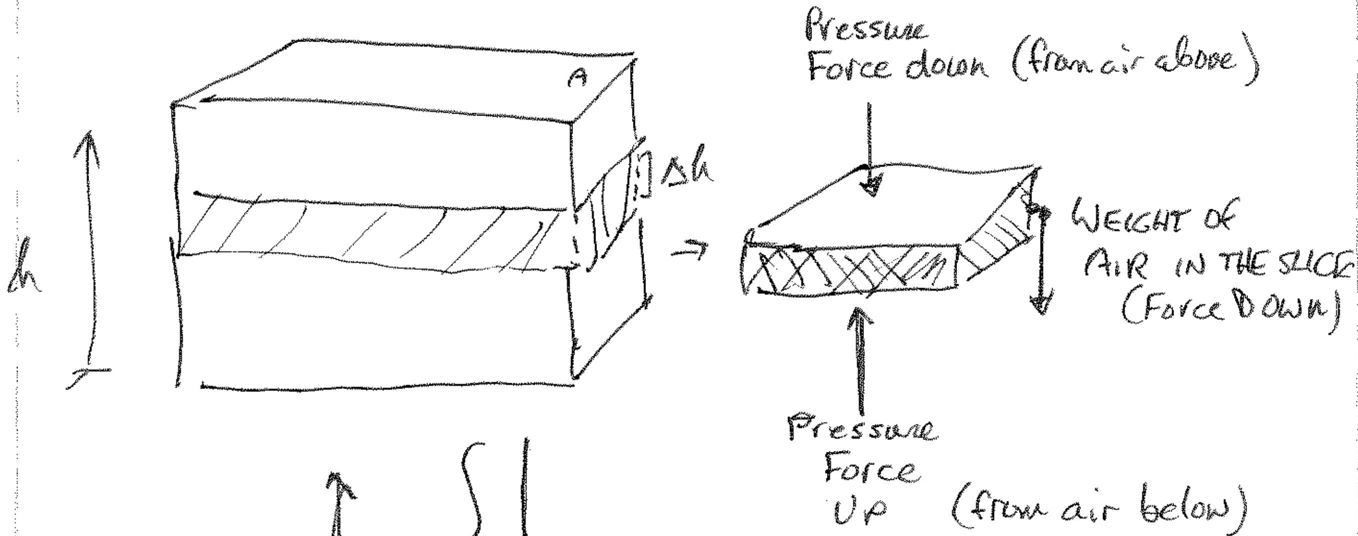
$\rho \simeq$ constant

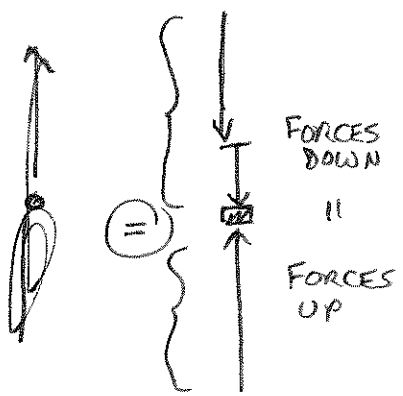


OK $\rho = f(h)$ so can't use just $p = \rho g h$

FORCE BALANCE will get us right answer then

(3) Let's NOT assume anything about weight of air above us. Just work in terms of pressures





← Want an eqn for these forces

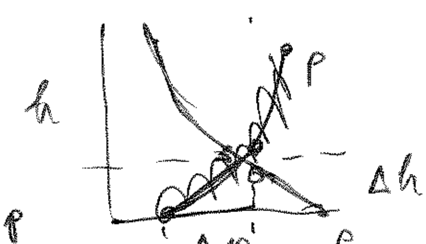
(4) (1) Suppose we know p at the bottom (e.g. start at sealevel ∴ bottom p = 1.0 atm)

p-Force Up $= pA$

(2) We also know weight of air down:

$$F_{g(down)} = mg = (\rho V) g = \rho A \Delta h g$$

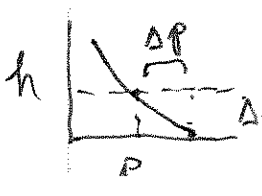
(3) What's pressure down at top of slice? We don't know it yet but we can write an eqn for it:



$$\cancel{p_{top} = p_{(bottom)} + \frac{\Delta p}{\Delta h}}$$

$$p_{top} = p_{(bottom)} + \Delta p$$

$$p_{top} = p + \frac{\Delta p}{\Delta h} \cdot \Delta h$$



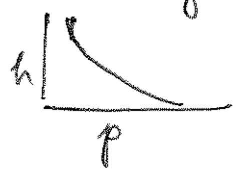
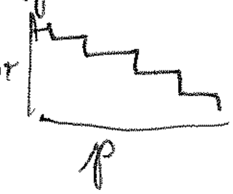
(slope of p vs. h) × (thickness Δh)

5 $$\underbrace{F_{up}}_{pA} = \overbrace{F_{down}}^{\left(p + \frac{\Delta p}{\Delta h}\Delta h\right)A + \rho A \Delta h g}$$

$$pA = \left(p + \frac{\Delta p}{\Delta h}\Delta h\right)A + \rho A \Delta h g$$

Now do 2 things

(1) Make use of calculus. Want slice to get very, very thin ($\Delta h \xrightarrow[\lim]{} 0$) so that our eqn. is continuous, not just stepwise.

We want  NOT 

$$pA = \left(p + \frac{dp}{dh} \cdot dh\right)A + \rho A g\, dh$$

AND (2) Use Ideal Gas Law $pV = nRT$

$$\frac{n}{V} = \rho^{mol} = \frac{p}{RT}$$

And convert from

$$\underset{\rho^{mol}}{\frac{mol}{L}} \Rightarrow \underset{\rho}{\frac{Mass}{L}} = \frac{mol}{L} \cdot MW$$

$$\rho = \frac{p \cdot MW}{RT}$$

OK Lets combine these steps & do some algebra simplifying

6 $$p\cancel{A} = (p + dp)\cancel{A} + \frac{p \cdot MW}{RT}\cancel{A} g\, dh$$

$$\cancel{p} - (\cancel{p} + dp) - \frac{p \cdot MW}{RT} g\, dh = 0$$

Divide thru by p
$$-\frac{dp}{p} - \frac{MW \cdot g}{RT} dh = 0$$

Can get rid of derivatives (differentials, really) by INTEGRATING

$$\int \frac{1}{p} dp = -\left(\frac{MW \cdot g}{RT}\right)\int dh$$

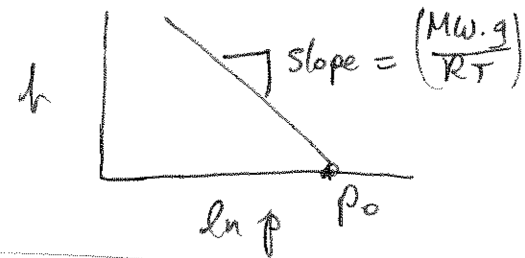
$$\ln p = -\left(\frac{MW \cdot g}{RT}\right) h + \text{constant}$$

$\ln p = \ln p_o$ (sea level) at $h = 0$

$$\ln p = -(MW \cdot g / RT) h + p_o$$

~~We want~~
So could say $\ln p = -\left(\frac{MW \cdot g}{RT}\right) h + \ln p_0$

7

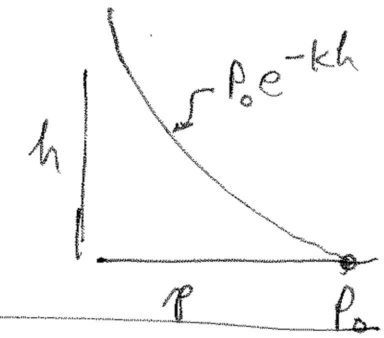


or take exp of

7b $\exp(\ln p) = p = e^{-\left(\frac{MW}{RT}\right) h} \, e^{\ln p_0}$

$e^{\ln p}$

$$\boxed{p = p_0 e^{-\frac{MW}{RT} h}}$$



8 $MW = 29.0$ g/mol $\quad g = 9.8$ m²/s

$T = 15° = 288$ K

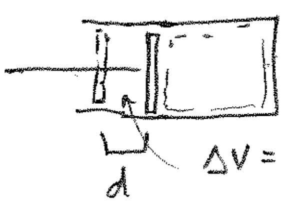
$k = 1.2 \times 10^{-6}$ cm^{-1} $= 1.2 \times 10^{-4}$ m^{-1}

But: this only holds exactly if T = const w/ h (not true in lower atm. in general

9 $p\cancel{\Delta}V$ = ~~Heat~~ Internal Energy If ad..

WORK

$p\Delta V$ = Int. Energy of Mols.



$\Delta V = dA$

$W \equiv F.d$

$F = pA$

$W \equiv pAd = p\Delta V$

"Adiabatic" No heat crosses boundary. Simplifies because energy conserved w/in the control volume. Also, is reasonable for our system (air parcel surrounded by ~ same temp.

Gr: A - dia - batos

No thru passable

10 Compression: Air heats up

Expansion: " cools off Even though we ~ air rises & exp

Lets look @ compression first: Easier to visualize. Same results either way

$p\Delta V$ = Increased "Internal Energy" E (Heat of molecules) ← Temp rises But

$\Delta E \propto \Delta T \rightarrow \Delta E = C_v \Delta T\, m$

Heat capacity; mass of air in piston

$p \frac{\Delta V}{m} = C_v \Delta T$

$\rho = \frac{m}{V}$ $\quad \cancel{p\Delta\left(\frac{1}{\rho}\right)}$

$p\,\Delta\left(\frac{1}{\rho}\right) = C_v \Delta T$

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11 Now what about h? Want $\frac{\Delta T}{\Delta h}$ or at limit $\frac{dT}{dt}$

Go to differential scale: $\Delta h \rightarrow dh$

$\Delta \frac{1}{\rho} = d\left(\frac{1}{\rho}\right)$ ← (4-7)

$p \frac{\partial}{\partial h}\left(\frac{1}{\rho}\right) = C_v \frac{\partial T}{\partial h}$ ↖ Id. Gas Law $pV = nRT$ $\quad \rho = \frac{p\,MW}{RT}$

$T = \frac{p(MW)}{R\rho}$

Take deriv. of T fcn w/r/t h: 1st × deriv 2nd, 2nd × deriv 1st

subst.

$\frac{\partial T}{\partial h} = \left(\frac{MW}{R}\right)\left[p \cdot \frac{\partial}{\partial h}\left(\frac{1}{\rho}\right) + \frac{1}{\rho}\frac{\partial p}{\partial h}\right]$ [4-9]

consts

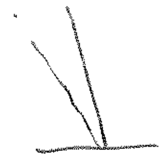
Now we go back to begin of class

$\frac{dp}{p} = -\left(\frac{MW}{RT}\right) g\, dh \Rightarrow \frac{dp}{\partial h} = -\rho g$ subst. rearrange

$\frac{\partial T}{\partial h} = \frac{-g(MW)}{R + C_v(MW)}$ ⇒ Fill in Constants (p. 299) $= \boxed{9.8\ K/km}$

— CONSTANT —

(2) Go over - std. dry ad. L.R.

- Stable 

- Unstable 

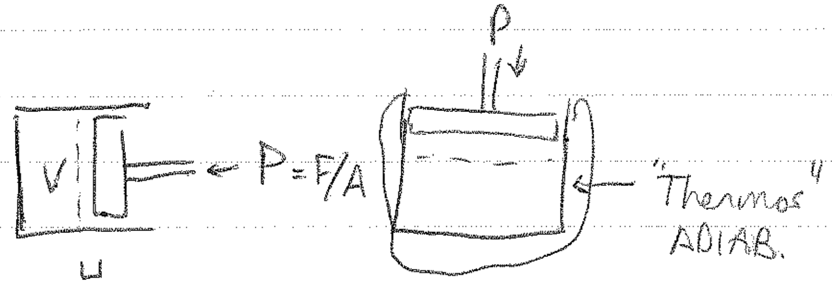
Slides

1 ⇒ Review WET lapse rate: - Air rises
- Cools off
- H_2O condense
- Air heats back up partially
- Lapse rate LESS step
(smaller T drop! non linear

(1)

D.A.L.

$9.8^\circ C / 1000 m \quad \sim 1^\circ C / 100 m$



$\Delta V = A \cdot \Delta X$

$W = p\Delta V = \frac{F}{\cancel{A}} \cdot \cancel{A} \cdot \Delta x = F \Delta x \Leftarrow$ So if air does work

loses energy, ADIABATIC.

What's TEMP change for given energy change?

(1) $\frac{\Delta \text{ HEAT}}{\text{UNIT MA}} = C_v \, \Delta T$

$\leftarrow$ Heat cap. $\frac{L^2}{T^2 K}$

$\frac{L^2/T^2 M}{M} = \frac{L^2}{T^2}$ OK

(2) VOLUME CHANGE per UNIT MASS

$\forall$ = sp. volume = $\frac{\text{Vol}}{\text{mass}} \leftarrow \frac{1}{\text{DENSITY}} \leftarrow \frac{\text{MASS}}{\text{VOL.}}$

$\Delta \forall = \frac{1}{\rho}$

(3) Look at incremental change in P & $\forall$ as go up increment in height (dh) $\quad dP, d\forall, dh \; (\Delta P, \Delta\forall, \Delta h)$

$\downarrow$

$$p\,\Delta v = \Delta \underset{\text{mass}}{\text{heat}} = -C_v \Delta T$$

per dh

<u>Eq. 4-7</u> $$p \frac{dv}{\partial h} = -C_v \frac{\partial T}{\partial h}$$

$$p \frac{\partial}{\partial h} \frac{1}{\rho} = -C_v \frac{\partial T}{\partial h}$$

Eq. 4-2 (Id. Gas Law) Rearrange $T = \frac{(MW)}{R} \frac{P}{\rho}$

or $T = \frac{P}{R\rho}$ ← IF R IN MASS NOT MOLE UNITS

Differentiate Eq. 4-2
Make substitutions ...

Eq 4-12 $$\frac{\partial T}{\partial h} = \frac{-g\,(MW)}{R + C_v(MW)}$$ ← $C_v(MW) = \frac{5}{2} R$ from gas physics

$$\frac{\partial T}{\partial h} = \frac{(981\ cm/s^2)(28.96\ g/mol)}{(1 + \frac{5}{2})(8.3 \times 10^{+7}\ erg/mol\,K)}$$

Lapse Rate = 9.8 K/km = 9.8 °C/km

REAL ATMOSPHERE: Lapse Rate Bigger OR SMALLER

AIR parcels "TRY" to follow adiabatic rate
Respond to surroundings