A Brief Summary of Some Atmosphere Properties

Temperature and temperature units

- Temperature is an indication of and can be used as a measure of stored heat
- Temperature is measures on either a relative or absolute scale

Relative: °C (Celcius)
Absolute: °K (Kelvin)

Conversion: °K = °C + 273.15

Atmosphere

Composition of the atmosphere

The atmosphere is composed of a mixture of gases and some particulate matter
The predominant components are
Nitrogen
Oxygen
Argon
Carbon dioxide
Water vapor
Suspended particles

Proportion of each gas in dry air (excluding water vapor)

	% by volume
Nitrogen	78.09
Oxygen	20.95
Argon	0.93
Carbon dioxide	0.03
Water vapor	Varies from 0.5 to 4.0

Vertical structure of the atmosphere

The vertical structure of the atmosphere can be described in terms of both:
- Temperature variation
- Chemical composition
- Absorption of radiant energy

(See figure 4.3)

Zone	Depth	Upper Boundary	Characteristics
Troposphere	11 km (varies from 10 to 16 km)	Tropopause	Temperature decreases with elevation
Stratosphere	50 km	Stratopause	Temperature increases (slightly) with elevation
Mesosphere	90 km	Mesopause	Temperature decreases with elevation
Thermosphere	700 km		Temperature increases with elevation
Exosphere	80,000 km		

Note: the earth's weather occurs primarily within the troposphere

Composition of the atmosphere (continued)

Component	
Nitrogen	Predominant constituent Occurs up to 100 km Primarily as N_2 below 50 km Primarily as N between 50 and 100 km Source of nitrogen for the nitrogen cycle – ESR 321
Oxygen	Occurs up to 120 km Primarily as O_2 below 60 km Primarily as O between 60 and 120 km Ozone occurs in small amounts - but absorbs SW (UV) radiation
Carbon dioxide	
Water vapor	
Suspended particles	

Temperature variation in the troposphere

In general, temperature decreases with elevation
This can be described by:

$$T = T_0 - \alpha z$$

where T is the air temperature at elevation z above the point where the temperature is T_0 and α is the lapse rate, usually expressed in °C/km.

The lapse rate, α, depends on the moisture condition of the atmosphere.

The lapse rate expresses how air cools as it rises, if the cooling is **adiabatic**
Adiabatic cooling requires that no energy in the form of heat is lost in the process
Then the temperature variation is simply due to the change in pressure that results
For a perfect gas, pressure, density and temperature are related
In the atmosphere, pressure decreases with elevation

The **dry adiabatic** lapse rate, Γ, is 9.8°C/km (lapse rate for dry air)
The presence of moisture decreases the lapse rate by as much as half
When the air becomes saturated the lapse rate can be reduced to about 5°C/km
The average lapse rate is about 6.5°C/km

Atmospheric Pressure

- Pressure at a point in a static fluid results from the weight of fluid above that point (under natural circumstances)
- Pressure is a force exerted over a unit area ,

$p = \frac{F}{A}$ so $F = pA$ (direction – normal to A)

Pressure units

Standard pressure units:

Unit system	Units	Name or abbreviation
SI	N/m^2	pascal
	$1000\ N/m^2$	kilo-pascal (kp)
BG	lb/ft^2	psf
	lb/in^2	psi

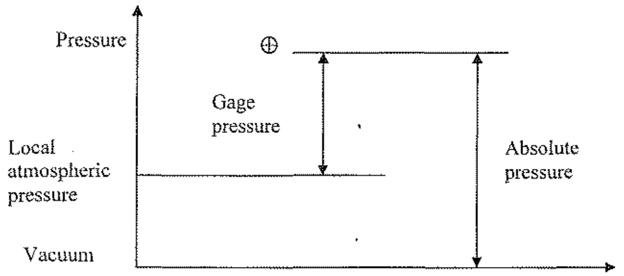
Other units of pressure:
Atmospheres: 1 atm = 101325 pascals – based on the US standard atmosphere
Bar (or millibar): 1 mb = 100 pascals
Height of a liquid – e.g. inches of mercury

Pressure scales

Pressure is commonly measured using two pressure scales that differ only in the datum

Pressure scale	Datum	Used for
Absolute pressure	vacuum	Atmospheric pressure
Gage pressure	local atmospheric pressure	Water pressure

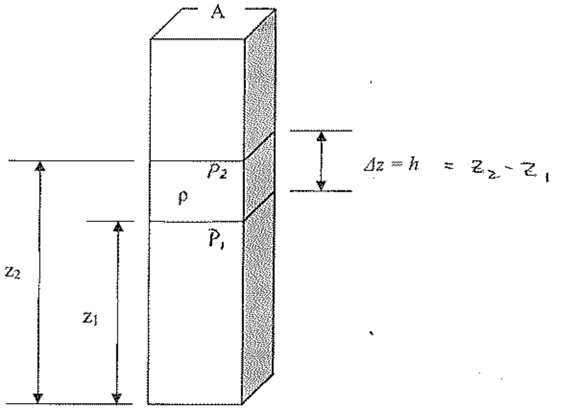
Illustration of the relationship between absolute and gage pressure



Pressure exerted by a column of fluid

Principles:
- Pressure at a point in a **static** fluid results from the weight of fluid above that point (under natural circumstances)
- Pressure is related to the fluid density since unit weight is related to the density
- Pressure increases with depth in a fluid

Consider the increase in pressure over a small vertical distance Δz in a column of fluid with cross sectional area A. The density of the "slice" of fluid is ρ. The pressure at z_2 is p_2 and the pressure at z_1 is p_1



Looking at this problem as one of the force increasing as we move down in the column of fluid from z_2 to z_1, the forced exerted on the plane at elevation z_1 is the weight force exerted on the at elevation z_2 plus the weight of the "slice of fluid. This can be written as:

$$p_1 A = p_2 A + \Delta W$$

where ΔW is the weight of fluid in the slice. Recall $\gamma = \frac{W}{V}$

The weight of an object can be written in terms of its unit weight, γ. In this case:

$\Delta W = \gamma \Delta V$ where ΔV is the volume of the slice of fluid

so: $P_1 A = P_2 A + \gamma \Delta V$

The volume of the slice of fluid can be written as:

$\Delta V = A(z_2 - z_1) = A\Delta z$ so $P_1 A = P_2 A + \gamma A \Delta z$

Recall that the unit weight is a function of the fluid density:

$\gamma = \rho g$ where ρ is the density so: $P_1 A = P_2 + \rho g A \Delta z$

Then the equation for the downward force on the plane area at z_1 becomes:

$p_1 A = p_2 A + \rho g A \Delta z$ $P_1 \cancel{A} = \cancel{A}(P_2 + \rho g \Delta z)$

Dividing through by the area of the column, A, gives:

$$p_1 = p_2 + \rho g \Delta z$$

This can also be written as:

$$p_2 - p_1 = \Delta p = -\rho g(z_2 - z_1)$$

This is one form of the **hydrostatic equation** showing how pressure changes with elevation in a fluid at rest.

The hydrostatic equation can also be written in **differential form**. If we let Δz and Δp shrink to an infinitesimally small quantities the hydrostatic law becomes a differential equation:

$$dp = -\rho g dz \quad \text{or} \quad \frac{dp}{dz} = -\rho g$$

gradient of pressure $z \uparrow$

The interpretation is the same but we can now use it to address the issue of changing density with pressure using the perfect gas law

Summary:
- Δp is the increase in pressure moving downward in a fluid column a distance $h = \Delta z$
- The pressure increase depends on the density of the fluid and the distance itself.
- The hydrostatic equation only strictly applies when the density is constant.
- pressure is constant on a horizontal plane

Pressure change in a liquid

Liquids are considered incompressible under most natural conditions so that the density is considered to be constant. As a result the hydrostatic equation applied directly so that changes in pressure only depend on changes of depth in the liquid.

Examples of pressure computations:

Pressure variation in a liquid (including the idea of gage pressure):

If the local atmospheric pressure is 1013 mb, what is the height of the column of mercury in a mercury barometer? The density of mercury is 13.6 times that of water.

[redacted]

Pressure changes in a gas

Gasses are compressible fluids and many, including mixtures such as air, follow the perfect gas law. As a result, the density is not constant but depends on the pressure as:

$p = \rho RT$ where p is the pressure, ρ is the density, R is the perfect gas constant and T is the absolute temperature

As a result of this characteristic, the hydrostatic law cannot be applied directly

. The density is related to pressure as:

$$\rho = \frac{p}{RT}$$

Substituting this into the differential form of the hydrostatic equation gives:

$$dp = -\frac{p}{RT} g dz$$

The solution of this equation is a function, in this case pressure, *p*, as a function of elevation, *z*. But there is one additional complication. The temperature is also known to vary with elevation and is therefore not a constant if the elevation changes a significant amount.

So, we can solve this equation in two ways depending on our assumption about the temperature variation.

Constant temperature:

The hydrostatic equation is the simplest case of a differential equation and it can be solved by a technique called the separation of variables where we can simply use what we know about integration to solve the problem.

Since R and g are constants and T is assumed constant, the first step is to separate the variables (p and z) and their differentials onto separate sides of the equation. This gives:

$$\frac{dp}{p} = -\frac{g}{RT}dz$$

With the equation separate, we can simply integrate both sides with the appropriate limits of integration; that is let the pressure at the earth surface where the elevation is $z=0$ be p_0 the pressure at any arbitrary elevation, *z*, be *p* so that:

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{RT}\int_{0}^{z}dz$$

The result of this integration is:

$$\ln p - \ln p_0 = -\frac{g}{RT}z$$

The equation can also be written in terms of p, as opposed to *ln p*, by exponentiating both sides to give:

$$p = p_0 e^{-\frac{g}{RT}z}$$

So, under the assumption of a constant temperature, pressure decreases exponentially as elevation increases.

Temperature varies with elevation:

The more realistic situation is where the variation of temperature with elevation is considered. In this case the temperature is give as:

$$T = T_0 - \alpha z$$

Substituting this express for temperature into the differential form of the hydrostatic equation and separating the variables gives:

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{R}\int_{0}^{z}\frac{dz}{(T_0 - \alpha z)}$$

The integration is similar given a relationship between pressure and elevation as:

$$\ln p - \ln p_0 = \frac{g}{R\alpha}z[\ln(T_0 - \alpha z) - \ln(T_0)]$$

This can be rearranged and simplified by exponentiating both sides to give:

$$p = p_0\left(1 - \frac{\alpha z}{T_0}\right)^{\frac{g}{\alpha R}}$$

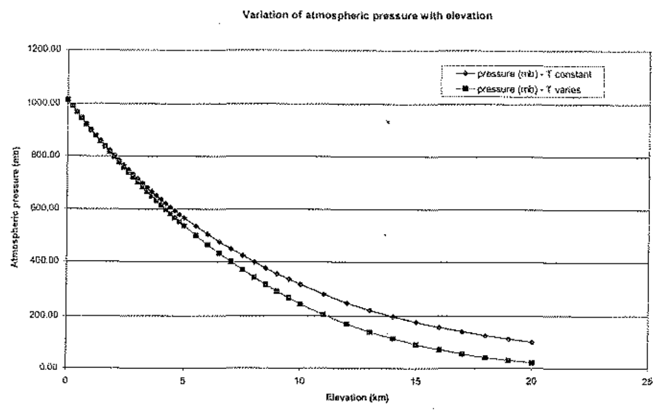
Comparison of the two "models" of pressure variation

These two equations are slightly but fundamentally different:

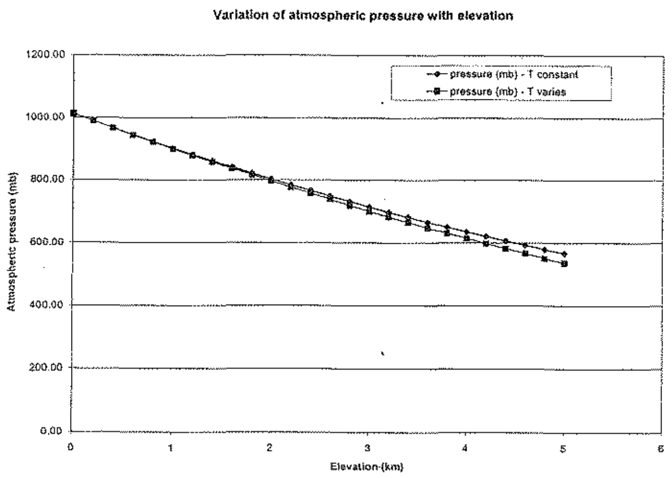
- One is derived assuming the temperature is not a function of either pressure or elevation
- One varies exponentially with elevation while the other varies as the power function

A comparison of the two models, starting with standard atmospheric pressure (1013 mb) at the surface (elevation $z = 0$) shows that:

- The model computations diverge as elevation increases
- They are nearly the same close to the earth surface -- within 2 km
- The pressure elevation relationship is nearly linear in the first 2 km



Comparison of the computed variation of atmospheric pressure with elevation up to 20 km above the surface using the two models of pressure variation.



Comparison of the computed variation of atmospheric pressure with elevation up to 5 km above the surface using the two models of pressure variation.

Example: *Pressure variation in the atmosphere*

What is the atmospheric pressure at Vail Pass if the pressure in Denver is 24,8 in Hg. The elevation in Denver is 5280 ft and the temperature is 80F while the elevation of Vail Pass in 10,600 ft and the temperature is 62F.

T const. 69,500 Pa
T varies 69,700 Pa

Continue with the composition of the atmosphere