CHAPTER TWO

Engineering Calculations

The ability to solve problems by using engineering calculations represents the very essence of engineering. While certainly not all engineering problems can be solved by using numerical calculations, such calculations are absolutely necessary for the development of technical solutions. Engineering calculations make it possible to describe the physical world in terms of units and dimensions that are understood by all those with whom communication takes place.

The first section of this chapter is devoted to a review of units and dimensions used in engineering. The second part describes some basic principles of performing some "back-of-the-envelope" calculations in the face of incomplete or unobtainable information, and the last section is about how to manage information.

2.1 ENGINEERING DIMENSIONS AND UNITS

A *fundamental dimension* is a unique quantity that describes a basic characteristic, such as force (F), mass (M), length (L), and time (T). *Derived dimensions* are calculated by an arithmetic manipulation of one or more fundamental dimensions. For example, velocity has the dimensions of length per time (L/T), and volume is L^3.

Dimensions are descriptive but not numerical. They cannot describe *how much*; they simply describe *what*. *Units* and the *values* of those units are necessary to describe something quantitatively. For example, the length (L) dimension may be described in units as meters, yards, or fathoms. Adding the value, we have a complete description, such as 3 meters, 12.6 yards, or 600 fathoms.

Three systems of units are in common use: the SI system, the American engineering system, and the cgs system. Developed in 1960 in an international agreement, the SI system(for System International d'Unites) is based on meter for length, second for time, kilogram for mass, and degree Kelvin for temperature. Force is expressed in Newtons. The tremendous advantage of the SI system over the older English (and now American) system is that it works on a decimal basis, with prefixes decreasing or increasing the units by powers of ten. Although the SI units are now used throughout the world, most contemporary American engineers still use the old system of feet, pounds (mass), and seconds, with force expressed as pounds (force). To ensure facility and familiarity, both systems are used in this book.

2.1.1 Density

The *density* of a substance is defined as its mass divided by a unit volume, or

$$\rho = \frac{M}{V}$$

where ρ = density
M = mass
V = volume.

In the SI system the base unit for density is kg/m^3 while in the American engineering system density is commonly expressed as lb$_M$/ft^3 {where lb$_M$ = pounds (mass)}.

Water in the SI system has a density of 1×10^3 kg/m^3, which is equal to 1 g/cm^3. In the American engineering system, water has a density of 62.4 lb$_M$/ft^3.

2.1.2 Concentration

The derived dimension *concentration* is usually expressed gravimetrically as the mass of a material A in a unit volume consisting of material A and some other material B. The concentration of A in a mixture of A and B is

$$C_A = \frac{M_A}{V_A + V_B} \tag{2.1}$$

where C_A = concentration of A
M_A = mass of material A
V_A = volume of material A
V_B = volume of material B.

In the SI system the basic unit for concentration is kg/m^3. However, the most widely used concentration term in environmental engineering is milligrams per liter (mg/L).

EXAMPLE 2.1

Problem Plastic beads with a volume of 0.04 m^3 and a mass of 0.48 kg are placed into a container, and 100 liters of water are poured into the container. What is the concentration of plastic beads, in mg/L?

Solution Using Equation 2.1, where A represents the beads and B the water:

$$C_A = \frac{M_A}{V_A + V_B}$$

$$C_A = \frac{0.48 \text{ kg}}{0.04 \text{ m}^3 + (100 \text{ L} \times 10^{-3} \text{ m}^3/\text{L})}$$

$$C_A = 3.43 \text{ kg/m}^3 = 3.43 \frac{10^6 \text{ mg/kg}}{10^3 \text{ L/m}^3} = 3{,}430 \text{ mg/L}$$

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Note that in the previous example the volume of water is added to the volume of the beads. If the plastic beads with a volume of 0.04 m³ are placed into a 100-liter container and the container filled to the brim with water, the total volume is

$$V_A + V_B = 100 \text{ L}$$

and the concentration of beads, C_A, is 4,800 mg/L. The concentration of beads is higher because the total volume is lower.

Another measure of concentration is *parts per million* (ppm). This is numerically equivalent to mg/L if the fluid in question is water because one milliliter (mL) of water weighs one gram (i.e., the density is 1.0 g/cm³). This fact is demonstrated by the following conversion:

$$\frac{1 \text{ mg}}{\text{L}} = \frac{0.001 \text{ g}}{1000 \text{ mL}} = \frac{0.001 \text{ g}}{1000 \text{ cm}^3} = \frac{0.001 \text{ g}}{1000 \text{ g}} = \frac{1 \text{ g}}{1{,}000{,}000 \text{ g}}$$

or one gram in a million grams, or one ppm.

Some material concentrations are most conveniently expressed as percentages, usually in terms of mass:

$$\Phi_A = \frac{M_A}{M_A + M_B} \times 100 \tag{2.2}$$

where Φ_A = percent of material A
M_A = mass of material A
M_B = mass of material B.

Φ_A can, of course, also be expressed as a ratio of volumes.

EXAMPLE 2.2

Problem A wastewater sludge has a solids concentration of 10,000 ppm. Express this in percent solids (mass basis), assuming that the density of the solids is 1 g/cm³.

Solution

$$10{,}000 \text{ ppm} = \frac{1 \times 10^4 \text{ parts}}{1 \times 10^6 \text{ parts}} = \frac{1}{100} = 0.01 \text{ or } 1\%$$

This example illustrates a useful relationship:

10,000 mg/L = 10,000 ppm (if density = 1) = 1% (by weight).

Many wastewaters are assumed to be dilute, so their density can be assumed to be approximately 1.

In air pollution control, concentrations are generally expressed gravimetrically as mass of pollutant per volume of air at standard temperature and pressure. For example, the national air quality standard for sulfur dioxide is 0.05 μg/m³ (one microgram = 10^{-6} gram). Occasionally, air quality is expressed in ppm, and in this case the calculations are in terms of volume/volume, or one ppm = 1 volume of

a pollutant per 1×10^6 volumes of air. Conversion from mass/volume (μg/m³) to volume/volume (ppm) requires knowledge of the molecular weight of the gas. At standard conditions—0°C and 1 atmosphere of pressure—one mole of a gas occupies a volume of 22.4 L (from the ideal gas law). One mole is the amount of gas in grams numerically equal to its molecular weight. The conversion is, therefore,

$$\mu\text{g/m}^3 = \frac{1 \text{ m}^3 \text{ pollutant}}{10^6 \text{ m}^3 \text{ air}} \times \frac{\text{molecular weight (g/mole)}}{22.4 \times 10^{-3} \text{ m}^3/\text{mole}} \times 10^6 \ \mu\text{g/g}$$

or simplifying:

$$\mu\text{g/m}^3 = (\text{ppm} \times \text{molecular weight} \times 10^3)/22.4 \quad \text{at } 0°\text{C and 1 atmosphere.}$$

If the gas is at 25°C at one atmosphere, as is common in air quality standards, the conversion is

$$\mu\text{g/m}^3 = (\text{ppm} \times \text{molecular weight} \times 10^3)/24.45 \quad \text{at } 25°\text{C and 1 atmosphere.} \tag{2.3}$$

2.1.3 Flow Rate

In engineering processes the *flow rate* can be either *gravimetric (mass) flow rate* or *volumetric (volume) flow rate*. The former is in kg/s or lb_M/s while the latter is expressed as m³/s or ft³/s. The mass and volumetric flow rates are not independent quantities because the mass (M) of material passing a point in a flow line during a unit time is related to the volume (V) of that material:

[Mass] = [Density] × [Volume].

Thus, a volumetric flow rate (Q_V) can be converted to a mass flow rate (Q_M) by multiplying by the density of the material:

$$Q_M = Q_V \rho \tag{2.4}$$

where Q_M = mass flow rate
Q_V = volume flow rate
ρ = density.

The symbol Q is almost universally used to denote flow rate.

The relationship between mass flow of some component A, concentration of A, and the total volume flow (A plus B) is

$$Q_{M_A} = C_A \times Q_{V_{(A+B)}} \tag{2.5}$$

Note that Equation 2.5 is not the same as Equation 2.4, which is applicable to only one material or one component in a flow stream. Equation 2.5 relates to two *different* materials or components in a flow. For example, a mass flow rate of plastic balls moving along and suspended in a stream is expressed as kg of these balls per second passing some point, which is equal to the concentration (kg balls/m³ total volume, balls plus water) times the stream flow (m³/s of balls plus water).

EXAMPLE 2.3

Problem A wastewater treatment plant discharges a flow of 1.5 m^3/s (water plus solids) at a solids concentration of 20 mg/L (20 mg solids per liter of flow, solids plus water). How much solids is the plant discharging each day?

Solution Use Equation 2.5.
[Mass flow] = [Concentration] × [Volume flow]

$$Q_{M_A} = C_A \times Q_{V_{(A+B)}}$$

$$= \left[20\ \frac{\text{mg}}{\text{L}} \times \frac{1 \times 10^{-6}\ \text{kg}}{\text{mg}}\right] \times \left[1.5\ \frac{\text{m}^3}{\text{s}} \times \frac{10^3\ \text{L}}{\text{m}^3} \times 86{,}400\ \frac{\text{s}}{\text{day}}\right]$$

$$= 2592\ \text{kg/day} \approx 2600\ \text{kg/day}$$

EXAMPLE 2.4

Problem A wastewater treatment plant discharges a flow of 34.2 mgd (million gallons per day) at a solids concentration of 0.002% solids (by weight). How many pounds per day of solids does it discharge?

Solution Use Equation 2.5.
[Mass flow] = [Concentration] × [Volume flow]

$$Q_{M_A} = C_A \times Q_{V_{(A+B)}}$$

Assume that $\rho = 1$ g/cm^3, so 0.002% = 20 mg/L, and assume the stated volume flow rate includes solids plus water. Then

$$Q_{M_A} = \left[20\ \frac{\text{mg}}{\text{L}} \times 3.79\ \frac{\text{L}}{\text{gal}} \times 2.2 \times 10^{-6}\ \frac{\text{lb}}{\text{mg}}\right] \times \left[34.2 \times 10^6\ \frac{\text{gal}}{\text{day}}\right] = 5700\ \frac{\text{lb}}{\text{day}}$$

The preceding example illustrates another convenient conversion factor:

$$3.79\ \frac{\text{L}}{\text{gal}} \times 2.2 \times 10^{-6}\ \frac{\text{lb}_M}{\text{mg}} \times 10^6\ \frac{\text{gal}}{\text{million gal}} = 8.34 \left[\frac{\text{L}}{\text{million gal}}\right]\left[\frac{\text{lb}_M}{\text{mg}}\right]$$

This factor, 8.34, is very useful in conversions wherein the flow rate is in mgd, the concentration is in mg/L, and the discharge is in lb/day:

$$\begin{bmatrix}\text{Mass flow}\\ \text{rate in}\\ \text{lb/day}\end{bmatrix} = \begin{bmatrix}\text{Volume flow}\\ \text{rate in mgd}\end{bmatrix} \times \begin{bmatrix}\text{Concentration}\\ \text{in mg/L}\end{bmatrix} \times 8.34 \qquad (2.6)$$

EXAMPLE 2.5

Problem A drinking water treatment plant adds fluorine at a concentration of 1 mg/L. The average daily water demand is 18 million gallons. How much fluorine must the community purchase?

Solution Use Equation 2.6.

$$18\ \text{mgd} \times 1\ \frac{\text{mg}}{\text{L}} \times 8.34 \left[\frac{\text{L}}{\text{million gallons}}\right]\left[\frac{\text{lb}}{\text{mg}}\right] = 150\ \frac{\text{lb}}{\text{day}}$$

2.1.4 Retention Time

One of the most important concepts in treatment processes is *retention time*, also called *detention time* or even *residence time*. Residence time is the time an average particle of the fluid spends in the container through which the fluid flows (which is the time it is exposed to treatment or a reaction). An alternate definition is the time it takes to fill the container.

Mathematically, if the volume of a container, such as a large holding tank, is V (L^3), and the flow rate into the tank is Q (L^3/t), then the residence time is

$$\bar{t} = \frac{V}{Q} \qquad (2.7)$$

The average retention time can be increased by reducing the flow rate Q or increasing the volume V, and decreased by doing the opposite.

EXAMPLE 2.6

Problem A lagoon has a volume of 1500 m^3, and the flow into the lagoon is 3 m^3/hour. What is the retention in this lagoon?

Solution Use Equation 2.7.

$$\bar{t} = \frac{1500\ \text{m}^3}{3\ \text{m}^3/\text{hour}} = 500\ \text{hours}$$

2.2 APPROXIMATIONS IN ENGINEERING CALCULATIONS

Engineers are often called on to provide information not in its exact form but a approximations. For example, an engineer may be asked by a client, such as city manager, what it might cost to build a new wastewater treatment plant for th community. The manager is not asking for an exact figure but a "ball park" estimate Obviously, the engineer cannot in a few minutes conduct a thorough cost estimate She would recognize the highly variable nature of land costs, construction costs

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required treatment efficiency, etc. Yet, the manager wants a preliminary estimate—a *number*—and quickly!

In the face of such problems the engineer has to draw on whatever information might be available. For example, she might know that the population of the community to be served is approximately 100,000. Next, she estimates, based on experience, that the domestic wastewater flow might be about 100 gallons per person per day, thus requiring a plant of about 10 mgd capacity. With room for expansion, industrial effluents, storm inflow and infiltration of groundwater into the sewers, she may estimate that a 15-mgd capacity may be adequate.

Next, she evaluates the potential treatment necessary. Knowing that the available watercourses for discharging the effluent are all small streams that may dry up during droughts, a high degree of treatment is required. She figures that nutrient removal will be needed. Such treatment plants, she is aware, cost about $3,000,000 per million gallons of influent to construct. She calculates that the plant would cost about $45 million. Giving herself a cushion, she could respond by saying, "about $50 million."

This is exactly the type of information the manager seeks. He has no use for anything more accurate because he might be trying to decide whether to ask for a bond issue of around $100 million or $200 million. There is time enough for more exact calculations later.

2.2.1 Procedure for Calculations with Approximations

Problems not requiring exact solutions can be solved by

1. carefully defining the problem
2. introducing simplifying assumptions
3. calculating an answer
4. checking the answer, both systematically and realistically.

Defining the Problem

The engineer in the previous case is asked for an estimate, not an exact figure. She recognizes that the use of this figure would be for preliminary planning purposes, and thus, valid approximations are adequate. She also recognizes that the manager wants a *dollar* figure answer, thus establishing the units.

Simplifying

This step is perhaps the most exciting and challenging of the entire process because intuition and judgment play an important role. For example, the engineer has to first estimate the population served and then consider the average flow. What does she ignore? Obviously, a great deal, such as daily transient flows, variability in living standards, and seasonal variations. A thorough estimate of potential wastewater flows requires a major study. She has to simplify her problem and choose to consider only an estimate of the population and an average per capita discharge.

Calculating

In the case of this problem the calculations are straightforward.

Checking

More important is the process of checking. There are two kinds of checks: systematic and realistic. In systematic checking the units are first checked to see if they make sense. For example,

$$[\text{persons}] \times \left[\frac{\text{gallons}}{\text{persons}}\right] = [\text{gallons}]$$

makes sense, whereas

$$\frac{\text{persons}}{[\text{gallons/person}]} = \frac{[\text{persons}]^2}{\text{gallons}}$$

is nonsense. If the units check out, the numbers can be recalculated to check for mistakes. It is wise always to write your units as you do your calculations, making this check as you go.

Finally, a reality check is necessary. Possibly no practicing engineer will explicitly recognize that they perform reality checks day in and day out, but such checks are central to good engineering. Consider, for example, if the engineer had made a mistake and thought (erroneously) that a wastewater treatment plant of the type needed by the community costs $3,000 per million gallons of influent. Her calculations would have checked, but her answer would have been $50,000 instead of $50 million. Such an answer should have immediately been considered ludicrous, and a search for the error initiated. Reality checks, when routinely performed, will save considerable pain and embarrassment.

2.2.2 Use of Significant Figures

Finally, note that significant figures in the answer reflect the accuracy of the data and the assumptions. Consider how silly it would sound to say that the treatment plant would cost *about* $5,028,467.19. Many problems require answers to only one significant figure or even to an order of magnitude. Nonsignificant figures tend to accumulate in the course of calculations like mud on a boot and must be wiped off at the end.[1]

Suppose you are asked to estimate the linear feet of fence posts needed for a pasture and are told that there will be 87 posts with an average height of 46.3 inches. You multiply and get 335.675 ft. Now it is time to scrape the mud off since the most accurate of your numbers has only three significant figures while your answer has six significant figures. So you report 336 feet. Or more likely you say 340 ft, recognizing that it is better to err slightly on the high side than to run out of fence posts.

Significant figures are those that transfer information based on the value of that digit. Zeros that merely hold place are not significant because they can be eliminated ts: systematic if they make

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without loss of information. For example, in the number 0.0036 the two zeros are only holding a place and can be eliminated by writing $0.0036 = 3.6 \times 10^3$.

Zeros at the end of a number are a problem, however. Suppose the newspaper reports that there were 46,200 fans attending a football game. The last two digits (zeros) may be significant if every person was counted, and indeed there were *exactly* 46,200 fans in the stadium. If, however, one were to estimate the number of people as 46,200 fans, then the last two zeros are simply holding places and are not significant. To avoid confusion when reporting numbers, it is useful to say "about" or "approximately" if that is what is meant. When using numbers of unknown significant figures, erring on the side of caution (fewer significant figures) is usually best.

EXAMPLE 2.7

Problem A community of approximately 100,000 people has about 5 acres of landfill left that can be filled to about 30 feet deep with refuse compacted to somewhere between 600 and 800 lb_M per cubic yard. What is the remaining life of the landfill?

Solution Using the procedure outlined above, the first step is to define the problem. Clearly, the answer does not require high precision as the data are not precise. In addition, the definition of the problem requires an answer in time units.

The second step is to simplify the problem. There is no need to consider commercial or industrial wastes. Estimate only refuse generated by individual households.

The third step is to calculate an answer. All the necessary data are available except the per capita production of refuse. Suppose a family of 4 fills up 3 garbage cans per week. If each can is about 8 cubic feet and if we assume the uncompacted garbage is at about one fourth of the compacted density, say at 200 lb_M/yd^3, it is reasonable to calculate the per capita production as

$$8\,\frac{ft^3}{can} \times \frac{1\ yd^3}{27\ ft^3} \times \frac{3\ cans}{4\ people} \times 200\,\frac{lb_M}{yd^3} \times \frac{1\ week}{7\ days} = 6.3\,\frac{lb}{person/day}$$

If there are about 100,000 people, the city produces

$$6.3\,\frac{lb}{person/day} \times 100{,}000\ people \times \frac{1\ yd^3}{700\ lb_M} = 900\ yd^3/day$$

The total available volume is

$$5\ acres \times 43{,}560\,\frac{ft^2}{acre} \times 30\ ft \times \frac{1\ yd^3}{27\ ft^3} = 242{,}000\ yd^3$$

Thus, the expected life is

$$\frac{242{,}000\ yd^3}{900\ yd^3/day} = 268\ days \approx 270\ days$$

Remember the fourth step. Is this reasonable? Pretty much so. The calculations may have overestimated refuse production as the average national per capita production is closer to 4 lb/capita/day, so in actuality the landfill may last a year. But considering the extreme difficulties of siting additional landfills, the town is clearly already in a crisis situation.

In professional engineering it is necessary to carry around in one's head a suitcase full of numbers and approximations. For example, most people would know that a meter is a few inches longer than a yard. We may not know *exactly* how much longer, but we could make a pretty fair guess. Similarly, we know what 100 yards looks like (from goal line to goal line). In a similar way, an environmental engineer in practice knows instinctively what a flow of 10 mgd looks like because he or she has been working in plants that received that magnitude of flow. Such knowledge becomes second nature and is often the reason why engineers can avoid stupid and embarrassing mistakes. A "feel" for units is a part of engineering and is the reason why a change of units from mgd to m^3/s for American engineers is so difficult. It would be an unusual American engineer who would know what a flow of 10 m^3/s looks like (without doing some quick mental approximate conversions!).

INFORMATION ANALYSIS